MISCELLANEA:

SIVE

LUCUBRATIONES MATHEMATICÆ.

SAMUELIS FOSTER,

Olim LONDINI in Collegio GRESHAMENSI *Astronomiæ* Professoris Publicæ.

Omnia in lucem edita, & pleraque Latine reddita, opera & Studio

JOHANNIS TWYSDEN, *C.L.M.D.*

Qui etiam ex suis nonnulla adjunxit.

Quorum omnium CATALOGUM versa Pagina exhibebit.

LONDINI,

Ex Officina LEYBOURNIANA.

M. DC. LIX.

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J-Barnard

MISCELLANIES:

OR,

MATHEMATICAL LUCUBRATIONS,

OF

Mr. SAMUEL FOSTER,

Sometime publike Professor of
Astronomie in GRESHAM
Colledge in LONDON.

Published, and many of them translated into
English, by the care and industry of

JOHN TWYSDEN. *C. L. M. D.*

The CATALOGUE of them shall be
declared in the following Page.

LONDON,
Printed, by R. & W. LEYBOURN.
M. DC. LIX.

CATALOGUS
Tractatuum hujus Libri.

A CATALOGUE
Of the Treatises of this Book.

Tractat. Ang.	*Treatises in English.*
XII. Quadrati Geometrici fabrica & usus.	XII. The construction and use of the Geometrical Square.
XIII. Planispherii Horizontalis fabrica & usus, in	XIII. The construction and use of the horizontal Planisphere, in
XIV. Horologiographia pro / XV. radiis Refractis.	XIV. Projective Dialling. / XV. Refracted Dials.
☞ XVI. Horologiographia Catoptrica, sive Instrumentum novum facillimum, quo horologia Catoptrica cum eorum apparatu sine molestia describantur.	XVI. Catoptrical Dialling, or a new & easie Instrument whereby Reflected Dials with their furniture, may be described without trouble. ☜
☞ XVII. De Architectura Militari Tractatulus.	XVII. A small Treatise of Architecture Military, or Fortification. ☜

INstrumenta omnia quorum descriptiones in hoc Libro continentur, ut & alia qualibet Mathematica, ex ligno orichalco, aut aliâ qualibet materia concinne fabricantur *Londini* ab *Anthonio Thompson*, in Vico appellato *Hosier-lane*, apud quem prostant venalia.

The Mathematical Instruments described in this Book, as all others, are neatly made, either in wood or brass by Mr. Anthony Thompson, *at his house in* Hosier-lane *in* London, *where they are to be sold.*

In an Appendix, added by the Printer.

XVIII. A short Declaration of Reflected Dialling in a method differing from the former.

XIX. A new Triquetrum, or the Parallattick Instrument improved, fitted for the taking of altitudes to Centesim's of degrees: the sights whereof doe with much delight and preciseness direct the plumb-line to the smallest parts of a degree.

XX. Æquations arising from a Quantity divided into two unequal parts, and the Second Book of *Euclides Elements*, demonstrated by Symbols

HONO-

HONORABILI

Doctissimoque Viro, Domino

HENRICO YELVERTON

BARONETTO.

JOHANNES TWYSDEN *S. P. D.*

INfœlici huic sæculo, id debemus (charissime consanguinee) quod cum bonæ artes exulant, mali mores excoluntur. Hinc exorta dissidia, simultates, bella, quæ cum omnia tantum non verterunt in ruinam, nil mirum si virtus ipsa, cum doctrina squaluerunt. Sed in ipso squallore sunt venustæ, & luctuosus licet sit illis, & lugubris amictus, sub obscura splendent Eclipsi, imò in densissimis tenebris facem semper præferent quâ cultores, & alumnos suos ducent in asylum. Emines

tu inter primos (Vir Clariſſime) *qui in multifario literarum genere paucis ſecundus bonos protegis, doctos foves.*

Has igitur Lucubrationes Mathematicas *eruditiſſimi* SAMUELIS FOSTER *hic illic, inſtar examinis apum ſparſas in Adverſariis, ego in unum alveare coegi; & cum pauca etiam, ex meis adjeciſſem tuo nomini conſecratas volui: ut inſtructiſſimâ, quam poſſides Bibliothecâ ſemper ad manus ſit aliquid quod tibi revocet in memoriam doctiſſimum Authorem, & mei erga te ſinceri amoris cum è vivis exceſſero ſit perenne teſtimonium.*

Faxit *Deus O. M. ut tibi omnia fauſta ſuccedant in terris, & ut tandem ætate gravis, ſenio attamen vegeto confectus deducaris ad proavos.*

TO THE RIGHT

HONOURABLE

LADY

SUSANNA LONGUEVILLE,

Baronesse GREY, RUTHIN, HASTINGS, WASHFORD, and VALENCE.

Wife unto the

HONOURABLE

Sr. HENRY YELVERTON BARONET.

MADAM,

ALthough the subject of this Book be such as few Ladies spend much time in, yet my desire to expresse in some measure the respects I owe to your Noble Family, in which I have the honour

to ſpend much of my time, hath made me præfix your name to it, that I might neither divide you from your dear husband, nor let the World believe, that though my relations to him are neerer, my affection to your Ladiſhip, is leſſe, who hath made our Family happy, by being now made one in it.

The enſuing Treatiſes, I confeſſe, are wholly Mathematical, and may therefore be thought unfit for your Ladiſhips peruſal, yet are they neither beyond the reach of your Sex, or your Self, whoſe Soul is large enough to comprehend whatſoever you are willing to undertake, and ſhall never, when you pleaſe to command it, want the aſſiſtance of him whoſe honour it is to be called

Madam, *Your Ladiſhips moſt humble Servant,*

and moſt affectionate Unkle,

JOHN TWYSDEN.

LECTORI CANDIDO

JOHANNES TWYSDEN

S. P. D.

PUblici Juris nunc tandem facta sunt (Candide Lector) *opera hæc posthuma* Samuelis Foster *Viri Industrii, eruditi, in Mathematicis versatissimi. Limatiora prodiissent modo Author ipse ultimam iis admovisset manum. Sed cum hoc nobis negavit Deus, nos nostras potius obstetricantes qualescumque præbuimus manus, quàm vel silentio perirent quæ prælo digna judicavimus, vel literarius orbis tanti viri genuinâ prole diutiùs privaretur.*

Tractatus sequentes cum sint varii generis, sed nunc Latine, nunc Anglice, aliquando mixtim scripti; nos ex iis Astroscopium, Instrumenta Planetaria, & alia nonnulla in Latinum *vertimus, & duplici columnâ excudi curavimus, ut ex nostratibus qui minus linguam callent Latinam suâ gaudeant vernaculâ, exteri vero inventionibus novis, & utilibus non destituantur.*

Alios aliquot absque versione edidimus, cum profecto variis implicatus à meipso otium in illis transferendis impetrare non potui: si quando secunda adornabitur editio, nec alius prævenerit, in hoc etiam fortasse laborabimus. Elegantiam styli nullibi sum sectatus, sed Authoris sensum idoneis verbis, & (ferente id linguâ Latina) ad literam, clare, & distincte conatus sum explicare. Observationes Eclipsium aliquot, nuperi Cometæ motum, & insuper alia ex nostris adjunximus: quæ cum per se typis non fuerint digna, malui sub umbrâ tanti Viri delitescerent. Nevero docto Authori sint injuriæ,

juriæ, omnia nostra in fronte libelli discriminata, in libro ipso indicatoriâ manu ad marginem affixâ distingues.

Restat ut ultimo loco de harum Scientiarum origine, & utilitate pauca dissererem, sed cum hoc alii abunde fecerunt; & unaquæque dies id nobis continuo magis aperit, nos ne crescat in molem hæc Præfatio hoc onere libenter sublevabimur. Nec profectò longâ refutatione sunt digni qui Geometriam, qui Astronomiam, qui Scientias denique Mathematicas, vel ex ignorantiâ suggillant, vel ex incuriâ non excolunt.

Nos tantum illos nostræ Nationis qui adhuc vivunt, & in utrâque Palæstrâ fœliciter desudarunt honoris causâ nominabimus. Ut posteri intelligant etiam nostro hoc sæculo superfuisse aliquos qui has artes neque ignorarunt, ut vanas, nec damnarunt, ut curiosas.

Inter omnes merito primum obtinebit locum, & qui nostram superat laudem venerandæ canitiei, & eximiæ pietatis senex Gulielmus Oughtred, Ætonensis, *qui Geometriæ abdita facili methodo, & admiranda Clave reseravit: qui accuratâ Trigonometriâ, & Instrumentorum plurium supellectile tum Geometriam, tum Astronomiam ditavit. Hunc sequantur Insignissimi Viri* Johannes Wallisius, & Sethus Wardus S. T. D. D. *alter Geometriæ, alter Astronomiæ in Academia* Oxoniensi *Professores* Saviliani. *Quorum primus operum Mathematicorum tomos jam edidit duos. Astronomiam Geometricam hactenus desideratam Juris Publici fecit secundus. Opera quæ nulla ætas corrumpet, & præsenti sequens plus admirabitur. In recondita harum scientiarum quam alte penetravit* D. Johannes Pellius *exinde facile conjicias, quod ingentes Celeberrimi* Longomontani *conatus, & annorum multorum molimina de vera Circuli mensura, pagellâ unicâ, & tramite ab aliis non trito novit evertere. Alia nobis promisit & longe majora perficere par est. Elementorum* Philosophiæ *Sectiones duas, cùm variæ eruditionis alijs evulgavit* Thomas Hobbes Malmsburiensis, *inter doctiores certè numerandus: sed hoc accidit erudito seni humanum, quod pluribus bonis inutilia etiam non pauca admiscuit, & erronea. Planisphærium Catholicum* Gemmæ Frisij *à* Johanne Blagrave *aranea sua instructum; diversas Sphæræ projectiones simul exhibens*

nova Methodo plane novum fecit mihi amicissimus Johannes Palmer, Ectonensis Ecclesiæ *in agro* Northamtoniensi *Rector Doctissimus; radio insuper Astronomico novo ad captandas Stellarum distantias perquam expedito rem Mathematicam, locupletiorem reddidit.*

Nec prætereundus est sine laude Typographus noster Leibournus, *qui scientiis Mathematicis delectatus in Gæodæticis benè meruit. Possem, & alios numerare Mathematicos dicam, an plagiarios, qui aliqua nostri Authoris quæ (pro ea qua erat effusa bonitate, & indolis candore) aliis communicavit; post ejus excessum larvata facie pro suis venditarunt. Sed hoc etiam illi redundabit in honorem quod quæ ipse pro nugis æstimavit hi habent pro thesauris unde vanam sibi ipsis gloriolam conantur aucupari.*

Ultimo loco monendus es (Benevole Lector) *me nihil hic tibi obtrusisse quod non prius fuerit ex autographis Authoris adversariis decerptum, quæ mihi communicavit Doctissimus Theologus* D. Gualterus Foster, S. T. B. *& in his studiis satis versatus; cui de jure incubuit defuncto fratri hosce liberos excitasse, nisi infirma quâ fruitur valetudo, & res domesticæ ruri agentem abhinc longo itinere distinuissent.*

Sed diutius non te sistam in limine. Adi librum ipsum si quid boni acceperis Erudito Fostero *hoc debes; sin quid aliter acciderit meæ quæso adjicias rationi, qui tamen otio non meo sed tuæ utilitati consului.* Vale.

THE PREFACE TO THE READER.

Courteous Reader,

WEh ave at last made publick these Posthumous Works of that learned, industrious, and most skillful Mathematician, Mr. *Samuel Foster*. They would have come out more polished, and with greater lustre, had himself lived to have added his last hand unto them. But since it hath pleased God to deny this unto us, we have rather made choiceto bring them to their birth, with our hands, such as they are, then suffer those things to perish, which we judged worthy of the Presse, or that the learned world should be longer deprived of the genuine off-spring of so worthy a Person.

The Treatises themselves are of different kinds, some of them written by the Author in Latine, some in English, others promiscuously in both languages. The Astroscope, Planetary Instruments, and some others, we have translated into Latine, and caused them to be printed in a double column; to the end that those of our own Nation, who are not much skil'd in the Latine tongue, may read them in their mother language. But strangers not remain deprived of the knowledge of new and profitable inventions.

Some others of them are put out without any version, because, in truth, being employed in other things, I could not get leasure enough to do them; Peradventure if they shall bear a second impression, and no body else prevent me, I may labour in that also.

I have in no place affected elegancy of style, but have endeavoured to expresse the Authors sense perspicuously, in as proper words as I could think of, and literally, where the Latine phrase would bear it.

The Observations of some Eclipses, the motion of the late Comet, with some other things, I have added of my own, which being of themselves not worthy the presse, I have made choice to hide under the shadow of so great a Person.

Yet

Yet least they might be a wrong to our learned Author, you shall find them all distinguished at the beginning of the Book, and in the Book it self, by an indicatory Hand affixed to the margine.

It remains that I should adde something touching the beginning, and use of these Sciences, but since others have before me, abundantly done that, and every day more openeth it unto us, I shall willingly be eased of that burthen: neither indeed deserve they any long confutation, who either out of ignorance, calumiate Geometry, Astronomy, nay, all Sciences Mathematical, or out of negligence bestow no time in them.

I shall only, to their honours, name some of our own Nation yet living, who have happily laboured upon both stages. That succeeding ages may understand that in this of ours there yet remained some who were neither ignorant of these Arts, as if they had held them vain, nor condemn them as superfluous. Amongst them all let Mr. *William Oughtred*, of *Æton*, be named in the first place, a Person of venerable grey haires, and exemplary piety, who indeed exceeds all praise we can bestow upon him. Who by an easie method, and admirable key, hath unlocked the hidden things of Geometry. Who by an accurate Trigonometry and furniture of Instruments, hath inriched, aswell Geometry, as Astronomy. Let D. *John Wallis*, and D. *Seth Ward*, succeed in the next place, both famous Persons, and Doctors in Divinity, the one of Geometry, the other of Astronomy, *Savilian* Professors in the University of *Oxford*. The first of them hath already printed two Tomes of Mathematical things: the other hath put out the hitherto desired, and wanted, Geometrical Astronomy. Works which no age shal consume, and the following will more admire then the present. How far Mr. *John Pell* hath pierced into the depths of these Sciences, you may from thence easily conjecture that he hath been able, and that in a way not troden by others, and within the compasse of one page, to overthrow the endeavours, & many years attempts of that famous *Longomontanus* touching the true measuring a Circle. He hath promised us other things and is fit to undertake far greater then this was.

Thomas Hobbes of *Malmsbury*, hath made publick the first and second Section of Philosophy, with divers others things

of

of various learning. Certainly a Person to be reckoned among the more learned, but this of humane frailty hath happened to the deserving old Man, that amongst more good things, he hath also mingled, not a few, uselesse, and erroneous.

My especial friend Mr. *John Palmer*, the learned Rector of the Church of *Ecton* in *Northamptonshire*, hath under a new method made clearly new the Universal Planisphere of *Gemma Frisius*, furnished with its Reet by *John Blagrave*, exhibiting at one view several projections of the Sphere, and hath farther inriched the Mathematical substance, with a new Cross-staff of very ready use in the taking the distances of Stars.

Neither is our Printer Mr. *Leybourn* to be passed over without his due praise, who being delighted in the Mathematicks, hath written well of Surveying.

I might name some others, shall I call them Mathematicians, or Plagiaries. Who having got into their hands some things of our Authors, which (out of that diffusive goodness, and candor of disposition, that was in him) he communicated to others, have under a disguised face, vented as their own. Yet shall this return also to the honour of our Author, that what he esteemed as trifles, they reckon as a treasure; from which they endeavour, to snatch unto themselves, the vain, and empty name, of glory.

In the last place, let me admonish thee (*Courteous Reader*) that I have here obtruded nothing upon thee which was not first taken out of the Authors Adversaries, written with his own hand, which were communicated to me, by that learned Divine Mr. *Walter Foster*, B. in Divinity, skilful also in these Studies: to whom of right it belonged to have raised up this seed to his deceased Brother, had not his infirm health, and domesticke affaires, held him in the Countrey, a great many miles distant from this place.

But I will not longer detein thee on the threshold, Peruse the Book it self, if in it thou find'st ought good, thou owest it to the learned Mr. *Foster*, if any thing happen otherwise, put it, I intreat thee, upon my account, who notwithstanding have not consulted my own ease, but thy profit.

Fare-well.

JOHN TWYSDEN.

STELLÆ FIXÆ,

[Quas *Tycho*, ad mille, in Catalogum congessit,]
[Et *Keplerus* Tabb. *Rudolphinarum* operi adnexuit.]

AD ANNUM INCARNATIONIS 1671.

[Adservatis eisdem Latitudinibus]

QUOAD LONGITUDINES,

[Ex additione nimirum gradus unius integri]

CORRECTÆ;

Et à Polo Eclipticæ ad Polum Mundi,

[Quorum distantia est 23 grad. 31 min.]

REDUCTÆ:

HOC EST

In Ascensiones rectas & Declinationes,

[Eidem Anno debitas]

CONJECTÆ.

A SAMUELE FOSTERO, olim Astronomiæ Professore in Collegio *Greshami*, *Londini*.

LONDINI,
Ex Officina LEYBOURNIANA.

MDCLIX.

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*

Ursa Minor, Cynosura.

IN extremo caudæ, vulgò polaris	2	84 02½	66 02	B	7 53	87 33½	B
Penultima caudæ	4	86 36	69 50½	B	289 22	86 28	B
Quæ in caudæ radice	4	94 24	73 50	B	260 36	82 29	B
Superior duarum in □ sequentium	4	112 29	75 00	B	239 16	78 50	B
Earundem inferior	5	115 52	77 38½	B	246 22	76 32	B
Superior duarum in □ præcedentium	2	128 16½	72 51½	B	222 46	75 36	B
Earundem inferior	3	135 41	75 23½	B	231 14	73 15	B
Informis duarum australior ad caput ursæ	6	123 54	71 23	B	211 36	77 04	B
Quæ supra hanc (linâ rectâ cum polo.	6	118 20	70 18	B	212 46	79 02	B
Informis, principium earum quæ sunt in(	6	78 17	35 50	B	71 34	58 37	B
Secunda	6	78 28	37 20	B	71 24	60 06	B
Tertia obscura	6	78 45	40 13	B	70 51	62 59	B
Quarta	6	79 03	42 56	B	70 16	65 41	B
Prima informis circà polarem	6	112 38	57 55	B	151 22	76 32	B
Secunda	6	82 55	70 42	B	298 24	85 05	B
Tertia	6	85 31	69 03	B	300 34	86 51	B
Quarta	6	76 07	68 04	B	337 06	84 25	B
Quinta	6	68 22	67 43	B	341 42	81 32	B
Sexta	6	70 57	67 22	B	344 13	82 31	B
Vicissima Polari	6	87 30	63 55	B	66 44	87 13	B

Ursa Major, Helice.

Quæ in rostro	4	108 36½	40 02½	B	120 52	61 34	B
Sub oculo sinistro	4	108 10	43 55½	B	122 37	65 23	B
Contigua sub hâc	5	107 08	44 22	B	121 14	66 02	B
Supra oculum dextrum	4	109 25	47 50½	B	128 06	68 48	B
Supra oculum sinistrum	4	110 44½	47 44½	B	130 14	68 22	B
Ad aurem sinistram	5	115 42½	51 36½	B	143 25	70 24	B
Infima & præcedens in parvo △ colli	5	114 50	42 30	B	132 00	62 26	B
Sequens in eodem triangulo	4	116 02	45 03	B	135 58	64 27	B
Suprema in apice ejusdem trianguli	5	119 00	46 21½	B	141 40	64 45	B
In collo, dicto △ succedens	4	121 38	42 36	B	141 44½	60 33	B
Sequens infrà hanc	4	124 38½	38 15½	B	142 17	55 39	B
In genu sinistro anteriori	3	121 32½	34 34½	B	135 56	53 10	B
Duarum in dextro pede Borealior	3	116 56	29 15½	B	127 20	49 20	B
Australior	3	118 10	28 38	B	128 34	48 21	B
Infrà genu dextrum	5	118 07	33 36	B	130 52	53 05	B
In ipso genu dextro	5	118 26	36 06	B	132 44	55 28	B
Superior præcedentium in □ majori	2	130 34	49 40	B	160 48	63 32	B
Inferior ejusdem □	2	134 43½	45 03½	B	160 18½	58 08	B
Superior sequentium quadrati	2	146 25½	51 37	B	179 48	58 51	B
Inferior earumdem	2	145 45	47 06½	B	173 59	55 33	B

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
Superior ſiniſtri pedis poſteriorum	4	134 56½	29 51½	B	149 17	44 33	B
Sequens & auſtralior	4	136 04½	28 45	B	149 55	43 08	B
In genu præcedentis pedum poſteriorum	4	143 33	35 14	B	161 54	46 16½	B
Præcedens duarum in dextro pede poſter.	4	151 55	26 14	B	165 05	35 01	B
Sequens & auſtralior	4	152 36	24 54	B	165 04	33 33	B
Ante penultima caudæ	2	154 10	54 18	*B*	189 53	57 47	B
Penultima	2	160 56½	56 22	*B*	197 37	56 41	B
Ultima caudæ	2	172 12	54 25	*B*	203 37	51 00½	B
Informis inter caudas hujus & Leonis	2	158 43½	40 06	*B*	189 04	40 34	B
Illa quæ in dorſo.	4	149 10	41 30	*B*	172 15½	49 32	B
In ſiniſtro pede poſteriori (tis ♌ primam	5	142 02	33 01	*B*	158 52	44 52	B
Inform's inter urſæ prioreˉ pedem & capi-(	3	127 17	17 55	*B*	135 15	35 45	B
Illa quæ ſuprâ hanc ad ortum	4	129 10	20 42	*B*	138 26	37 51	B
Illa quæ hanc præcedit	4	126 00	20 05	*B*	134 36	38 10½	B
Sequens duarum antè has	4	122 57	20 51	*B*	131 22½	39 44	B
Earum præcedens	4	120 42	23 41	B	129 46	43 02½	B
Inter extremum pedem & caput ♌	4	135 12	21 53	B	145 36	37 03½	B
Sequens borealis	4	139 55	25 04	B	152 11	38 24	B
Sequens auſtralis	3	140 57	24 50	B	153 08	37 49½	B
Præcedens duarum in baſi oxigonii	3	144 22	21 28	B	155 06	33 30	B
Sequens	3	147 09	20 44	B	157 36	31 48	B
Tertia borealis in oxigonio	4	146 19	24 58	B	158 48	35 59½	B
Quæ inter crura Urſæ	5	163 16	40 30	B	184 33	43 04	B
Prima inter caudam & corpus	6	112 29	58 08	B	151 45	76 45	B
Secunda	6	114 55	47 14	B	136 28½	66 46	B
Tertia	6	110 49	47 30	B	128 53	67 30½	B
Prima inter Urſam & caput Leonis	6	114 17	46 50	B	135 03½	66 35	B
Secunda	6	124 58	47 55	B	151 38	64 07	B
Tertia	6	157 00	48 40	B	185 54	52 19½	B
Quarta	6	157 30	49 42	B	186 22½	52 57	*B*
Quinta	6	157 19	49 42	B	187 12	53 01	B
Sexta	6	170 05	49 00	B	197 00½	47 29	B
Septima	6	169 01	49 27	B	196 32½	48 15½	B
Octava	6	176 42	48 11	B	201 30	44 19	B
Nona	6	167 02	52 25	B	197 47	51 22½	B
Parvula quæ contingit coxam	6	122 41	35 40	B	138 03	53 51	B

Draco.

Quæ eſt in linguâ	4	229 56½	76 17	B	254 36	54 55	B
In ore	4	245 14½	78 15½	B	261 21	55 29	B
Duarum lucidarum in capite præcedens	3	247 19½	75 21	B	260 46	52 34	B
Quæ ad genam	4	260 03	80 21½	B	266 57½	56 57	B
Sequens lucidarum, vulgò lucida capitis	3	263 24	75 03½	B	267 15½	51 36	B
In prima colli inflexione trium borealior	5	288 04	81 53	B	274 50	58 40	B
Auſtralis	5	295 31	77 57	B	279 05	55 14½	B
Media earundem	5	291 33½	79 51½	B	276 48	56 51	B
Quæ ſequitur ad ortum	4	310 29	80 53½	B	281 31	59 01	B
Quæ eſt propè ſecundam flexuram	4	359 33	81 51	B	289 42	65 08	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Aſc. Rect.	Declin.	Pl.
Borea quadrati ſecundæ fluxuræ	3	13 26½	82 49	B	288 14	67 07½	B
Borea lateris ſequentis	4	16 21	78 09½	B	300 15½	67 00	B
Auſtralis ejuſdem lateris	3	28 47	79 25	B	297 21½	69 30	B
Sequentis Trianguli præcedens	4	46 18	83 05	B	284 45	70 55	B
Quæ ſequitur ad Auſtrum	4	50 40½	86 38	B	290 20½	72 44	B
Quæ ſuprà hanc	4	27 44	80 54	B	293 12	69 11	B
In reliquo Triangulo ſequens	4	97 34½	83 04½	B	266 49½	73 19	B
Auſtralis ejuſdem	4	92 28	83 28½	B	269 02½	73 00	B
Præcedens ac Borealis Trianguli	4	66 31	84 48	B	276 25	72 09	B
Quæ in flexura nodi tertii	3	150 44½	81 04½	B	247 22	69 24	B
Polo Zodiaci proxima	4	127 26	86 53	B	263 27	68 53	B
quæ 24 ſequitur	5	179 21	83 18	B	253 33	65 40½	B
Succedens huic	5	179 22	81 41	B	249 34	65 32	B
Polo vicinior mediocriter lucida	3	177 51½	84 46	B	256 59	66 08	B
Præcedens ante penultimã ab extr. flexione	3	188 55	78 32	B	244 54	62 25	B
Ante penultima flexuram præcedens	3	193 28½	74 11½	B	239 04	58 58½	B
Penultima ad flexuram	3	180 22	71 04	B	229 27	60 03	B
Quæ flexuram ſequitur, ſecunda	5	150 17	65 18	B	205 26	66 18	B
Quæ flexuram proximè ſequitur	2	153 10½	66 36	B	209 40	65 56	B
Penultima caudæ	3	131 26	61 33	B	184 13	71 34	B
Ultima caudæ	3	125 37½	57 07	B	167 15	71 05	B
Inter 11 & brach. Cephei informis	5	2 04	77 31½	B	299 26½	63 57	B

Cepheus.

In cingulo	3	31 13	71 07	B	321 03	69 09	B
Lucida in humero dextro	3	8 13	68 54	B	317 38	61 10	B
In ſiniſtro humero	4	28 53½	62 35	B	339 34	64 31	B
Quæ in tiara ſequitur ad Boream	4	9 29	61 03	B	329 59	56 32	B
Auſtralis	4	8 53½	59 59	B	330 56	55 34	B
Quæ verſus Ortum	4	14 39	58 46	B	336 14	56 45½	B
Duarum in flexu brachii, Auſtralis	4	0 21	71 49	B	309 36	60 41	B
Borealis	4	0 54	74 00½	B	305 29½	62 02	B
Illa quæ in humeris	5	19 46	65 42	B	328 39	63 02	B
In dextro pede	4	58 33	75 27	B	304 40	76 40	B
In ſiniſtro pede	3	55 23	64 28	B	351 59	75 41	B

Bootes, Arctophylax.

Trium in ſiniſtra manu præcedens	4	175 09½	58 53	B	210 21½	53 22	B
Secunda	4	176 33	58 51	B	211 13	52 51½	B
Tertia	4	177 59½	60 05	B	213 28	53 18½	B
Quæ in ulna ſiniſtra	4	182 18	54 40	B	210 58	47 38	B
In humero ſiniſtro	3	194 05½	49 33½	B	215 29	39 24½	B
In Capite	3	199 43½	54 15½	B	222 32½	41 44	B
In dextro humero ſuprà coronam	3	208 29½	49 01	B	225 34	34 34	B
In coxendice infrà brachium dextrum	3	203 29½	40 40	B	217 40	28 29	B
Infima duarum in dorſo	4	199 16	42 11	B	225 07½	31 12½	B
Superior earum	4	198 17½	42 35½	B	214 35	31 94	B

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl*
Quæ in crure dextro	3	208 26½	27 57	B	216 24	15 11	B
Superior cruris	3	194 42	28 09	B	204 45	20 05	B
Media	4	193 25	26 33	B	202 58	19 05½	B
Infima	4	194 37	25 14	B	203 25½	17 27	B
In fimbria, ARCTURUS	1	199 39	31 02½	B	210 13	20 58	B
Infima trium informium circà genu dex-(trum.	4	207 13½	30 27½	B	216 20½	17 54	B
Media	4	208 11	31 22	B	217 30	18 27	B
Superior	4	208 52	33 52	B	219 03	20 33	B
Præcedens ex quatuor in dextra manu	5	209 11	40 14½	B	221 57	26 21	B
Sequens Australis	5	210 40	40 31½	B	223 14	26 11	B
Borealis	5	208 53	42 16	B	222 37	28 18	B
Quæ hanc ſequitur	6	210 16	41 55	B	223 31½	27 34½	B
Præcedens Auſtral. duarum in coloboro	5	210 34½	45 06	B	225 11	30 25	B
Sequens	5	212 26½	46 52	B	227 24	31 31	B
Superior in coloboro	4	208 32	53 27½	B	228 01	38 33	B
Informis circa hanc	4	183 35	54 00	B	241 17	46 39	B
Informis è duabus ſuprà caput	6	192 49	60 40	B	223 08	49 07	B
Secunda ipſarum.	6	193 33	60 57	B	223 50	49 08	B

Corona Borea.

Lucida Coronæ	2	217 38½	44 23	B	230 12½	27 51	B
Præcedens	4	214 37	46 08	B	228 56½	29 44	B
Illa quæ ſuprà hanc	5	214 10½	48 25	B	229 55½	31 28½	B
Quæ ſequitur ad Septentrionem	6	219 02	50 21	B	233 44	33 04	B
Quæ ſequitur Lucidam	4	220 14½	44 33	B	232 14	27 21½	B
Proximè ſequens	4	222 25	44 52	B	233 59	27 08	B
Quæ hanc rurſus comitatur	4	224 32	46 09½	B	236 02½	27 52½	B
Omnium ultima	6	224 02	48 24	B	236 31	30 06	B

Engonaſi, Hercules.

In capite	3	251 31	37 23	B	254 12	14 50	B
In humero dextro	3	236 27½	42 48	B	245 12	22 16	B
Penultima dextri brachii	3	234 36	40 05½	B	241 52	19 59	B
Infima in dextro brachio	4	233 06½	37 19	B	238 20	17 59½	B
In ſiniſtro humero.	3	250 10	47 47	B	255 24	25 16	B
In ſiniſtro brachio	4	255 22	49 23	B	259 25	26 24	B
Præcedens in exuviis Leonis	4	260 36	51 16½	B	263 22	27 59	B
Sequens in Triangulo exuviarum	4	268 19	52 19	B	268 49½	28 48½	B
In baſi Trianguli ad Boream	4	264 57	53 46	B	266 33	30 19	B
Media earum quæ in exuviis	4	264 38	52 47	B	266 17	29 20	B
Quæ in coxa ſiniſtra	3	237 02	53 10½	B	247 19	32 14	B
Hàc Orientalior in femore ſiniſtro	3	243 45½	53 21	B	251 58	31 28	B
Præcedens trium contiguarum in femore	4	247 21½	59 38	B	255 52	37 14	B
Media	4	248 19	60 11½	B	256 35	37 41	B
Sequens	4	250 47½	60 13½	B	258 07	37 28	B

In

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
In genu sinistro	3	263 56	60 47	B	266 17	37 21	B
quæ in sinistra sura propè caput Draconis	3	255 17	69 22	B	262 34	46 14	B
Præcedens trium obscurarum in pede (sinistro	6	248 05½	71 20	B	259 36	48 36½	B
Media earundem	6	252 07	71 13½	B	261 28	48 14½	B
Ultima	*Neb.*	259 00	71 05	B	264 43	47 46	B
In superiori femore dextro	3	234 08½	60 22½	B	247 56	39 35	B
borealior in eodem femore	4	228 39½	63 14	B	245 57	43 08½	B
quæ est in dextro genu	4	219 43½	65 55	B	242 31	47 09	B
Quæ est in superiore sura	4	216 57	63 51	B	239 38	45 50½	B
quæ in crure	4	213 43	64 23	B	238 11½	46 58½	B
Præcedens in dextro crure.	5	227 32	62 29	B	244 55	42 38	B
Quæ in tibia dextri pedis	4	213 28½	60 15½	B	235 17	43 23½	B
Extrema in dextro pede	4	208 06	57 15½	B	230 02	42 02	B

Lyra, vultur Cadens.

LUCIDA LYRÆ	1	280 43	61 47½	B	276 27	38 30	B
Quæ suprà lucidam ad Aquilonem.	5	284 14	62 27	B	278 27	39 21	B
Quæ infra lucidam ad Eurum	5	283 36	60 26	B	278 24	37 19	B
Quæ in medio eductionis cornuum	4	287 10½	59 26	B	280 46	36 33½	B
Duarum contiguatum ad Boream	5	295 32½	60 46	B	285 38	38 38	B
Quæ ad Austrum	5	269 02	59 41	B	286 15	37 38	B
Duarum præcedentium in jugo Borealis	3	284 16½	56 05	B	279 27	33 02	B
Parva sub hâc	6	284 03½	55 16	B	279 25	32 12	B
In jugo duarum sequentium Borea	3	287 11	55 06	B	281 32	32 16	B
Parva quæ hinc subest	6	287 20	54 31½	B	281 44	31 43	B
Quæ in medio ferè corpore	5	291 52	58 06	B	284 01	35 39	B

Olor, Cygnus.

In rostro	3	296 44	49 02	B	289 23	27 18½	B
In capite	5	300 20	50 42	B	291 33	29 26	B
In medio colli	4	308 33	54 19	B	296 05½	34 15	B
In pectore	3	320 25	57 09½	B	302 39	39 13	B
In cauda	2	330 53½	59 56½	B	307 32	44 05	B
Prima & lucidissima in ancone superioris (alæ	3	311 53	64 28	B	293 44½	44 22½	B
Trium in superiore alâ Australis	4	314 21	69 42	B	291 57	49 33	B
Penultima superioris alæ	4	313 39½	71 31	B	290 24	51 05	B
Extrema superioris alæ	4	310 36½	73 50½	B	287 26	52 47	B
Quæ in ancone inferioris alæ	3	323 09½	49 26	B	308 14	32 45	B
In medio ipsius	4	325 18	51 41½	B	308 41	35 22	B
Extrema inferioris alæ	3	328 43	43 44	B	314 54	28 57½	B
Præcedens in inferiori pede	4	331 32	54 59	B	311 08	39 56	B
Quæ sequitur in inferiori genu	4	336 21½	56 36	B	313 17	42 39	B
Australis & præced. duarũ cont. in sup. pede	4	323 50	63 37	B	300 58	45 47½	B

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Aſc. Rect.*	*Declin.*	*Pl.*
Superior earundem, & Borealis	4	325 34½	64 17½	*B*	301 29	46 45	B
Inferior duarū informiū dextrū alā ſequens	4	334 03½	50 33	*B*	315 25½	36 40	B
Superior earundem	4	335 53½	51 31	*B*	316 10	38 03	B
Infra alam verſus pedem Pegaſi	3	335 33	38 39	*B*	322 20	26 20½	B
Duarum verſus Lyram præcedens	4	290 57	66 15	*B*	281 28	43 43	B
Sequens Borealior	4	295 49½	68 52	*B*	283 12	46 37	B
Ad volam alæ parvula	4	314 31	69 35	*B*	292 07	49 28½	B
	6	329 44	25 11	*B*	323 03	12 01	B
	6	329 22	35 35	*B*	318 50	21 36	B
Ad inferiorem alam	6	319 15	53 12	*B*	303 48	35 19½	B
Ad ſuperiorem	6	314 18	69 42	B	291 56	49 33	B
Nova Anni 1600, in pectore Cygni		317 16	55 30	B	301 24	37 02	B

De duabus, quarum nomina hic omittuntur vide quid Keplerus *adnotavit ad finem hujus Anteriſmi.*

Caſſiopeia.

In capite	4	30 35	44 40½	*B*	4 46	52 06	B
In pectore. *Schedir*	3	33 17½	46 35½	*B*	5 32	54 45	B
In cingulo	4	35 38	47 05	*B*	7 22	56 05	B
In flexura ad coxas	3	39 27½	48 46	*B*	9 22½	58 57	B
Ad genu	3	43 21	46 22	*B*	16 10½	58 30	B
In crure	3	50 13½	47 29	*B*	22 51	62 01	B
Extrema pedis	4	57 39	48 54	*B*	30 37	65 52½	B
In brachio ſiniſtro	4	37 14½	43 06½	*B*	22 49½	53 25	B
In cubito ſiniſtro	5	36 16	43 28	*B*	11 30	53 20	B
In cubito dextro	6	25 39	49 24½	*B*	355 39	53 58	B
In eductione ſedis	4	38 06	52 14	B	3 41	61 07	B
Lucida Cathedræ	3	30 35½	51 14½	B	357 59	57 22	B
Extrema Cathedræ	6	26 34	51 08	B	354 37	55 41	B
Quæ juxta hanc juxta extremitatem ſtellæ	6	26 32	52 39	B	352 48	56 50	B
Quæ in recta ferè lineâ cum 11 & 17	6	50 28	52 48	B	15 55	66 24½	B
Extrema Scabelli	6	53 21	56 13	B	13 09	70 04	B
Media Scabelli	6	23 23	54 27	B	348 02	56 56½	B
Hanc Longimontanus *ſic exprimit.*	6	23 32	54 27	B	348 09	57 00	B
In Scabello proximè ad plantam pedis	6	52 58	52 08½	B	19 57	66 51	B
Quæ ſequitur genu	6	43 57½	44 57½	*B*	18 18	57 33	B
Quæ genu præcedit	6	41 00	45 04½	B	14 57	56 31	B
Gyrus umbilici	6	37 52	47 31½	B	11 00	57 06½	B
Parvula ad crines	6	30 10	45 38	B	3 29	52 43	B
Sequens ex duabus Borealibus in Virga	6	30 32	41 15	B	7 39	49 12	B
Præcedens earundem	6	28 57	41 25½	B	5 59	48 43	B
Penultima Virgæ	6	27 56	39 15½	B	6 37	46 28½	B
Extrema Virgæ	6	26 54½	38 09	B	6 29	45 06½	B
Infra ſcabellum trium præcedens Septen-(	6	62 46	53 16	B	31 31	71 16	B
Sequens Septentrionalis (trionalis	6	67 12	53 32	B	38 14	72 57	B
Auſtralis	6	61 11	52 04	B	31 08	69 45	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Quæ suprâ has versus polum	6	67 45	59 08	B	25 32½	77 34	B
Inter Cassiopeiam & Ericthonium prima	6	78 17	35 50	B	71 35	58 36	B
Secunda	6	88 19	35 48	B	87 19	59 18	B
Tertia	5	93 33	34 49	B	95 33	58 16	B
Quarta	6	94 00	30 22	B	95 51	53 48	B
Trium in Boream prima	6	91 45	44 10	B	93 18	67 40	B
Secunda	6	91 57	45 32	B	93 49	69 01½	B
Tertia	6	87 15	45 32	B	84 37	69 00	B
Quæ magis in Boream, prima versus(	6	91 10	54 43	B	93 18	78 13	B
Secunda (ursam	6	88 45	56 15	B	86 06	79 45	B
Tertia	6	95 13	56 55	B	106 50	80 08	B
Quarta	6	90 58	59 18	B	93 56½	82 48	B
Quinta	6	98 54	60 47	B	128 46	83 04	B
Sexta	6	101 14	62 04	B	143 00	83 26	B
Septima	6	100 37	62 46	B	145 39	84 08	B
Octava	6	111 58	63 17	B	171 16	80 12	B
Nova Anni 1572		37 54	53 45	B	1 23	62 11	B

Perseus.

In extrema dextræ manus involutione	6	49 31	39 00½	B	29 57	54 23	B
In cubito dextro	4	54 09½	37 28½	B	36 45	54 33	B
In dextro humero	3	55 26½	34 30	B	40 17	52 12	B
Quæ in sinistro humero	4	50 04½	31 34½	B	35 30	47 48	B
Quæ in capitis vertice	5	52 50	34 26	B	37 07	51 20	B
Quæ in dorso	4	54 33	30 36½	B	41 23	48 18	B
Fulgens in dextro latere	2	57 17	30 05	B	44 16	48 36	B
Quæ proximè infrà sequitur	5	58 04½	27 59	B	46 57	46 50	B
Hanc sequens parva	5	59 13½	27 55	B	48 24	47 04½	B
Quæ est ad flexuram ejusdem lateris	3	50 15	27 14	B	38 00	43 49	B
Quæ est in cubito sinistro	4	53 06	26 04	B	41 52	43 36	B
Caput Medusæ, sive ALGOL	3	51 37	22 22	B	41 46	39 39	B
Quæ sub Algol	5	51 31	20 54	B	42 15	38 14	B
Hanc præcedens	4	50 18	20 33	B	41 02	37 32½	B
Præcedens ad Boream in eodem capite	4	49 20	21 35	B	39 31	38 13½	B
In poplite dextro	5	67 13½	28 22½	B	58 26	49 25	B
Quæ dextrum genu præcedit	4	65 11½	28 50	B	55 36½	49 25	B
Flexuram genu præcedens	5	46 55	26 11	B	56 14½	46 48	B
Media in genu dextro	4	66 14	26 39	B	57 45	47 32	B
Quæ infra genu dextrum	6	67 00	24 35	B	59 26	45 41	B
Quæ est in planta pedis dextri	5	69 01	18 56	B	63 32	40 32	B
Quæ in sinistro femore	4	59 11	22 06	B	50 41	41 29	B
Quæ in sinistro genu	3	61 08	19 04	B	54 01½	39 02	B
Quæ in crure sinistro	5	60 23½	14 53½	B	54 26	34 49	B
Quæ in sinistro calcaneo	4	56 33	12 08	B	50 56	31 13	B
Sequens sinistri pedis	3	58 36	11 17½	B	53 27	30 54	B
Informis suprà caput	5	57 45	42 26	B	37 35	60 12	B
Quæ in superiori parte femoris dextri	5	63 32	29 31	B	53 09½	49 42	B
Informis præcedens caput Medusæ	4	47 16	20 53	B	37 31	36 55	B
Quæ facit lin. rect. cū polo & lucid. Persei	6	63 18	45 10	B	42 55	64 22	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Secunda illarum	6	65 12	48 07	B	42 44	67 35½	B
Meus (id est) *Kepleri* Catalogus habet	6	65 02	48 07	B	42 27	67 33	B
Tertia	6	65 41	49 27	B	41 53	68 55½	B
Quarta	6	67 25	53 37	B	38 28	73 05	B
Meus (*Kepleri*) Catalogus	6	67 15	53 37	B	38 10	73 02	B

Auriga, Heniochus, Ericthonius.

Præcedens & superior duarum in capite	6	84 38	32 15	B	81 57	55 37	B
Inferior & sequens	4	85 14	30 50	B	82 59	54 14	B
In sinistro humero fulgens, CAPELLA	1	77 16	22 50½	B	73 07½	45 37	B
Lucida in dextro humero	2	86 52	21 27½	B	85 53	44 56	B
Meus (*Kepleri*) Catalogus	2	85 28	21 27½	B	84 02	44 53	B
In dextro brachio	4	85 28	13 44	B	84 28	37 10	B
Meus & *Longim.* & *Progymnas.*	4	84 58	13 44	B	83 52	37 08½	B
In sinistro cubito	4	74 09	20 52	B	69 29	43 15½	B
Præcedens hædus	4	74 05½	18 08½	B	69 57	40 33	B
Sequens hædus	4	74 49½	18 11½	B	70 51	40 42	B
In superiore pede	4	72 04½	10 22	B	68 57	32 35	B
Superior ad lucidam in dextro humero	5	85 25½	27 27	B	83 34	50 52	B
Duarum in lumbis borealis	6	77 52½	18 34½	B	74 36	41 27	B
Australis	5	77 06	16 59	B	73 52	39 47	B
Hac inferior ad occasum	5	75 58	15 21¼	B	72 44	38 02	B
Sequens	6	78 09	14 04	B	75 34	36 59	B
Ad nates	5	73 00	15 03	B	69 12	37 20½	B
Præcedens duarum in dextro brachio	5	83 12½	15 42½	B	81 35	39 01½	B
Sequens	5	83 24	15 43	B	81 48½	39 03	B
Meus Catalogus habet	5	83 44	15 43	B	82 13	39 04	B
Sub hac in dextro crure	6	83 35	13 49	B	82 10	37 09½	B
In sinistra tibia	5	77 39½	11 15	B	75 20	34 08	B
In dextro pede	5	79 34	8 51	B	77 48	31 55½	B
Præcedens duar. informiū circà *Ericthonium*	5	71 04½	14 51	B	66 56	36 51½	B
Seqnens Australis	5	71 31	14 02	B	67 37	36 07	B
Borealis informium inter *Ericthonium* & (	4	88 47	6 04	B	88 36½	29 35	B
Secunda (ped. ♊	4	83 58	4 06	B	83 13	27 28½	B
Sub ista ad ortum	4	84 58	2 26	B	84 25	25 51	B
Harum præcedens	4	80 52½	2 28	B	79 52	25 40	B
Ultima omnium	4	82 55	1 06	B	82 13	24 25½	B

Ophiuchus, Serpentarius.

In capite	3	257 50	35 57	B	259 55	12 52	B
In dextro humero	3	260 45	28 01	B	261 49	4 46	B
Inferior & sequens in dextro humero.	3	262 05	26 11	B	262 53½	2 52	B
Præcedens in sinistro humero	4	245 39½	32 35½	B	249 35	10 46	B
Sequens in eodem humero.	4	247 16	31 56	B	250 33	9 57	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Quæ in sinistro cubito	4	241 03	23 39½	B	243 39	2 46	B
In sinistra manu Borealior	3	237 44½	17 19	B	239 19½	2 49	A
Sequens Australior	3	238 57	16 30½	B	240 17	3 51	A
In dextro ancone	4	260 33	15 19	B	260 48	7 54	A
Meus Catologus	4	260 03	15 19	B	260 19	7 52	A
Australior & præcedens in dextra manu	4	265 13½	13 47	B	265 18	9 39	A
Borealior & sequens in eadem manu	5	266 14½	15 20	B	266 20	8 08	A
In dextro genu	3	253 24	7 18	B	252 55	15 14	A
Correxi (inquit *Kepleri*) *in lib. de stel. novâ.*	3	253 20½	7 18	B	252 52	15 14	A
Quæ in sinistro genu.	3	244 39	11 30	B	244 84	9 49	A
Quæ in dextra tibia	3	255 23	2 12	B	254 23	20 31½	A
Quinta informium in via lactea	4	267 31	33 02½	B	267 53	9 33	B
Supra lucidam in collo Serpentis	4	227 48	26 36	B	232 37	8 25	B
Post coxas Ophiuchi	4	255 49	10 21	B	255 42½	12 28	A
Sequentium duarum Australis	3	259 57	8 04	B	259 41½	15 05	A
Borealis	4	260 48	10 35	B	260 44	12 38	A
Illa quæ supra hanc	4	259 45	15 18	B	260 01	7 52	A
Inter sinistram manũ & genu Ophiuchi	5	241 57	13 19	B	242 30½	7 33	A
Informis circa humerum Borealem	4	265 30	27 55	B	266 01	4 28	B
Media ipsarum	4	265 38	26 23	B	266 05	2 56	B
Australis trium	4	265 53	24 50	B	266 16	1 22	B
Sequens tres illas	4	266 58	26 10	B	267 16	2 41	B
Præcedens ex quatuor in dextro pede	3	255 01	2 16	B	254 00	20 25	A
Sequens	4	256 42	1 32	B	255 42½	21 19½	A
Tertia	4	257 23	0 20	B	256 19	22 35	A
Alia sequens	5	258 12	0 29	B	257 13	22 30½	A
Illa quæ contingit calcaneum	5	258 36	0 58	B	257 47½	20 50	A
In crure dextro	5	257 50	7 10	B	257 26½	15 55½	A
Informis extra crus	6	262 45	4 20	B	262 21	19 00	A
Sequens duarum in manu	5	241 07	23 34	B	243 41	2 39½	B
In coxa Ophiuchi	5	256 00	10 18	B	255 53	12 32	A
Sequens Australis	4	260 02	8 05	B	259 46	15 05	A
In dextra manu	5	261 04	10 40	B	261 00	12 34	A
Borealis	5	260 05	15 06	B	260 20	8 05	A
Serpens Ophiuchi.							
Præcedens in ore	5	222 35	38 12	B	231 46	20 47	B
Quæ in ore est	3	225 24½	39 06½	B	234 19	20 58	B
Quæ in temporibus	3	228 06½	35 25	B	235 21	16 49	B
In eductione colli	3	225 21½	34 27½	B	232 49	16 32½	B
Quæ ad sinistrum oculum	4	226 10	37 28½	B	234 24	19 14	B
Quæ ad nares	4	227 32	42 37	B	238 04	23 51	B
Secunda in collo infra caput	3	223 46½	28 58	B	229 50	11 41	B
In medio nexu colli	2	227 30	25 35	B	232 04	7 30	B
Longimontanus habet	2	227 49	25 35	B	232 21	7 26	B
Australior trium	3	229 46½	24 05½	B	233 41	5 31	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Quæ est in secunda flexione	4	231 26½	16 26½	B	233 15	2 17	A
Ante penultima caudæ	3	265 34½	19 57	B	265 50	3 30	A
Meus Catologus	3	265 34½	19 37	B	265 49	3 50	A
Penultima	3	271 12½	20 37½	B	271 08	2 53	A
Ultima	3	281 10	26 59	B	279 57	3 51	B

Sagitta, *sive* Telum.

Superior & Orientalior	4	302 32	29 13	B	298 22	8 51½	B
Media seu hanc præcedens	5	298 55	38 58½	B	293 15	17 46	B
Parvula, quæ est suprà mediam	6	299 31	39 31	B	293 37	18 24	B
Superior duarum contiguarũ in glyphide	4	296 30½	38 53	B	291 21	17 19	B
Inferior earundem	4	296 39	38 18	B	291 34	16 45	B
Informis & inferior suprà Sagittam	4	301 13	42 43	B	294 13	21 48	B
Superior informium	4	302 36	44 02	B	294 57	23 18½	B
Tertia in oxygonio informium	4	294 57	46 03	B	288 43	24 09	B

Aquila *seu* Vultur Volans.

Quæ in capite	6	300 28½	27 08½	B	297 01	6 27	B
In collo	3	297 53	26 49½	B	294 48	5 41	B
Lucida in scapulis	2	297 09	29 21½	B	293 41	8 03	B
Parva, quæ supra lucidam	6	296 33	30 54½	B	292 53	9 29	B
Quæ in sinistro humero	3	296 26	31 18	B	292 43	9 54½	B
Quæ sequitur parva	5	297 08½	31 59	B	293 11	10 38	B
Superior, & præcedens in inferiori ala	4	292 16½	28 46½	B	289 33	6 46	B
Inferior, & sequens in ala	5	293 14	26 35	B	290 44	4 44	B
Cauda Vulturis	3	285 15½	36 16½	B	282 36	13 25½	B
Quæ proxime caudam præcedit informis	3	283 44	37 40	B	281 12	14 41	B
Suprema informiũ sup. caud. quæ ex tri-(bus præcedit	4	280 12	43 32½	B	277 52	20 18	B
Media informium	4	280 17½	41 05	B	278 08	17 51½	B

Antinous.

In manu sinistra	3	300 21½	18 48	B	298 36	1 45	A
In latere dextro	3	291 17½	20 14½	B	289 56	1 47	A
In genu	3	290 17	14 28	B	289 48	7 40	A
In dextro brachio	3	289 01	24 56	B	287 12	2 33	B
In pectore	3	295 50	21 38	B	293 54	0 14½	B
In pede dextro	3	282 46	17 41	B	282 12½	5 17½	A
Præcedens hanc informis	4	281 29	16 57	B	281 02½	6 08	A

Delphinus.

Lucida caudæ	3	309 32	29 08	B	304 24	10 14	B
Quæ caudam sequitur	6	310 48	28 52½	B	305 33½	10 16	B
Quæ infra caudam	6	310 42	27 34	B	305 49	8 59	B
In Rhomboide præcedentis lateris Au-(stralior	3	311 56	31 57½	B	305 40	13 30	B
Ejusdem lateris Borealior	3	312 50½	33 05	B	306 06	14 48	B

Sequenti

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc.Rect.	Declin.	P
Sequentis lateris Australior	3	314 36½	32 00	B	307 54	14 11	B
Quæ est in capite	3	314 52	32 47	B	307 53	15 00	B
Quæ in præcedente latere 4 contig. anteit	5	311 17	32 08½	B	305 04	13 32	B
Præcedens duarũ infimarũ in Rhomboide	6	310 18	30 41½	B	304 38½	11 54½	B
Sequens earundem	6	311 42	30 41	B	305 50	12 13	B

Equuleus, Equisectio.

Præcedens capitis	4	318 32½	20 12½	B	314 50	3 57	B
Sequens capitis	4	320 54½	21 06	B	316 40	5 28	B
Præcedens oris	4	318 54	25 16	B	313 36	8 52	B
Sequens oris	4	319 54½	24 52	B	314 37	8 46	B

Pegasus, Equus Alatus.

Os Pegasi	3	327 22	22 07½	B	322 03	8 24	B
Caput	4	332 15½	16 25	B	328 24	4 39	B
Quæ ad Austrum in capite	5	330 45½	15 43	B	327 18	3 29	B
Inferior, & sequens in juba	6	344 00	14 30½	B	339 40½	7 04½	B
Superior, & præcedens in juba	6	343 44	15 43½	B	338 58	8 06	B
Lucida colli	3	341 39½	17 41	B	336 21	9 08½	B
Sequens in collo	5	343 25	18 29	B	337 36	10 31½	B
Sinistrum crus	4	334 23	36 42½	B	322 25	24 11½	B
Sinistrum genu	4	339 50	34 19	B	327 55	23 47	B
Dextrum crus	4	345 03	41 00½	B	328 52	31 36	B
Præcedens duarum in pectore	4	348 29½	28 49	B	337 40	21 51	B
Sequens	4	349 53½	29 24½	B	338 36	22 54½	B
Dextrum genu	3	351 10½	35 07½	B	336 54	28 31	B
In eodem genu ad Austrum	5	350 25	34 24½	B	336 38	27 36	B
Præcedens duarum in ala	6	356 33	25 35	B	346 09	21 59	B
Sequens in ala, & Australior	6	358 06	24 50½	B	347 51	21 55	B
Prima alæ. Marchab	2	348 56½	19 26	B	342 07	13 28	B
Eductio cruris. Scheat	2	354 49	31 07½	B	341 59	26 18	B
Extrema alæ	2	4 38	12 35	B	359 08	13 22	B
In collo Pegasi	4	337 28	20 51	B	331 25	10 34	B
Infra os & supra pedem	4	325 51	33 21	B	316 48	18 29	B
Hâc superior	4	329 47	36 11	B	318 55	22 17	B
Primum sequens	4	346 15	23 16	B	338 09	15 58	B
Meus Catalogus	4	356 15	23 16	B	346 56½	19 46	B
Fortè	4	326 15	23 16	B	320 41	9 07	B

Andromeda.

Caput	2	9 47	25 42	B	357 54	27 18	B
Infima in scapula dextra	5	18 06½	27 06½	B	4 51	31 53	B
Inferior in sinistro humero	4	16 25	23 03½	B	5 19	27 34½	B
In dextro brachio trium Australior	5	15 58	31 33	B	0 26	34 59	B

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Borea	4	16 45½	33 20½	B	2 37	36 48	B
Media	5	17 07	32 14½	B	1 05½	36 03	B
Australior in superiori manu	4	11 28	40 56½	B	350 29	41 22	B
Borealior	4	12 46	41 44	B	351 05	42 33	B
Obscura ibidem	5	15 23	42 08	B	353 00	43 55	B
Suprema omnium in Boreali manu	4	13 47	43 49½	B	350 24½	44 43	B
Præcedens & superior duarum in sinistro (brachio	4	16 09	17 48	B	7 32	22 42	B
quæ in sinistro cubito	5	17 53½	15 58	B	10 00½	21 43	B
Australior in cingulo	2	25 49	25 59	B	12 49	33 55	B
Media	4	25 06½	30 33½	B	9 45½	37 42	B
Borea	4	24 36	32 30½	B	7 58	39 16	B
In Australi pede lucida	2	39 39	27 46½	B	25 57	40 44	B
Extrema in superiori pede	5	40 06½	36 49½	B	20 54	49 03	B
Lucidior, & præcedens in dextro pede	4	37 52	35 21½	B	19 30	46 55	B
Suprema in sinistra sura	5	36 06	28 59	B	21 33	40 32	B
Inferior	5	34 23	27 54½	B	20 22⅓	38 56	B
Quæ ad genu dextrum	5	31 56	36 20	B	12 43	45 30	B
Quæ in extremo catenæ annulo	4	355 00	57 19	B	324 50	48 51	B
Clarior, & superior in sinistra scapula	3	17 19½	24 20	B	5 33	29 05	B

Triangulus, Deltoton.

In apice Trianguli	4	32 19	16 49½	B	23 37	28 00½	B
In basi ad Boream	4	37 49½	20 33	B	27 35½	33 26	B
Media	5	38 59	19 29	B	29 16	32 51	B
Australior in basi	4	38 58	18 57	B	29 29	32 21	B

Coma Berenices.

In cuspide primi & Borealis Trianguli	3	169 17	28 25	B	182 39	30 06	B
Superior contingentium hanc ad Austrũ (sequens	4	169 42	27 23½	B	182 32	29 01	B
Inferior earundem	4	169 46	27 20	B	182 31	28 57	B
Quæ contiguas duas sequitur	4	170 19	27 07	B	182 00	28 36	B
Præcedens duarũ Australium contiguarũ	4	169 25	25 51	B	181 27	27 45½	B
Altera contigua ad ortum	4	169 48½	26 07	B	181 56½	27 50½	B
Omnium præcedens ad Austrum	4	169 00	23 30	B	179 24	25 48½	B
Suprema trium contiguarum sequentium	4	172 10	25 16	B	183 40	26 08	B
Altera & præcedens	4	171 51	24 56	B	183 12	25 58	B
Infima & sequens	4	173 52	24 00½	B	184 34	24 20	B
Meus Catalogus	4	172 52	24 00½	B	183 41	24 44	B
Postrema in extensione comæ	4	179 58½	32 46	B	194 23	29 46	B
Quæ hanc præcedit	4	178 49½	31 42	B	192 49	29 16	B
Quæ inter has & primam in cuspide	4	175 17	30 16	B	188 58	29 22½	B
Quæ est in Australi cuspide △ parvi.	5	179 15	28 32	B	191 35	26 16	B

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
ARIES.							
Australis in præcedente cornu	4	28 37	7 08½	B	23 54½	17 40½	B
Borealis & sequens in eodem cornu	4	29 23	8 29	B	24 08	19 12	B
Lucida in vertice capitis: principalis	3	33 06	9 57	B	27 12½	21 54	B
In rictu duarum Borea	6	33 34	7 23	B	28 39	19 40	B
Quæ magis ad Austrum	6	34 20	5 42½	B	30 01	18 22	B
Quæ in cervice	5	28 57	5 24	B	24 54	16 10½	B
In renibus	6	39 36	6 07	B	35 07	20 31½	B
Quæ in eductione caudæ	5	43 57	4 08½	B	40 09	20 01½	B
Præcedens trium in cauda	4	46 15	1 46½	B	43 14	18 27	B
Media	5	47 24	2 50	B	44 04	19 48	B
Ultima	6	48 50½	2 36	B	44 24	19 59	B
In femore	6	42 22	1 12	B	39 31	16 44½	B
In poplite	6	40 35	1 07	B	37 47	16 06	B
In genu sinistro	6	40 23	1 30	A	38 26	13 33½	B
In genu dextro	6	38 52	0 39	A	36 41	13 53	B
Parvula in alvo	6	39 46	4 01	B	36 00	18 35½	B
Quæ est infrà lucidam capitis.	6	32 41	9 13	B	27 05	21 04	B
Supra dorsum 4 informium præcedens	5	41 35	10 50½	B	35 26	25 38	B
Sequens sci. ad basin Occi. △ & sequentib.	4	42 23	11 16	B	36 06	26 17½	B
Orientalis in basi Trianguli	3	43 40	10 24	B	37 44½	25 53	B
In apice ejusdem Trianguli ad Boream	4	43 51	12 25½	B	37 12	27 51	B
Taurus.							
Suprema in sectione	5	49 00	5 57	A	48 12	11 48	B
Altera post ipsam	6	48 30	7 29	A	48 07½	10 11	B
Tertia	4	47 18	8 49½	A	47 20	8 34½	B
Quarta maximè Austrina	4	46 35½	9 22½	A	46 48½	7 51	B
In dextro armo	5	52 46	8 41	A	52 35	10 07	B
In pectore	4	56 01	8 03	A	55 37	11 29	B
In genu dextro	4	58 59	12 13½	A	59 26	8 03	B
In suffragine dextra	4	55 19	14 30½	A	56 25½	5 02	B
In genu sinistro	5	65 09	9 32	A	64 57	11 50½	B
In suffragine sinistra	5	64 11	11 48	A	64 24	9 27	B
In facie, suculatum prima in naribus	3	61 12	5 46½	A	60 17	14 49	B
Inter hanc & oculum Boreum	3	62 16½	4 02	A	61 01	16 44	B
Quæ inter eandem & oculum Australem	4	63 22	5 53	A	62 29½	15 07	B
In Austrino oculo, ALDEBARAN (Palilicium	1	65 12½	5 31	A	64 17½	15 48½	B
In Boreo oculo	3	63 53	2 36½	A	62 23	18 26	B
Ad radicem cornu Australis	6	69 12	3 40	A	68 05	18 17	B
In eodem cornu duarum Australior	6	73 13½	2 30½	A	72 08	19 58	B
Quæ magis in Boream	4	72 04	1 49½	A	70 49	20 30	B
In extremitate ejusdem	3	80 12	2 14	A	79 30½	20 56	B
In origine cornu Septentrionalis	5	67 35	0 40	B	65 39½	22 18	B

In

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl
In extremitate, communis cũ dextr. pede	2	77 59½	5 20	B	76 23	28 21	B
In aure duarum Borea (Heniochi	5	63 54	1 04	B	61 40	22 03	B
Australior	4	63 38	0 35	B	61 28	21 35	B
In collo duarum præcedens	5	58 51	1 12	B	56 19½	21 08	B
Quæ sequitur	6	61 28½	0 46½	A	59 30½	19 46	B
In cervice quadrilateri præcedentium(	6	61 04	5 16	B	57 43	25 35	B
Ejusdem lateris Borea (Austrina	5	60 45½	7 55	B	56 44	28 06½	B
Sequentis lateris Australis	5	63 34	3 57	B	60 42½	24 49	B
Hujus lateris Borea	5	63 25½	5 45½	B	60 09½	26 33	B
Occidentalis lucidior trium in Pleiadibus	5	54 13½	4 11	B	50 43	22 56	B
Meus Catalogus	6	54 50	4 11	B	51 21½	23 06	B
Infima, & Occidentali proxima	5	55 03	4 02	B	51 37½	23 00	B
Media & lucida Pleiadum	3	55 24	4 00	B	52 00	23 03½	B
Quæ est in cuspide ad ortum	6	55 47	3 55	B	52 26	23 04	B
In ungula pedis sinistri	6	50 57	13 30	A	52 01	5 10	B
Stellula in talo pedis sequentis	6	61 10	12 02	A	61 30	8 40	B
Quæ in armo dextro	5	62 58½	8 41	A	62 38	12 18	B
Præcedens trium infra suculas	5	62 42	6 56½	A	62 01	13 57	B
Media earundem	5	64 28	7 04½	A	63 49½	14 09	B
Sequens	6	65 55	6 17½	A	65 09	15 10	B
Parvula in australi cornu	6	76 02	1 04	A	74 57	21 43	B
Sequens in eodem cornu	6	77 55½	1 20	A	77 00	21 38	B
Parvula sequens quatuor in sectione	6	48 33	9 34½	A	48 44	8 11	B
Duarum in quadrilatero colli præcedens	5	60 22½	6 33	B	56 40	26 41½	B

Gemini.

In superiori capite, Castor, Apollo	2	105 41	10 02	B	108 24½	32 33	B
In inferiori capite, Pollux; Hercules	2	108 43	6 38	B	111 19	28 46	B
In sinistra manu præcedentis Gemini	5	96 32	10 58	B	97 46	34 18½	B
In sinistro brachio	4	100 54	7 43	B	102 36	30 45½	B
In scapulis ejusdem	4	104 24	5 42½	B	106 20½	28 24½	B
In dextro humero ejusdem	5	106 47	5 10	B	108 57	27 35	B
In sinistro humero sequentis Gemini	4	109 06	3 03	B	111 10	25 10	B
In latere dextro præcedentis Gemini	6	104 18	2 56	B	105 53	25 40	B
Stellula in sinistro cubito superioris ♊	6	105 10	6 00½	B	107 15	28 37	B
In Boreali & supremò genu	3	95 22	2 11	B	95 57	25 35½	B
In sinistro genu sequentis	3	100 26	2 06½	A	101 11	21 00	B
Quæ in ventre Meridionalis Geminorum	3	103 56	0 13½	A	105 07	22 34	B
In poplite inferioris Gemini	4	104 13	5 41	A	104 49	17 06	B
In pede præcedentis ♊ antecedens	4	88 53	0 58	A	90 48	22 33	B
Sequens in eodem pede, dicta calx	3	90 44	0 53	A	88 48	22 38	B
In extremitate pedis dextri præcedentis ♊	4	92 14	3 08	A	92 23	20 22	B
Lucida pedis	2	94 31	6 48½	A	94 41	16 38	B
In infimo pede sequentis Gemini	4	96 29½	10 09	A	96 34	13 13	B
In calce pedis ejusdem	6	98 56	9 41	A	99 04	13 33	B
Quæ est supra genu inferioris Gemini	6	97 23	1 12	A	97 58	22 07	B

In

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc.Rect.	Declin.	Pl
In femore superioris Gemini	6	99 37½	1 31	B	100 36	24 41	B
Quæ infra caput inferius, in manu	6	110 42	5 44	B	113 23	27 35	B
Parvula inter utrumque caput	5	108 04½	7 24	B	110 44	29 37	B
Ad aurem superioris Gemini	5	104 29	9 42	B	106 58	32 22	B
Præcedens ad summũ pedẽ: Propus græcè	4	86 22	0 13	A	86 07	23 15	B
Præcedens è quatuor infrà Geminos, &	6	108 02½	5 52	A	108 45	16 29	B
Sequens supra istam (infima	6	109 06	3 48½	A	110 07½	18 23	B
Tertia	6	110 30½	2 42	A	111 46	19 17	B
Quarta	6	112 28	0 57½	A	114 06	20 41½	B

Cancer.

Nebulosa in pectore, quæ præsepe vocat.	Neb	122 46½	1 14	B	125 22½	20 48	B
Borea præcedentium in quadrilatero(	5	120 49	1 31½	B	123 24	21 32	B
Australior (Cancri	5	121 09½	0 47½	A	123 13	19 11½	B
Asellus Boreus	4	122 57	3 08	B	126 02	22 37	B
Asellus Austrinus	4	124 08	0 04	A	126 28	19 13	B
In brachio Austrino	3	129 03½	5 08	A	130 07	13 06	B
In brachio Boreali	5	121 44	10 23	B	126 39	29 56	B
In extremitate pedis Borei	5	114 56	1 15½	B	117 08	22 27	B
In extremo pedis Austrini	5	116 04	7 05	A	116 43	14 02½	B
Quæ in radice caudæ lucidior	4	116 45½	2 18½	A	118 20	18 36	B
Proximæ sequens in dorso	6	119 12½	1 04	A	121 08	19 20	B
Borealis trium in brachio Australi	6	127 47½	1 54	A	129 43	16 33	B
Australis in eodem	5	131 36	5 36	A	132 30	11 58½	B
Duarum in rostro Septentrionalis	6	126 27	7 14	B	130 51	25 43	B
Inferior & Australis	6	128 36½	5 20	B	132 34	23 18½	B

Leo.

In naribus	4	130 41½	10 23	B	136 23	27 38	B
In hiatu	4	133 16½	7 52	B	138 13	24 25	B
In capite duarum Borealior	4	136 51	12 21	B	143 30	27 34	B
Australior	3	136 05	9 40	B	141 44½	25 16	B
In collo trium Borea	3	142 57½	11 50	B	149 34½	25 02½	B
Media & lucida colli	2	144 59	8 47	B	150 26½	21 29	B
Australis	3	143 20	4 52	B	147 22	18 22½	B
Cor Leonis. REGULUS. Basiliscus	1	145 17	0 26½	B	147 43½	13 33	B
In pectore Australior	5	145 50½	1 25½	A	147 37	11 36	B
Antecedens Regulum proximè	4	142 43½	0 00½	B	145 06	13 59½	B
Quæ hanc præcedit in genu dextro	5	138 54½	0 16	B	141 26	15 27½	B
In drace dextrâ	4	137 07	3 10	A	138 36	12 44	B
Sequens in altero pede	4	139 40	3 47	A	140 53	11 22	B
In drace sinistra	4	144 46	3 55	A	145 44½	9 37	B
In sinistra axilla	4	151 48	0 08	B	153 52	10 59½	B
In ventre trium antecedens	6	143 24	2 10	B	146 29½	15 48½	B
Sequentium Borealior	6	153 06	5 56	B	157 17	15 55	B
Australior	6	155 05	2 49½	B	157 59	12 18	B

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
Præcedens duarum in lumbis	5	154 14	12 53	B	161 09½	21 56	B
Quæ ſequitur lucida	2	156 41	14 20	B	164 08	22 20	B
In clune duarum præcedens & Borea	3	158 50	9 41½	B	164 15	17 14½	B
Sequens Auſtrina	6	160 08	7 50½	B	164 44	15 02	B
In femore	3	162 58½	6 07	B	166 43	12 21	B
In genu poſteriori	4	164 08½	1 40	B	166 03	7 48	B
Media in pede	4	166 57	0 33	A	167 47	4 40	B
Infima in pede	4	170 27	3 02½	A	170 02	1 00	B
In extremo CAUDÆ LUCIDA	1	167 03	12 18	B	173 04	16 25	B
Extrema in ungula pedis ſiniſtri	6	137 32	4 48	A	138 30	11 03	B
In ungula alterius pedis præcedentis	5	137 01½	5 43	A	137 44	10 20	B
Quæ in medio corpore ferè	6	151 14	10 17	B	157 11	20 39	B
Parvula in capite	6	137 13	10 47½	B	143 18½	25 57½	B
Præcedens duarum in ſiniſtro pede po-(	4	166 53	7 39	A	164 58	1 51	A
Sequens (ſteriore	5	169 50	5 41	A	168 26	1 11	A
Meus Catalogus habet	5	169 05	5 41	A	167 44	0 54	A
Præcedens duar. informium ſupra dorſum	5	147 22½	17 40	B	156 26½	28 54	B
Sequens	5	150 75	16 30	B	159 29½	26 30	B
Supra lucidam dorſi	5	155 54½	16 47	B	164 27	24 53	B
Supra caudam	4	164 22	17 19	B	172 44	22 04	B
Borealis trium ſub ventre	4	159 58	1 20½	B	162 01½	9 06	B
Media	5	159 30	0 09½	A	161 01	7 53	B
Auſtralis trium	5	160 20	2 29	A	160 54½	5 25	B

Virgo.

Borealis præcedentium in quadrilatero(	5	168 44	6 06½	B	171 47½	9 51	B
Auſtralis (capitis	5	169 33	4 37	B	172 14	8 23½	B
Sequentium duarum in vultu Borea	5	173 07	8 33½	B	177 07½	10 35	B
Auſtralis	5	172 58	6 10	B	176 01	8 27½	B
In extremo alæ Auſtrinæ & ſiniſtræ	3	172 32	0 43	B	273 26	3 38	B
Præcedens quatuor in ſiniſtra ala	4	180 16	1 25	B	180 49	1 11½	B
Altera ſequens	3	185 35½	2 50	B	186 15	0 22	B
Penultima parva	6	190 28½	2 23½	B	190 34	1 58	A
Ultima	4	193 37	1 45	B	193 12	3 47	A
In dextro latere ſub cingulo	3	186 55	8 41	B	189 48	5 13	B
In dextra & Boreali ala trium, præcedens	5	180 53	13 36½	B	186 19	12 06½	B
Reliquarum duarum Auſtrina	6	182 52	11 37	B	187 18	9 30	B
Borealior, Vindemiatrix vocata	3	185 23½	16 15½	B	191 30	12 45	B
In ſiniſtra manu, SPICA VIRGINIS	1	199 16	1 59	A	196 56½	9 31	A
Sub perizomate, in clune dextra.	3	196 22½	8 10	B	198 13	1 05	B
In ſiniſtra coxa, borealiſſima	6	198 58½	3 11	B	198 43	4 30½	A
Sequentium duarum borealior	6	202 09½	1 45½	B	201 08½	7 01½	A
Auſtralior	6	200 44	0 19½	A	199 01	8 25	A
In genu ſiniſtro	6	205 44	2 24½	B	204 44	7 44	A
Borealior in ſuprema fimbria duarum	5	208 49	11 02½	B	210 41	0 45	A
Media trium in fimbria	4	209 09	7 18½	B	209 41	4 22	A
Iufima & Auſtralis	4	209 51	2 57½	B	208 48½	8 41	A
Auſtralior duarum in ſuperiori fimbria	4	210 51½	11 48	B	212 49	0 44	A

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
In Australi pede	4	212 22	0 31½	B	210 21	11 50	A
In Boreali, seu dextro pede (cingulum	4	215 30	9 49	B	216 27½	4 08	A
Inferior duar. inter Vindemiatricem & (	6	182 21	10 26	B	186 20½	8 37½	B
Sequens illam quæ in clune dextra	6	202 37½	9 40½	B	204 30	0 09½	B
Quæ est in cervice	6	178 45½	4 59½	B	180 51½	5 04	B
Parvula sequens Vindemiatricem	6	189 25	16 14	B	195 00	11 10	B
Præcedens trium in recta lin. alæ Boreæ	5	191 11	12 40½	B	195 15	7 13	B
Media earundem	6	195 46	12 34½	B	199 22	5 23	B
Sequens	5	203 11	13 07½	B	206 17	3 10	B
Quæ est inter quartam & quintam	6	173 56½	3 22½	B	175 47	5 31	B
Informis sub brachio sinistro	5	187 38	3 25	A	185 39	6 10½	A
Media	5	191 39	3 23	A	189 22	7 44	A
Sequens	5	195 08½	3 13½	A	192 40	8 57	A
Sequens trium sub Spica	5	198 13	7 51	A	193 42	14 24½	A
Media versus Austrum	5	200 35	9 16	A	195 22	16 37½	A
Sequens Orientalis	5	201 35½	6 16	A	197 31½	14 15	A

Libra.

Lanx Austrina	2	220 31	0 26	B	218 13	14 37	A
Quæ est supra Australem lancem	5	219 42	1 55	B	217 54	12 57	A
Lanx Borea	2	224 48	8 35	B	224 52	8 07	A
quæ supra borealem lancem ad occasum	4	220 40½	8 18½	B	221 09	7 10	A
Prima ab Austrina lance ad ortum	5	223 26½	1 14	B	221 21	14 45	A
Secunda ab eadem lance ad ortum	6	227 19	2 58½	B	225 42	14 12	A
Tertia ab eadem lance ad ortum	3	230 33	4 28	B	229 19	13 38	A
Quæ est infra hanc ad ortum	4	232 84½	4 04	B	231 28	14 36	A
Quæ infra eandem ad occasum	4	230 27	2 21	B	228 39	15 39	A
Quæ est infra boream lancem ad ortum	4	226 46	8 07	B	226 37	9 07	A
Informis duarum infra lancem Austri-(	4	233 11	0 02½	B	230 47	18 35	A
Earum inferior (nam superior	4	236 03½	0 07	B	233 45	19 13	A
Præcedens trium sequentium	4	235 16	3 33	B	233 48	15 42	A
Media	4	235 48	6 10	B	234 57½	13 16½	A
Superior Orientalis	4	236 41½	9 19	B	236 34	10 24½	A
Sequens	5	238 19	10 57	B	238 31	9 09½	A
Sub boreali lance in sinistro brachio m	3	226 27	7 37	A	221 35	24 05	A
Sequens	3	226 17	1 48	A	223 15	18 29	A

Scorpius.

Suprema in fronte	2	238 36	1 05	B	236 36	18 51	A
Media in fronte	3	237 59	1 54½	A	235 15	21 38	A
Australis trium in fronte lucidiorum	3	238 25	5 22½	A	234 50	25 06½	A
Quæ ad huc magis ad Austrum, in pede	4	238 43½	8 27½	A	234 22	28 11	A
Borealissima in fronte	4	240 03½	1 42	B	238 15	18 34	A
Parvula in △ cum lucida frontis & quinta	5	239 07	0 14	B	236 56	19 48	A
Lib. de Stel. nova, correxi sic	5	238 57	0 14	B	236 46	19 46	A
Fortè melius sic.	5	239 02	0 14	B	236 51	19 47	A

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
Præcedens cor ad Boream	4	243 11	3 55	A	240 18	24 42	A
In medio rutilans. ANTARES COR ♏	1	245 13	4 27	A	242 23	25 37	A
Quæ cor ſequitur ad Auſtrum	4	246 53	5 50	A	243 56	27 16½	A
In præcedentibus inferioribus pedibus	5	241 46½	6 37½	A	238 09½	27 04	A

Sagittarius.

In cuſpide Sagittæ	3	266 30	6 54	A	265 58½	30 22	A
In manubrio ſiniſtræ manus	3	266 51	6 50	A	269 50	30 21	A
In Boreali parte arcus duarum Auſtralior	4	271 47½	2 00	A	271 59	25 30	A
Borealior in eadem parte arcus	4	268 41½	2 27½	B	268 36	21 03	A
In ſiniſtro humero	4	277 51	3 31	A	278 47	26 47½	A
Antecedens hanc in jaculo	5	275 40	3 50	A	276 22	27 13½	A
Trium in capite præcedens	4	278 56½	1 44½	B	279 37	21 28½	A
Media	4	280 28	0 59	B	281 18½	22 07	A
Ultima	4	281 43	1 31	B	282 36	21 29	A
Prima in contactu	6	283 44	3 06½	B	284 35	19 43	A
In Boreo contactu, media	4	284 54½	4 17	B	285 41	18 25	A
Sequens, & ſuperior (ſubjuncta	5	285 11	6 09½	B	285 46	16 32	A
Hac Orientalior duab. obſcuris forma △	6	290 08½	5 08	B	291 00½	16 55	A
Orientalis & ultima in ſuperiori contactu	6	293 52	5 12	B	294 49	16 17	A
Obſcura in inferiori contactu ad ortum	6	290 24	1 25	B	291 51	20 34	A
Obſcura in dextro cubito	6	287 26	3 08	A	289 21	25 29	A

Capricornus.

Borealis trium in cornu præcedente	3	299 18	7 02	B	299 58	13 29	A
Media	6	299 51	6 53	B	300 33	13 31	A
Auſtralis	3	299 31	4 41	B	300 40½	15 44	A
Nebuloſa ſuperius cornu præcedens	*Neb.*	298 08	7 16	B	398 45	13 29	A
Nebuloſa Occidentalis, baſis △ in fronte	*Neb.*	299 57	0 48½	B	301 58	19 26	A
Nebuloſa Orientalis	*Neb*	300 41	0 28	B	302 48	19 37	A
Suprema in eodem Triangulo	6	300 37	1 20	B	302 32	18 47	A
Nebuloſa præcedens in fronte	*Neb*	298 13	0 24	B	300 15	20 11½	A
In cervice duarum, Borea	6	303 49	3 25	B	305 20	16 11	A
Auſtralis	6	303 06	0 15	B	305 21	19 17	A
Præcedens in dextro genu obſcura	6	302 47	6 58	A	306 52	26 22	A
Sequens in ſiniſtro genu	6	303 28	9 02	A	308 10½	28 13	A
In ſiniſtro armo	6	307 13	8 08	A	311 55½	26 21	A
Infima in ventre (alvo	5	312 24½	6 56	A	317 01	23 46½	A
Sequens Borea duarum contiguarum ſub(	6	313 00	6 29	A	317 29	23 10	A
Trium in medio ventris Orientalior	6	310 23	4 25	A	314 08½	21 56½	A
Infima earum	6	308 31	4 27	A	312 13	22 29	A
Septentrionalis trium	5	308 18	3 01	A	311 35	21 10	A
Duarum in dorſo anterior	5	309 21	0 29	A	311 56	18 26	A
Sequens earundem in dorſo	5	313 07	1 16½	A	315 59	18 09	A
Antecedens duarum ad ilia	4	315 25	4 48	A	319 25	20 51	A
Sequens earundem.	5	317 06	4 49	A	321 07½	20 21	A

Duarum

Denominatio Stellarum.	*M*	*Longit.*	*Latit.*	*Pl.*	*Asc. Rect.*	*Declin.*	*Pl.*
Duarum lucidarum in cauda præcedens	3	317 14	2 26	A	320 29	18 02	A
Sequens	3	319 00	2 29	A	322 15	17 32	A
Antecedens in cauda ſuperiori	5	319 14	2 22	B	320 55	12 51	A
Reliquarum in ſuperiori cauda Auſtralis	5	321 27	0 14½	A	323 55	14 37	A
Præcedens hanc ad Septentrionem	6	321 16	0 10	A	323 43	14 37	A
Borea in extremo caudæ	6	320 54	4 17	B	321 48	10 02	A

Aquarius.

In capite	6	323 26½	15 23	B	320 46	0 48½	B
In humero dextro, clarior	3	328 49½	10 42	B	327 46	1 52½	A
Obſcurior & Auſtralior	5	327 36	9 11½	B	326 38½	3 42	A
In humero ſiniſtro	3	318 51	8 42	B	318 07½	6 56½	A
Quæ in dorſo, ſub axilla	5	319 38	6 00½	B	320 10	9 16	A
Sequens & inferior trium in ſiniſtra(	5	311 51	4 50	B	312 57	12 39	A
Media (manu	5	308 28½	8 19	B	308 43	10 10	A
Antecedens lucidior	4	307 12	8 10	B	307 30½	10 37	A
In cubito dextro	3	332 10	8 17½	B	332 30	2 59	A
In dextra manu Borealior	5	334 04½	10 31	B	332 09	0 14	A
Reliquarum duarum Auſtralium præ-(	4	334 23	8 52½	B	333 02	1 39½	A
Sequens (cedens	4	335 53	8 10	B	334 41	1 47	A
In cotyla dextra duarum præcedens	4	328 45	2 46	B	329 56	9 21	A
Sequens earum	6	329 31	2 29½	B	330 54	9 20½	A
In dextro femore	5	330 53	1 10	A	333 22	12 17	A
Quæ eſt ad clunes	4	324 13	2 00	A	327 14	15 23	A
Auſtralis in dextra tibia. Sheat,	3	334 22	8 10	A	339 22	17 31½	A
Borea, ſeu quæ ad genu eſt	5	334 05	5 37	A	338 06	15 16	A
In ſiniſtra coxa	6	330 40	5 40	A	334 50	16 34	A
In ſiniſtro genu, duarum Auſtralior	5	327 55½	10 48½	A	334 08	22 20	A
Borealior	6	330 50	9 57½	A	336 09	19 53	A
In effuſione aquæ, à manu prima	4	334 52	4 08½	B	335 12	5 53	A
Succedens Auſtralis	4	337 04	0 19½	A	338 56	9 15	A
Sequens in primo flexu aquæ	6	340 00	1 24	A	342 05	9 08	A
Quæ eam comitatur	5	342 38	1 00	A	344 23	7 46	A
In altero flexu Auſtrali	5	342 33	2 49	A	345 01	9 28	A
Præcedens & Borealior duarum ſequen-(	5	341 43	3 58½	A	344 41½	10 52	A
Sequens & Auſtralior (tium	5	342 11	4 10½	A	345 12	10 52	A
Propè hanc in Auſtrum declinans	5	342 14½	4 44	A	345 29	11 21½	A
Poſt hanc duarum contiguarũ præcedens	5	345 07	10 59	A	350 43	15 59	A
Sequens earundem contiguarum	5	345 38	11 33	A	351 27	16 18	A
In tertio aquæ flexu Borea trium	5	344 03	14 29	A	351 11	19 36	A
Media in tertio aquæ flexu	6	344 46	15 16½	A	352 13	20 02½	A
Sequens trium, & Auſtralis	6	345 44	16 23	A	353 36½	20 40	A
Sequentium trium Borealis	5	338 54½	14 45	A	346 27	21 51½	A
Media trium earundem,	5	339 21	15 30	A	347 12	22 22½	A
Auſtralis harum trium	5	340 50	16 31	A	349 03½	22 43½	A
In ultimo flexu trium ſuperior	5	335 25	14 25½	A	342 57	22 54	A
Media	5	335 02	15 40	A	343 07	24 11½	A
Infima, proxima Fomahant	5	334 17	15 53	A	342 29	24 40½	A
Ultima in effuſione : FOMAHANT	1	329 11½	21 00	A	339 46	31 17½	A

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Pisces.							
In ore Piscis austrini	5	344 02	9 04	B	341 49	2 04	B
Duarum in occipite, australis	4	346 50½	7 17½	B	345 03	1 30½	B
Borea in occipite	6	348 30½	8 54½	B	345 57	3 38	B
Præcedens duarum in dorso	5	350 42	9 03	B	347 53	4 37	B
Sequens in dorso	5	352 56½	7 13½	B	350 39½	3 49½	B
Præcedens in alvo	5	348 21	4 27	B	347 33	0 32	A
Sequens in alvo	5	352 05	3 25	B	351 24	0 27	B
In cauda	5	358 02	6 23½	B	355 44	5 09	B
Supra hanc ad ortum	6	359 27	7 27	B	356 31	6 36½	B
Sequens	6	3 29	5 28	B	1 00½	6 24	B
In lino australi lucidiorũ triũ præcedens	4	9 36	2 11	B	7 57	5 49	B
Earundem media	4	12 58	1 05½	B	11 30	6 08½	B
Sequens (cedens & Borea	4	15 19	0 57½	B	13 43	6 56	B
In flexu lini duarum exiguarum ante-(	6	13 25	1 31	A	12 56	3 55	B
Earundem sequens & austrina	6	14 46	4 19½	A	51 16	1 51	B
Post flexionem trium præcedens	5	18 33	3 03	A	18 16	4 28	B
Media	5	20 56	4 40½	A	21 07	3 45	B
Sequens ultima	5	22 57½	7 56	A	24 10	1 35	B
Lucidior in nexu amborum linorum	3	24 47½	9 04½	A	26 16½	1 11	B
In lino Boreo à connexu præcedens	5	23 12	1 38½	B	20 50	10 34	B
Post hanc trium australis	5	22 16	1 51½	B	19 52½	10 25	B
Media & lucidior in nexu Boreo	4	22 16	5 21	B	18 32	13 39	B
Borea trium & ultima in lino	5	22 36½	9 24	B	17 14	17 31½	B
Borea duarum in ore Piscis Borei	6	24 15	22 00	B	13 17	29 42	B
Australis	5	23 49½	20 43	B	13 30	28 22	B
Borealis Trianguli in capite	6	20 22½	20 55	B	10 05	27 12	B
Australis ejusdem Trianguli	6	19 06½	19 24	B	9 34	25 20	B
Media & antecedens Trianguli	6	18 03½	20 24	B	8 08	25 49	B
In australi spina trium præcedens propè(	5	18 56½	13 21	B	12 07	19 44	B
Media (sinist. cubitum Andromedæ	6	19 02½	12 21½	B	12 37½	18 52	B
Infima trium	6	19 09	11 21	B	13 09	17 59	B
In alvo, duarum Borea	5	24 18	17 26	B	15 27	25 33	B
Quæ magis ad austrum	5	21 58½	15 30	B	14 06	22 53	B
Sequens mediam trium in australi spina	5	20 00	12 27½	B	13 29	19 20	B
Sequens, Boreã in alvo, ad Septentrionem	6	25 11	18 31	B	15 54½	26 50	B
In occipite Borei Piscis	6	22 41	23 03	B	11 16	30 02½	B
Longimontanus habet	6	22 25	23 03	B	11 00½	29 56	B
Cete.							
Quæ in rostro	4	40 31	7 50	A	40 33	7 35	B
Lucida mandibulæ Ceti	2	39 47	12 37	A	41 21	2 48	B
Media in ore	3	34 53½	12 02½	A	36 37	1 50	B

Præ-

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl
Præcedens trium ad genam	3	33 02	14 32	A	35 44	1 07	A
Quæ infra oculum	4	32 54	5 52	A	32 42	7 00	B
Quæ est supra oculum	4	37 07	5 36	A	36 36½	8 38	B
In occipite	4	29 29½	4 19	A	28 57	7 17	B
In pectore quadrilateri præcedens Borea	4	25 09	25 17	A	32 34	13 47	A
Duarum inferiorũ præcedens ad austrum	4	25 32½	28 31	A	34 09½	16 39	A
Sequentium in pectore australis	4	29 11½	28 16½	A	37 10	15 15	A
Præcedens & Borealis	3	28 47½	25 58	A	35 58	13 12	A
In ventre media	4	13 25	25 01	A	22 18	17 41	A
Infima in ventre	4	14 50	31 04	A	26 10	22 41	A
Borea ventris	3	17 25	20 19	A	23 52	11 54½	A
Duarum lucidiorũ in dorso, Orientalior	3	11 42½	15 46½	A	16 58	9 52	A
Occidentalior earundem	3	7 11½	16 55	A	13 23	12 39	A
Borealis caudæ	3	356 23	10 01	A	0 43	10 37	A
Australis, seu lucida, caudæ	2	357 56	20 47	A	6 45	19 48½	A
Lucidã mandib. ad ortum sequ. informis	5	43 45	14 30	A	45 35	2 09	B
Boream ventris præcedens ad austrum	5	16 04½	21 55	A	23 20	13 52	A
Quæ in recta linea cum 3ª & 5ª Cap.	4	33 49½	9 12½	A	34 41½	4 09½	B

Orion.

Suprema trium conjunctarum in capite	4	79 11½	13 26	A	79 20	9 41	B
Occidentalior	5	79 06½	13 54	A	79 17	9 13	B
Tertia quæ ad ortum	5	79 33½	14 04½	A	79 45	9 04	B
Sequens seu lucidus humerus	2	84 12	16 06	A	84 23	7 18	B
Sinister, seu præcedens humerus	2	76 23	16 53	A	76 54	6 01	B
Sequens in sinistro humero	5	77 47	17 22	A	78 17	5 39	B
Quæ in dextro brachio	4	86 04½	14 51	A	86 10	8 37	B
In dextra ulna	6	89 30½	11 30	A	89 30	12 01	B
In manu dextra australior	4	88 23½	9 15	A	88 22	14 15	B
Præcedens in dextra	4	87 21	8 44	A	87 17	14 46	B
Proxima supremæ in dextra manu	6	88 22	7 20½	A	88 19	16 10	B
Suprema & ultima earum quæ in manu	6	89 08½	7 19	A	89 06	16 12	B
Præcedens duarum quæ in coloboro	5	84 09	3 12½	A	83 46	20 11	B
Sequens earundem (occasum	5	86 21½	3 21	A	86 08	20 07	B
Quæ est infra dextrum humerum ad (	5	79 56½	19 17½	A	80 29	3 53½	B
Ex duabus obscuris in dorso sequens	6	78 40	19 36½	A	79 19	3 29	B
Præcedens earundem	6	77 46	19 52½	A	78 29	3 09	B
Quæ ex quatuor in dorso præcedit	5	76 34	20 08½	A	77 23	2 47	B
In clypeo novem Borealissima	4	68 53	8 17	A	68 29	13 40	B
Secunda	4	69 48	9 07	A	69 31	12 58	B
Tertia	6	69 10	11 06	A	69 11	10 55	B
Quarta	4	69 00½	12 25½	A	69 13	9 35	B
Quinta	4	67 49	13 03½	A	68 11	8 12	B
Sexta	4	67 23	15 27	A	68 06	6 21	B
Septima	4	67 33	16 50	A	68 28½	5 01	B
Octava	4	67 58	20 02	A	69 21	1 55	B
Ultima	4	68 57	20 55½	A	70 24	1 10	B
Prima Balthei	2	77 50½	23 38	A	78 52	0 35	A

Denominatio Stellarum.	*M.*	*Longit.*	*Latit.*	*Pl.*	*Aſc.Rect.*	*Declin.*	*Pl*
Media	2	78 54	24 33½	A	79 55	1 26	A
Ultima	2	80 06½	25 21½	A	81 04	2 09	A
Quæ in manubrio enſis	3	75 37½	25 36½	A	77 03	2 44	A
Suprema trium in enſe	5	78 28	28 09½	A	79 48	5 31	A
Media enſis	3	78 24½	28 45	A	79 48	5 39	A
Auſtralis	3	78 27½	29 17	A	79 52	6 10	A
Præcedens duarum infrà enſem	4	77 20	30 37½	A	79 02	7 35½	A
Sequens duarum infrà enſem	5	79 23	30 38	A	80 48	7 27	A
Lucida in ſiniſtro pede. REGEL	1	72 17	31 11½	A	74 44	8 37	A
Quæ in ſiniſtro calcaneo	4	73 15½	29 53	A	75 25	7 13	A
Quæ in ſurâ ſiniſtri pedis	5	75 02	31 00	A	77 05	8 10	A
In genu dextro	3	81 49½	33 08	A	83 03½	9 49	A
Quæ ultimã balthei præcedit ad auſtrum	4	79 39	26 00½	A	80 42	2 50	A
Quæ ad dorſum eſt, hanc præcedens	6	75 34	19 40	A	76 24	3 10	B
Sequens duarum ſuper manubrium enſis	6	75 45	24 06	A	77 01	1 13½	A
Præcedens	5	74 59	23 32	A	76 15	0 44	A
In ſiniſtro latere ſuprà hanc	5	75 57	21 23	A	76 56	1 27	B
Sub brachio & ſcuto præcedens hanc	4	72 58	20 08	A	74 01	2 26½	B
Duarum in ſiniſtro latere præcedens	5	80 45	21 58	A	81 25½	1 16½	B
Sequens	5	83 25½	21 39	A	83 53	1 43½	B
Poſt hanc. Informis	5	85 10	22 57	A	85 33	0 29½	B
Superior trium in ſiniſtra manu	6	74 36½	11 45	A	74 39	10 57	B
Media	6	72 33½	13 08	A	72 47½	9 21	B
Auſtralis (præcedens	6	72 00	14 24	A	72 24½	8 01	B
Decem informium ſuprà Orionem (	4	89 44	29 31	A	89 46	6 00	A
Piſerus	4	119 44	29 31	A	115 53	8 38½	A
Sequens	4	93 43	29 49	A	93 15	6 20½	A
Suprà hanc	5	93 22	28 04	A	92 59	4 35	A
Præcedens trium in recta linea infer. (	4	92 08	18 47	A	92 02	4 43	B
Piſerus (ped. II	4	62 08	18 47	A	63 43	2 13	B
Media	4	93 58	15 56½	A	93 51	7 31	B
Piſerus	4	93 58	15 16	A	93 52	8 12	B
Borealis	4	95 50	13 15	A	95 46	10 09	B
Infra lineam rectam ad Auſtrum	5	93 58	18 24	A	93 46½	5 04	B
Suprà hanc ad Ortum	5	97 36	14 59	A	97 25	8 20	B
Præcedens duarum quæ ſuprà Canem(	4	98 14½	20 33	A	97 43	2 45	B
Sequens (majorem	4	105 00	22 47	A	103 48	0 01	B

Eridanus.

Quæ ad ſiniſtr.ped. Orionis in principio(	4	70 40	31 35½	A	73 24	9 11	A
Supra pedem Orionis in fluvio (fluvii	3	70 42	27 54½	A	72 56	5 32	A
Duarum aliarum ſequens	5	68 39	29 52	A	71 25	7 43	A
Præcedens	5	66 29½	27 51½	A	69 14	6 02	A
Sequens duarum ſuperiorum	4	64 45½	25 34	A	67 19	4 01	A
Præcedens earundem	4	62 14½	25 11½	A	65 00½	4 04	A
Poſt intervallum ſequens ex quatuor	3	49 18	33 13½	A	55 43	14 26½	A
Quæ præit hanc	4	46 22½	31 09	A	52 41	13 08	A

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Denominatio stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Quæ ad Septentrionem est, seu tertia (	3	46 07	28 46½	A	51 46	10 55	A
Quæ omnis quatuor antecedit(præcedens	3	43 45	27 47	A	49 27	10 34	A
Prima contiguarum Cete	3	34 00	24 34	A	40 08	11 01	A
Inter hanc & tertiam	4	36 36	23 58½	A	42 03	8 56	A
Tertia quæ sequitur	3	39 16	25 59	A	45 11	10 04	A
Præcedens & inferior	5	54 49	30 25	A	59 38½	10 33	A
Piserus	5	54 40	30 25	A	59 31	10 29½	A
Suprà hanc	4	54 53	27 32	A	59 01	7 44	A
Sequens	4	55 58	28 09½	A	60 06	8 07	A
Superior Orientalis	5	58 46	25 03	A	61 53	4 32½	A
Præcedens duarum inter Eridanum & (	4	47 25½	18 26	A	50 04	0 38½	A
Sequens Australis (Taurum	4	51 07	22 45	A	54 32	3 54½	A

Lepus.

Superior præcedentis auris	5	71 14½	34 34	A	74 18	12 04	A
Inferior ejusdem auris	5	71 20½	35 54	A	74 33	13 22	A
Superior sequentis auris	6	73 27	35 18	A	76 13	12 34½	A
Inferior sequentis auris	5	73 14	36 14	A	76 09	13 31	A
Quæ est in capite	5	70 49	39 04	A	74 34	16 35	A
Extrema anteriorum pedum	4	67 25½	45 00	A	72 52	22 49	A
Quæ in dorso seu medio corpore	3	76 49½	41 05½	A	79 35½	18 03	A
In armo sinistro	3	75 06½	43 57½	A	78 35	21 02	A
Australior duarum in posterioribus pe-(	3	80 21½	45 49½	A	82 32	22 33	A
Borealior earundem (dibus	3	82 36	44 18	A	48 20	20 56	A
Præcedens in dorso	4	81 26½	38 16	A	83 03	14 57½	A
Sequens in dorso	4	84 27½	37 40½	A	85 29	14 15	A
Ultima in cauda	4	87 22	38 26	A	87 52	15 32	A

Canis major.

In ore splendidissima, SIRIUS vocata	1	99 35½	39 30	A	97 42	16 14½	A
Quæ in fronte ad dextram aurem	4	110 01½	34 50	A	106 44	12 29	A
In media fronte	5	102 27	36 43	A	100 15	13 39	A
Quæ sub sinistra aure	3	105 06	38 02½	A	102 16	15 10	A
In collo	4	103 03	39 30	A	100 28	16 27½	A
In armo dextro anteriorum pedum	5	97 32½	42 12½	A	95 54	18 51	A
Quæ in extremitate pedis prioris	2	92 42½	41 18½	A	92 08	17 49	A
Quæ in dorso	5	106 30½	46 09½	A	102 23	23 21	A
Media in pectore	5	103 36½	46 39½	A	100 09	23 37	A
Quæ in ventre	3	108 55	48 30	A	103 48	25 53	A
In ventre inter posterioram femora	3	106 21½	51 24½	A	101 32	28 33	A
Inferior dextri pedis priorum	3	92 07	51 46½	A	91 29	28 16	A
Quæ in cauda	3	115 11½	51 24½	A	107 45	29 26	A

Canis minor, Procyon.

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
In collo	3	107 39½	13 33½	A	107 22	8 54	B
In femore: PROCYON	2	111 18½	15 57	A	110 34	6 03	B
Suprà lucidam colli	6	107 49	12 51	A	107 37	9 35	B
Informis suprà hanc	6	107 42½	9 46	A	107 53½	12 39	B
Sequens ad caudam Cancri	5	111 57½	10 19½	A	112 03	11 28	B

Argo Navis.

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Quæ in suprema puppi	3	126 53½	43 18½	A	118 25	23 22½	A
Suprema clypei navis	3	121 35½	44 58½	A	113 57	24 03½	A
Præcedens clypei	3	119 00	47 28	A	111 24	26 05	A
In velo	4	125 06½	32 07	A	119 53½	12 11	A
Informis ad Austrum	4	125 27	38 31	A	118 35	18 28	A
In malo trium inferior	6	133 26½	32 56	A	126 39	14 48	A
Suprà hanc	4	133 51½	30 18	A	127 46	12 22	A
Hàc ipsa altior	4	131 01½	24 29½	A	126 55	6 06	A
Duarum in Antenna præcedens	4	150 26	21 39½	A	144 54	8 53	A
Sequens	3	155 20½	22 29½	A	148 55½	11 22	A
Informis inter velum & lacteam	3	174 44	30 30	A	162 14	25 43	A

Hydra.

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Præcedens in capite	5	126 39½	14 37	A	125 25	4 30	B
Suprà primam ad Aquilonem	4	127 46	14 16½	A	126 33	4 31	B
Borealior in occipite	4	127 48	11 08	A	127 21	7 36	B
Quæ tertiam ad Austrum præit	5	128 22½	11 36	A	127 47	7 01	B
Omnium in capite Orientalior	4	130 00½	11 01	A	129 30	7 10	B
Quæ in collo præcedit	6	132 51½	11 05½	A	132 12	6 21	B
Sequens in eductione colli	4	135 41½	13 05	A	134 17	3 40	B
Media colli, & præcedens trium in nexu	5	141 11½	15 00	A	138 49½	0 15	B
Borea trium in flexu colli	4	143 04	14 17½	A	140 46	0 21	B
Australis in nexu	5	140 53½	16 46	A	138 00	1 21	A
LUCIDA HYDRÆ, sive COR	1	142 45½	22 24	A	137 54	7 15	A
Quæ proxime Cor sequitur	4	148 12	26 33½	A	141 14	12 49	A
Quæ hanc deinde sequitur	5	151 09	26 12	A	143 54	13 25	A
Præcedens ex duabus contiguis supra hanc	5	153 48	23 13	A	147 18½	11 31	A
Sequens earundem	4	154 53	21 51	A	148 46	10 36½	A
Quæ à corde quinta est	4	160 31½	24 38	A	152 36	15 09	A
Quæ in recta linea cum hac & sequente	5	163 41½	23 31	A	155 48½	15 15	A
Cratera proximè præcedens	4	165 51	21 48½	A	158 24½	14 29	A
Informis, caput proximè præcedens	4	125 45½	12 27	A	125 05	6 48	B
Sub basi Crateris, Borealis	4	174 01½	25 36	A	163 55½	21 01	A
Australis	5	174 49	30 17	A	162 24	25 33	A
Sub cauda Corvi	3	202 24	13 43	A	195 15	21 25	A

Hanc

Denominatio Stellarum.	M.	Longit.	Latit.	Pl.	Asc. Rect.	Declin.	Pl.
Hanc præcedens parvula	6	200 24	14 37	A	192 57	21 28	A
Informis ante caput Hydræ	3	119 44	10 19	A	119 43	10 11	B

Crater.

Quæ est in basi Crateris	4	169 13	22 41	A	161 00	16 33	A
Sequens duarum in medio	4	174 43	19 39	A	167 10	15 53	A
Præcedens earundem	4	172 10½	17 25	A	165 50	12 52	A
Præcedens duarum supra Craterem	4	171 27	13 10	A	166 56	8 42	A
Earum sequens	4	174 02	11 17	A	170 01½	7 59	A
Præcedens duarum inferiorum	4	179 30	18 10	A	171 51	16 25	A
Piferus facit	4	179 30	18 16	A	172 30	16 30	A
Sequens	4	181 33	16 02	A	174 52	15 17	A
In medio Cratere	5	175 55	14 09	A	170 33½	11 20	A

Corvus.

Quæ ad oculum	4	187 08	19 39	A	177 44½	20 48	A
Præcedens duarum superiorum in □	3	186 13	14 25	A	179 49	15 39	A
Sequens earundem	3	188 55	12 07	A	183 16	14 39	A
Sequens inferiorum in quadrato	3	192 49	17 59	A	184 20	21 33	A
In rostro	4	187 38	21 46	A	177 50	21 54½	A
In collo	5	189 14	18 14	A	180 55	20 21	A
In sinistrâ alâ suprà lucidam	5	189 21½	11 28	A	183 57	14 14	A

Centaurus, Chiron.

In capite de quatuor australissima	5	212 27	21 49	A	201 30	32 39	A
Quæ magis in Boream	5	212 59	19 08	A	202 16	30 01	A
Intermediarum duarum præcedens	5	211 12	20 51	A	200 41	31 18	A
Sequens & reliqua de quatuor	5	212 20	20 12	A	201 51	31 01	A

FINIS.

A Table of the Suns Right Ascension for every Degree of the Ecliptic.

Deg.	♈ D	′	♉ D	′	♊ D	′	♋ D	′	♌ D	′	♍ D	′	♎ D	′	♏ D	′	♐ D	′	♑ D	′	♒ D	′	♓ D	′
0	0	0	27	54	57	48	90	0	122	12	152	6	180	0	207	54	237	48	270	0	302	12	332	6
1		55	28	51	58	51	91	5	123	14	153	3	180	55	208	51	238	51	271	5	303	14	333	3
2	1	50	29	49	59	53	92	11	124	16	154	1	181	50	209	49	239	53	272	11	304	16	334	1
3	2	45	30	46	60	56	93	16	125	19	154	58	182	45	210	46	240	56	273	16	305	19	334	58
4	3	40	31	44	61	59	94	22	126	20	155	54	183	40	211	44	241	59	274	22	306	20	335	54
5	4	35	32	42	63	2	95	27	127	22	156	51	184	35	212	42	243	3	275	27	307	22	336	51
6	5	30	33	40	64	6	96	32	128	24	157	48	185	30	213	40	244	6	276	32	308	24	337	48
7	6	25	34	38	65	9	97	38	129	25	158	44	186	25	214	39	245	9	277	38	309	25	338	44
8	7	21	35	37	66	13	98	43	130	26	159	40	187	21	215	37	246	13	278	43	310	26	339	40
9	8	16	36	36	67	17	99	48	131	27	160	37	188	16	216	36	247	17	279	48	311	27	340	37
10	9	11	37	34	68	21	100	53	132	28	161	33	189	11	217	34	248	21	280	53	312	28	341	33
11	10	6	38	33	69	25	101	58	133	28	162	29	190	6	218	33	249	25	281	58	313	29	342	29
12	11	1	39	33	70	29	103	03	134	29	163	25	191	2	219	32	250	29	283	3	314	29	343	25
13	11	57	40	32	71	34	104	08	135	29	164	20	191	57	220	31	251	34	284	8	315	29	344	20
14	12	52	41	31	72	38	105	13	136	29	165	16	192	53	221	31	252	38	285	13	316	29	345	16
15	13	48	42	31	73	43	106	17	137	29	166	12	193	48	222	31	253	43	286	17	317	29	346	11
16	14	44	43	31	74	47	107	22	138	29	167	7	194	44	223	31	254	47	287	22	318	29	347	7
17	15	40	44	31	75	52	108	26	139	28	168	3	195	40	224	31	255	52	288	26	319	28	348	[illegible]
18	16	35	45	31	76	57	109	31	140	27	168	58	196	35	225	31	256	57	289	31	320	27	348	58
19	17	31	46	32	78	1	110	35	141	27	169	54	197	31	226	32	258	2	290	35	321	27	349	54
20	18	27	47	32	79	7	111	39	142	26	170	49	198	27	227	32	259	7	291	39	322	26	350	49
21	19	23	48	33	80	12	112	43	143	24	171	44	199	23	228	33	260	12	292	43	323	24	351	44
22	20	20	49	34	81	17	113	47	144	23	172	39	200	20	229	34	261	17	293	47	324	23	352	39
23	21	16	50	35	82	22	114	51	145	22	173	35	201	16	230	35	262	22	294	51	325	22	353	35
24	22	12	51	36	83	28	115	54	146	20	174	30	202	12	231	36	263	28	295	54	326	20	354	30
25	23	9	52	38	84	33	116	57	147	18	175	25	203	9	232	38	264	33	296	57	327	18	355	25
26	24	6	53	40	85	38	118	1	148	16	176	20	204	6	233	40	265	38	298	1	328	16	356	20
27	25	2	54	41	86	44	119	4	149	14	177	15	205	2	234	41	266	44	299	4	329	15	357	15
28	25	59	55	44	87	49	120	7	150	11	178	10	205	59	235	44	267	49	300	7	330	11	358	10
29	26	56	56	46	88	55	121	9	151	9	179	5	206	56	236	46	268	55	301	9	331	9	359	5
30	27	54	57	48	90	0	122	12	152	6	180	1	207	54	237	48	270	0	302	12	332	6	360	0

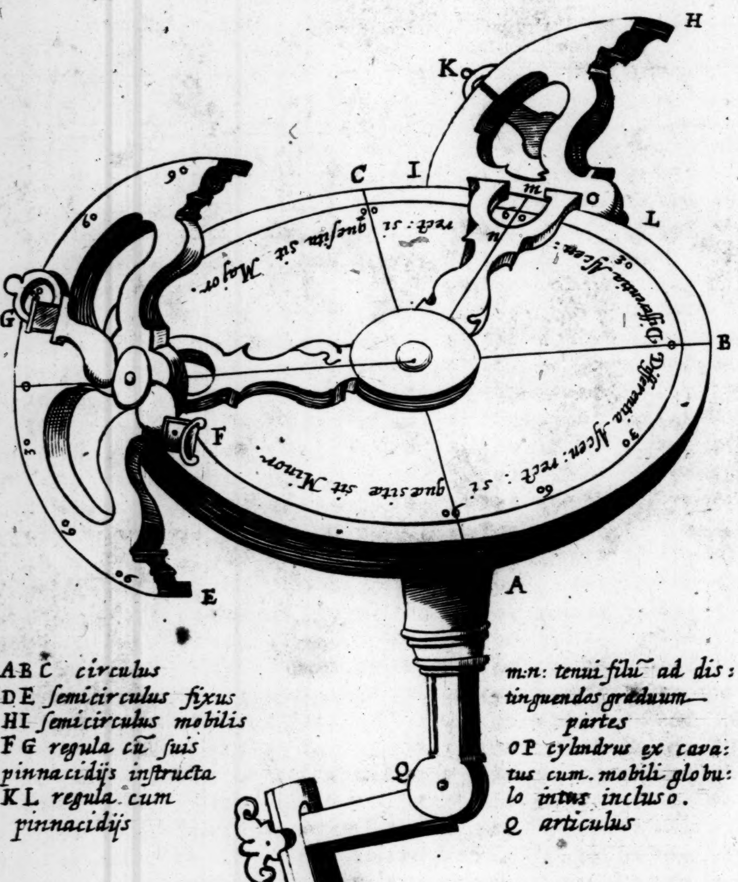
To find at what Time any Star (mentioned in the former Catalogue) will come to the South.

Subtract the Right Ascension of the Sun, from the Right Ascension of that Star whose time of coming to the Meridian is required, the remainer converted into Hours and Minutes, is the time of the Stars coming to the Meridian afternoon. But if the Right Ascension of the Star be lesse then the Right Ascension of the Sun, adde 360 degrees thereto, and subtract the Right Ascension of the Sun from that sum, and the remainer converted into Hours and Minutes, is the time of the Stars coming to the Meridian.

EXAMPLE.

October the fourth 1659, the Sun is in neer 21 Degrees of *Libra*, on which day, I desire to know when SIRIUS comes to the Meridian, by the the Catalogue you shall find the Right Ascension of SIRIUS to be 97 Degrees 42 Minutes, and the Right Ascension of the Sun for the 21th degree of *Libra* is 199 Degrees 23 Minutes. Now (because the Right Ascension of the Star, is lesse then the Right Ascension of the Sun) adde thereto 360 deg. and the sum will be 477 Degrees 42 Minutes, from which subtract the Right Ascension of the Sun 199 Degrees 23 Minutes and there remains 278 Degrees 19 Minutes; which converted into time is 18 Honrs 33 minutes afternoon, that is, at 33 minutes past 6 the next morning.

FINIS.



Pag I Astroscopium

ASTROSCOPIUM.

DE ASTROSCOPIO.

Usus ejus in genere hic est.

UT quævis in Catalogo præcedenti stella fixa dignoscatur In Cœlis: quarum ascensiones rectæ, & declinationes huic fini, ut uni expræcipuis supputantur.

SUPPOSITA,

1 Cognita supponitur stella polaris (seu potius punctum in Cœlis polare) & alia aliqua insuper stella à polo magis distans. Punctum polare ope stellæ polaris expeditissimè inveniatur, quæ non plus gradibus duobus cum triginta septem minutis distat à polo Boreo: hic vero inter Allioth (seu Radicem caudæ majoris Ursæ, & stellam Polarem penè situs est. Concipias igitur lineam Rectam à polari ad Allioth ductam, & imaginatione distinguas duas tertias distantiæ, proximæ stellæ in cauda Ursæ minoris à polari versus Allioth; ibi enim est ipsissimum punctum polare. Ut in schemate adjuncto pateat.

2 Declinationes, & Ascensiones Rectæ stellarum, tam cognitarum, quam incognitarum dantur.

3 Quod observator ita se disponat, ut instrumentum inter ipsum, & stellas semper locetur.

4 Quod stella quæ majorem habet Ascensionem Rectam versus siuistram posita est, in cœlis; quæ vero minorem habet Ascensionem Rectam sita est versus dextram. Unde sæpius in hoc casu 00 sumitur pro 360, 10 pro 370, 20 pro 380, &c.

5 Quod Ascensiones Rectæ duarum stellarum (cognitæ scilicet, & incognitæ non plus centum gradibus differant. Nam licet hoc non sit absolute necessarium valdè tamen est expediens. Sin vero differentia sit major assumenda est stella aliqua intermedia cujus ope quod quæritur inveniri potest.

6 Quod Semicirculus fixus ad sinistram, mobilis vero ad dextram positus sit.

7 Quod Semicirculus fixus ad cognitum, mobilis autem ad sidus incognitum dirigatur.

8 Quod ambarum stellarum Declinationes juxta graduationes, & denominationes in Semicirculis numerentur.

I Quomodo articulus debite disponatur ad usum.

Hac de causa duplici instruitur motu, altero super bacilli summitate, altero in ipsius articuli vertice.

Primo igitur, (bacillo firmiter in terram fixo) immobilis Semicirculi indicem super duos polos ad 90, & 90 pone; & Semicirculi hujus index sit paulò cæteris elatior, ut ad pinnacidia, quorum usus est, commodius diveniatur. Deinde instrumentum ope duplicis motus (ad bacilli summitatem & articuli verticem) move, & labora, usque dum fixi Semicirculi pinnacidia punctum polare directè aspiciant. Hoc facto, excavatum cylindrum in baculi summitate versatilem cochleâ sua sic figas ne amplius divagetur.

Hoc totum opus est, ut instrumentum ad observationes sub dio faciendas debite disponatur. Sin vero cylindrus, aut tale quid instrumento adaptetur, super quem verti possit, & loco idoneo (super firmum puta tignum, aut fenestræ transversarium) juste figatur, ut polum Boreum aspiciat, hâc rectificatione opus non erit.

II Quomodo observandum sit.

1 *Dirige duos indices, juxta stellæ declinationem, per septimum, & octavum suppositum.*

2 Si ascensio recta stellæ incognitæ sit $\left\{\begin{matrix}\text{major}\\\text{minor}\end{matrix}\right.$ *ascensione rectâ stellæ cognitæ; numera differentiam ascensionum recta-*

rectarum super 100 Æquinoctii gradus qui tibi sint {remotiores / proximiores} & cum mobilem Semicirculum istic posueris ibi maneat absque ulteriori alteratione. Deinde Æquinoctialem, aut maximum Circulum (qui duos reliquos secum portabit) move, donec per fixi Semicirculi pinnacidia in stellam cognitam collimaveris, & sic maneat.

Festina deinde, & mobilis Semicirculi pinnacidia aspice, quæ oculum ad requisitam stellam dirigent.

ULTERIOR ASTROCOPII USUS.

Quando differentia Ascensionum Rectarum duarum stellarum fuerit inter 90 & 180 gradus.

1 Quomodo minuenda erit differentia Ascensionum Rectarum duarum quarumlibet Stellarum ne excedat 180 grad. & quomodo dignoscendum sit utra illarum ad dextram, utra vero ad sinistram adparebit.

1 *Subduc Ascensionem rectam minorem ex majori. Si residuum 180 gr. non superaverit; stella ascensionis minoris ad sinistram, majoris ad dextram sita est.*

2 *Si residuum superet 180 gr. hujus residui supplementum ad 360 sumendum est pro ascensionum rectarum differentia. Et in hoc casu stella quæ minorem habet ascensionem rectam apparebit ad sinistram, quæ majorem ad dextram videbitur.*

2 Quomodo Ascensionum differentia super duos Æquinoctii quadrantes numeranda est.

1 *Cum differentia minor sit 100 gradibus numeranda est juxta numerationis ordinem in quadrantibus respective descriptam.*

2 *Cum differentia superat 100 gr. deinde numera 90 primos gradus ordine directo ut antea: postea ordine retrogrado à 90 ad 80 æstima pro 100 grad. ad 70 pro 110, ad 60 pro 120, ad 50 pro 130, 40 pro 140, 30 pro 150, 20 pro 160, 10 pro 170, 00 pro 180.*

3 Quo-

3 Quomodo duo Semicirculi ad quamlibet ascens. Differentiam, indices vero ad declinationes rectificandi sunt: & quomodo instituenda est observatio.

1 *SVpponitur instrumentum ad polos mundi rite positum per præceptum generale primum.*

2 Sit Semicirculus fixus semper ad sinistram cujus officium est ut stellam notam aspiciat: ita mobilis Semicirculi ad dextram usus est ut te dirigat ad stellam ignotam.

3 Apponantur indices ad stellarum declinationes in Semicirculis numeratas. Fixi scilicet ad declinationem Boream aut Meridionalem juxta titulos inscriptos. Mobilis autem Semicirculi index admoveatur declinationi stellæ requisitæ, quæ tamen titulis declinationum inscriptarum æstimanda est contraria.

4 Si sidus notum sit ad {sinistram / dextram}; numeretur differentia ascensionum rectarum super istum Æquinoctii quadrantem qui tibi {remotior / proprior} est, & mobilis Semicirculi indicem illic fige.

Gira deinde Circulum Æquinoctialem (cum duobus reliquis adnexis) donec per fixæ regulæ pinnacidia in cognitam stellam, uti communiter fit collimaveris. Subito postea quàm poteris adi mobilem Semicirculum, & ad latus adversum te siste (ita scilicet ut mobilis Semicirculus qui antea erat ad dextram, nunc ad sinistram sit) tunc aspecta pinnacidia ad stellam requisitam visum tuum dirigent.

FINIS.

ASTROSCOPIUM.

Concerning the ASTROSCOPE.

The use of it in general, is,

TO make known every fixed Starre in the Heavens, so farre as the Catalogue of Starres reacheth, whose right ascensions therefore, and declinations are calculated for that, as for one chiefe, purpose.

In the use of it, these things are presupposed.

1 THat you know the Pole Starre (or rather the Pole-point of the heavens) and some one Star besides, which is further distant from the Pole. The Pole-point may be the readiest way found by the North-star, which is within 2 deg. 37 minutes of the North-Pole: which lies very neer between *Allioth*, (or the root of the Great Bears tail) and the North-star: therefore, if you conceive a right line drawn from the Pole-star to *Allioth*, & by your imagination suppose two third parts of the distance of the next Star of the Little Bears tail from the Pole-star towards *Allioth*, for there is the very Pole-point.

2 That the Declinations and right ascensions, both of the known and unknown Stars, are given.

3 That you take your standing so, as the Instrument may alwayes be placed between you and the Sars.

4 That that Sarre, which hath greater right aſcenſion, is (in the heavens) towards your left hand, and that which hath leſs right aſcenſion is toward your right hand. And many times in this caſe, 00 muſt be taken for 360, 10 for 370, 20 for 380, &c.

5 That the 2 Starres (known and unknown) be not above 100 degrees differing in right aſcenſion. Though this be not neceſſary abſolutely, yet it is moſt expedient ſo to be. And if their difference be more, then muſt the help of ſome intermediate Starre be uſed, by meanes whereof you may come to find that which you look for.

6 That the fixed Semicircle ſtand on your left hand, and the moveable Semicircle on your right hand.

7 That the fixed Semicircle be put upon the known Star, the moveable upon the unknown.

8 That the declinations of the two Starres be counted according to the graduations and denominations upon the two Semicircles.

I *How to place the joint in a true poſition.*

FOr this purpoſe, you have two motions; one upon the head of the ſtaffe, the other upon the joint head.

Firſt, therefore, when your ſtaffe is firmly placed down, ſet the index of the fixed Semicircle upon the two Poles at 90 and 90, and let that index and Semicircle lie above the reſt, ſo, as you may moſt conveniently come to make uſe of the ſights. Then work the Inſtrument upon the two motions (at the head of the ſtaffe, and joint head) until you have punctually directed the two ſights of the fixed index, upon the Pole-point in the Heavens. When this is done, ſcrew the ſocket upon the head of the ſtaffe, ſo as not to ſtirre any more.

This work is to ſet the Inſtrument in a fit poſture for obſervation in the open aire. But if you have a Cylinder on purpoſe fitted for the Inſtrument to turn upon, and juſtly fixed in ſome convenient place (either fixed poſt or window) in ſuch wiſe that it may point up into the North-pole, then there will need none of this rectification here mentioned.

11 *How to observe.*

1 SEt the two Indexes according to the Starres declinations, by the seventh and eighth before.

2 If the Starre required be {more / less} in right ascension then the known Starre, Count the difference of their right ascensions upon the 100 degrees of the Equinoctial that are {furthest from / neerest to} you, and when you have thereunto placed the moveable Semicircle, let it so remain without any further alteration. Then turn the Equinoctial or great Circle (which carries upon it the two other Semicircles) till you may see the sights of the fixed Semicircle, upon the known Star, and there let it stand. After this you must instantly look upon the sights of the moveable Semicircle, & by direction of them you shall find the Star which you look after: for they will guide your eye upon it.

The further use of the ASTROSCOPE,

Where the difference of the right ascensions of the two Starres, is between 90 deg. and 180 deg.

1 *How to make the difference of the right ascensions of any two Stars, less then 180 deg. and to know which of them appears towards the right hand, and which to the left.*

SUbduct the lesser right ascension out of the greater. And 1, If the remainder be lesse then 180 deg. then the Starre of least right ascension is toward the right hand, and that of greatest right ascension is towards the left.

2 If the remainder be greater then 180 deg. then you must take the residue of that remainder to 360, and that must be counted for the difference of right ascensions. And in this case the Star of least right ascension will appear towards the left hand, and the greater towards the right hand.

2 *How to count the difference of right ascensions upon the two Quadrants of the Equinoctial Circle.*

1 WHen the difference is lesse then 100 deg. count it according to the order of numeration upon the Quadrants respectively.

2 When the difference of right Aſcenſions is more then 100 deg. then count 90 deg. the right way, as before: and from thence count back again from 90 to 80 as 100: to 70 as 110: to 60 as 120: to 50 as 130: to 40 as 140: to 30 as 150: to 20 as 160. to 10 as 170: to 0 as 180.

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3 *How to rectiſie the two Semicircles to any difference of right Aſcenſions; and their two Rulers to the Declinations of two Stars; and how to make your Obſervation.*

1 IT is ſuppoſed that the Inſtrument is rightly ſeated to the Pole of the World, by the former general direction given: the Axis thereof being levelled directly upon the Pole-point.

2 Let the fixed Semicircle be on your left hand alwayes: & let the office of it be, to look upon the known ſtar. And ſo on the other ſide, let the moveable Semicircle ſerve to direct to the unknown ſtar, and keep it on your right hand.

3 Set the Rulers to the ſtars declinations counted in the ſemicircles: that of the fixed ſemicircle, to the declination of the known ſtar, according to the titles of North and South declinations thereon inſcribed: But ſet the Ruler of the moveable ſemicircle, to the declination of the required ſtar, counted contrary to the titles of Declinations written thereon.

4 If the known ſtar be on the {left / right} hand of you; count the difference of right aſcenſions upon the Quadrant (of the Equinoctial) {remoteſt from / neereſt to} you, and to that place ſet the Index of the moveable ſemicircle, and let it not be thence ſtirred. Then turn the Equinoctial Circle (with the two ſemicircles faſtned upon it) till, by the fixed Ruler in the ordinary way of collimation, you may ſee the known ſtar. Then preſently goe to the moveable ſemicircle, and ſtanding on the other ſide of it (that is, ſo as the ſaid moveable ſemicircle may be on your left hand; whereas, before, it was upon your right hand) and looking to the ſights, you ſhall find them to point you upon the ſtar which you require.

FINIS.

DE INSTRUMENTIS PLANETARIIS.

Cui usui inserviunt, & quomodo sunt tractanda.

A SAMUELE FOSTERO, *olim Astronomiæ Professore in Collegio* Greshami, *Londini.*

OF THE PLANETARY INSTRUMENTS.

To what end they serve, and how they are to be used.

By SAMUEL FOSTER, *sometime Professor of Astronomie in* Gresham *Colledge*, *London.*

LONDINI,

Ex Officina LEYBOURNIANA.

M. DC. LIX.

DE

INSTRUMENTIS

PLANETARIIS.

Cui usui inserviunt, & quomodo sunt tractanda.

1 *Ad quod Systema Mundi fabricentur, & quibus Planetis accommodentur.*

HÆ Theoricæ ad Hypotheses *Copernicanas* instituuntur, in quibus cum Sol Centrum Mundi possideat, hujus apparentes motus, realiter existunt in terra. Unde hæc loco Solis inter septem Planetas numeratur.

De quinque tantum ex his septem eorumque locis investigandis hic dicemus. Nam Lunæ motus, & passiones quas conjunctim habet cum terra, quia plures reliquis admittit varietates non nisi per instrumentum particulare commode absolvi nequeunt, quare Lunam hic missam facimus.

Rursus locus terræ in his Theoricis non tam sui ipsius quam aliorum Planetarum causa requiritur; quorum loca in Zodiaco deprehendi nequeunt, nisi

OF THE

PLANETARY INSTRUMENTS.

To what end they serve, and how they are to be used.

1 To what Systeme of the world these Theorics are framed & to what planets they serve.

THese Theorics are framed according to Copernicus *his Hypothesis: in which the Sun is supposed to be in the Center of the World, and those motions that are apparently in the Sun, to be really in the Earth. And so the earth, in the Suns roome comes to be numbred among the 7 Planets.*

Of these 7 we shall properly enquire after the places of five onely. For, the perfect absolution of the Moones motion, and passions jointly with the Earth, being of more varieties then the rest, will require an Instrument alone, and so the Moon is dismissed hence.

Again, the earths place is required in these Theorics, not so much for it self, as for the other five Planets, whose places in the Zodiac

nisi prius in qua mundi parte terra sit (hoc est nos ipsi simus) dignoscatur. Interim tamen verus terræ locus respectu Ecclipticæ, & per consequens apparens solis, modo requiratur, hic inveniri poterit. Uti postea in octavâ Propositione indicabitur.

diac cannot be had in respect of us, unlesse we first know in what part or place of the World the earth (that is, our selves upon the earth) do stand. Yet the true place of the earth in respect of the Ecliptick, & consequently the apparent longitude of the Sun, may here likewise be found, when at any time it shall be required, as is shewed afterwards in the 8th Proposition.

2 *Quomodo tempus omne calculo accommodetur.*

UT tempus calculo accommodetur hæc sunt observanda.

1 Omnes motus colligendi sunt ad tempora completa.

2 Dies inchoatur in suo meridie completur vero in meridie die sequentis. Ita quod,

3 Meridies primi diei Januarii est terminus communis veteris, & novi Anni: periodus (sc.) præcedentis, & principium Anni sequentis.

2 How all time is to be fitted for computation.

FOr the accomodation of time to calculation, we may observe these things.

1 *All motions are to be collected for complete times.*

2 *A day begins upon its own noon, and ends upon the noon of the next day. So that,*

3 *The noon of the first day of January is the common term of the old and new years, being the end of the former and the beginning of the latter.*

3 *Quid sit locus Planetæ, cum methodo colligendi æquales Anomalias.*

HÆ Theoricæ, uti antea dictum est, præcipue instituuntur ad expeditam inventionē locorum *Saturni, Jovis, Martis, Veneris,* & *Mercurii*, a cujusque diei meridiem & in formâ quâ nunc sunt ad annum septingentesi-

3 What the place of a Planet is, with the manner of collecting the equal Anomalies.

THese Theorics (as is said before) do especially concern the 5 Planets, Saturn, Jupiter, Mars, Venus, *&* Mercury, *& are intended for the speedy finding out of their places for every day at noon. They will serve as they are now*

tesimum supra millesimum absque sensibili errore inservient.

Locus Planetæ est ejusdem situs ad planum Eclipticæ respectu longitudinis in illâ, latitudinisque ab eadem. Cui etiam intervallum seu distantia Planetæ á terra addi poterit.

Ad hæc invenienda primo dignoscendum est quænam tempori dato debeatur Anomalia tam terræ, quam Planetæ cujus locus inquiratur. Hæ vero Anomaliæ ex propriis Tabulis orbitæ cujusque Planetæ annexis excerpenda. Numerique Tabulares pro gradibus graduumque partibus centesimis æstimandi sunt.

His præmissis modus colligendi æquales Anomalias hujusmodi est.

Primo, Exscribe Epocham anni proxim præcedentis.

2 Sub ista Epochâ, seu numero scribe motus competentes tot annis, mensibus, & diebus quot ab anno Epochæ completis sint, hi ex propriis Tabulis sunt sigillatim sumendi, & invicem ordinatim subjicendi: quod ut fiat numerorum disunctio satis docebit.

3 Horum

now framed, till the year 1700 without any notable alteration.

The place of a Planet is the situation of it to the plain of the Eclipticke, in respect of longitude therein, and latitude therefrom. To which also may be added the interval or distance of it from the Earth.

To find these things, we must first know, what Anomaly is due, for the time assigned, both to the earth, and likewise to the Planet whose place is required. These are severally to be gathered out of their proper Tables, annexed to every Planets Orbit. And the numbers in those Tables are to be esteemed for degrees and centesimal parts of degrees.

The manner of collecting the equal Anomalies is this.

First, Exscribe the Epocha which belongs to that year, wch most neerly precedeth the year wherein you seeke the place of any Planet.

2 Under that Epocha or number, write the motions belonging to so many years, moneths, and dayes, as are completely expired since the year of the Epocha. Each of these numbers must be taken out of their proper Tables, & set orderly one under another which the disjunction of the numbers will give direction enough to doe.

3 All

3 Horum aggregatum dabit Anomaliam quæsitam, sin vero excedat circulum seu 360 gr. integer circulus quoties poterit rejiciendus est, & residuum sumendum pro Anomalia.

Hæc tam pro terra quam Planeta sigillatim facienda sunt. Qua de causa Anomaliæ terrestris Tabula bis repetitur, ut scilicet in quaque lamia semel in promptu sit, pro singulari instrumenti faciebus quæcunque illarum in usum venerit, & sine qua nec Planetæ locus, nec passiones aliquot quibus subjicitur inveniri possunt.

Sequitur jam

1 Longitudinem Planetæ in Ecliptica investigare,

2 Latitudinem ab Ecliptica investigare.

Huc rei centro instrumenti, hoc est centro Solis filum appendendum est. Insuper comparanda est tenuis e metallo regula cum linea fiduciali ejusdem (aut circiter) longitudinis cujus est diametrus instrumenti. Quæ solute sit oportet & mobilis nullo modo alligata, sed datis duobus quibuslibet instrumenti punctis applicabilis.

4 Cujus-

3 All these numbers must be added into one, and their summe shall give the Anomaly for the time assigned. If the sum rise to be above a Circle or 360 d. you must then cast away the said number of 360 as oft as you may, and the remaining number must be taken for the Anomaly.

These thinges are to be done both in the Earth and Planet severally. And for that purpose the Table of the Earths Anomaly is twice set down upon each plate once; that which soever of the plates you are to use, you may have the earths Table at hand: without which neither the Planets place, nor some of the passions thereto belonging can be found. Now it follows to be shewed,

1 How to find the Longitude of a Planet in the Ecliptic.

2 How to find the Latitude of a Planet from the Ecliptic.

And for this purpose you must have a thread fixed to the Center of your plate, which is the Center of the Sun. And besides, there must be a thin plate-ruler, with a streight or fiducial edge, of such length as may be neer about the Diameters of the plates. It must not at all be fastened to them, but be separate and loose, that it may be applyed to any two points prescribed upon the superficies of the plates.

4 How

4 *Cujuslibet e quinque Planetis longitudinem invenire.*

1 COllige Anomalias tam terræ, quam Planetæ cujus Longitudo inquiritur ex propriis Tabulis, uti antea præceptum est.

2 Numera Anomaliam Planetæ in Orbita ipsius, Anomaliam terræ super illam terræ Orbitam quæ in eadem instrumenti facie, qua etiam est Planetæ Theorica describitur. Hæc duo puncta observa nam in illis erit & Planetæ & terræ locus pro dato tempore.

3 His punctis lineam regulæ fiducialem ita applicabis ut eadem regulæ linea, & Solem respiciat, & limbum seu Zodiacum secet, vel prætergrediatur prout ratio postulet, & disponatur major ejus portio á terra versus Planetam, sæpius enim ad operationes sequentes illud requiretur.

4 Per circinum cape minimam distantiam inter Centrum Solis, & lineam regulæ fiducialem, & invariatâ aperturâ fige pedem unam super aliquem Zodiaci exterioris sive limbi gradum in eodem regulæ latere in quo erat Solis Centrum, & versus eam Zodiaci plagam

4 How to find the longitude of any of the 5 Planets.

1 G*Ather the Anomalies of the Earth and of the Planet whose longitude is required, each out of their own proper Tables: in such manner as was before shewed.*

2 *Count the Planets Anomaly upon the Planets Orbit, & the Earths Anomaly upon that Orbit of the earth which is drawn upon the same side of the plate with the course of your Planet, and observe these two points, for in them are the places of the earth and Planet, for the time assigned.*

3 *To both these points, apply the fiducial edge of your little plate-ruler, so, as that the same edge may look towards the Sun, and that it may also cut the limbe or Zodiac, and goe beyond it as occasion shall be: and let the greatest part of it lye from the earth towards the planet, for many times it will be requisite so to lay it, because of the work that next follows.*

4 *Measure with your Compasses the least distance between the Center of the Sun and the fiducial edge of the same ruler: and set one foot of this distance upon any part on the exteriour limbe or Zodiac of the plate, & on the same side of the ruler that the Suns Center is, and on that part*

plagam quæ à terra versus Planetam respicit. Quæ omnia ita dirigenda sunt ut alter pes circini lineam regulæ fiducialem tangat. Tunc enim pes iste super Zodiacam positus ostendet Planetæ Longitudinem in signis & partibus ejus.

Videas exempla post præceptum sequens.

5 *Cujuslibet è 5 Planetis Latitudinem investigare.*

1 COgnitis Anomaliis tam terræ quam Planetæ, applica filum Centro affixum Anomaliæ Planetæ in suâ Orbitâ numeratæ, & immoto filo cape minimam distantiam inter illud & istum Planetæ characterem (cujus locum inquiris) filo magis commodum, nam uterque aptus non erit: Et observa utrum filum Borealem an Australem inclinationem secuerit.

2 Metire istam distantiam in Scala pro inclinationibus Planetæ, facta & ei circinus inclinationem ostendet (plaga vero antea detecta est.)

3 Re-

part of the Zodiac which is from the Earth towards the Planet. All this must be done in such wise, that the other foot of the Compasses being turned about may justly touch the edge of the ruler. In this posture, that foot which standeth upon the Zodiac will there shew the signe and degrees of the Planets longitude.

See examples after the next Precept.

5 How to find the Latitude of any of the 5 Planets.

1 H*Aving found the Anomalies of the Earth and Planet, lay the threed that is fixed at the center upon the Planets Anomaly numbred in its proper Orbit. And to the threed so laid, take the least distance from that character of the Planet (whose place you seeke) that lyes fitted to the threed, for both will not: and observe whether the threed cut through the title of* North *or* South *inclination.*

2 *Measure the same least distance, upon that Scale which is made for the measure of the Planets inclination, and upon that Scale the Compasses will shew how much the* inclination *is: the coast or title of it being discovered before.*

3 *You*

3 Restant adhuc duæ distantiæ mensurandæ. Prima, est distantia Planetæ a terrâ, hoc est à punctis Anomaliarum quæ sunt loca eorum in ipsorum Orbitis. Secunda, est Planetæ à sole. Quæ fiunt applicando distantias in circino captas Scalæ huic rei factæ Scalæ (sc.) Decimali quæ in singulis Theoricis grad. 360 sive exterioris Planetæ punctum Aphelium secat. Hoc pacto distantias ipsas, vel saltem earum proportionem dignosces.

3 You are then to measure two distances more. The first, is from the Planet to the earth; that is, from the points of their Anomalyes, which are their places in their Orbits. The second, is from the Planet to the Sun. And these are done, by taking the said distances in your compasses, and applying those lengths to the Scale appointed for that purpose [namely that Decimal Scale, which on every Theoric passeth through 360, or the Aphelial point of the exteriour Planet.] By this meanes you shall know their distances, or the proportion of them at least.

4 Adi Scalam in partes 120 æquales divisam cum arcu graduationum sibi appendente, & super istum arcum numera Planetæ inclinationem prius inventam cui filum applica. Deinde super eandem Scalam numera Planetæ distantiam a Sole, & minimum abinde ad filum spatium per circinum cape, & serva. Denuo in eadem Scalâ Planetæ à terra distantiam nota, & circini pedem alteram istic fige. Filum verum ita move ut pes circini alter conversus invariata apertura filum exacte tangat. Sic demum filum super arcum appendentem ostendet Planetæ latitudinem quæsitam. Quæ semper ejusdem erit denominatio-

4 Next, goe to the equal Scale divided into 120, which hath an ark of graduations appendent to it. And upon that ark, Count the inclination of the Planet, which you found before, and thereto lay the threed. Afterwards, upon the Scale of 120 count the number of the Planets distance from the Sun, and take the least extent from that number to the threed, keeping it still in your compasses. Then again, upon the same Scale, count the distance of the Planet from the Earth, and there set one foot of the former extent, and apply the threed to the other foot, so, that the said other foot being turned about, may onely reach the

nationis cujus est inclinatio prius inventa.

the threed neither going beyond, nor falling short of it. So the threed, in this position, will shew upon the appendent arke the quantity of the Planets latitude. And for the coast or denomination of the Latitude it must alwayes be the same that the Inclination was, whether North or South.

Duo plenissima Exempla hic sequuntur. Longitudinis, Latitudinis, Distantiæque terræ reliquorumque 5 Planetarum. Unum ad quartum Octobris 1649 in Meridie. Alterum ad 19 Feb. 1651 in Meridie.

See two examples at large here following for the Longit. Latit. and Dist. of the earth and the other 5 Planets. One Example is for the 4th of October at noon 1649. The other is for the 19th of February at noon, 1651.

Locus Terræ reliquorumque 5 Planetarum ad quartum Octobris in merid. 1649.

The Places of the Earth and the other 5 Planets, Octob. 4th. at noon. 1649.

	Earth	♄	♃	♂	♀	☿	
Epocha 1644	194 80	119 90	229 28	299 78	238 78	61 55	*Epocha 1644*
Motus in 4 annis	359 96	48 86	121 40	45 59	180 69	218 86	*Motion in 4 years*
Sept. compl. an. com.	269 07	9 13	22 68	143 06	77 38	37 20	*Sept. compl. com. year*
Octob. dies 3 compl.	2 96	0 10	0 25	1 37	4 81	12 28	*Octob. 3 dayes complete*
Summa	826 79	177 99	373 61	489 80	501 66	329 89	*Summe*
Circuli subtrahendi	720		360	360	360		*Circles subtracted*
Anomaliæ æquales	106 79	177 99	13 61	129 80	141 66	329 89	*The equal Anomalyes*
Planetarum longit.	♈ 21 45	♋ 1 20	♎ 20 20	♐ 4 00′	♍ 7 15′	♏ 2 00′	*The Planets Longitudes*
Inclinationes		*aust.* 1 12′	*bor.* 1 18′	*aust.* 3 15′	[illegible] 0 45′	*aust.* 1 15′	*Inclination*
Distantiæ à Sole	68	77½	93½	50	49½	31½	*Distances from the Sunne*
Distantiæ à Terra		74	110	69	62⅓	95½	*Distances from the Earth*
Planetarum Latitud.		*aust.* 1 15′	*bor.* 1 07′	*aust.* 1 00′	*bor.* 0 37′	*aust.* 0 25′	*The Planets Latitudes*

Locus Terræ reliquorumque 5 Planetarum ad 19 Feb. in Meridie 1651.

The places of the Earth and the other 5 Planets upon the 19th of Febr. at noon. 1651

	Earth	♄	♃	♂	♀	☿	
Epocha 1644	194 80	119 90	229 28	299 78	238 78	61 55	*Epocha, 1644*
Motus in sex annis	359 45	73 27	182 06	68 13	270 23	326 24	*Motion in 6 years*
Janu. compl. an. com.	30 55	1 04	2 58	16 24	49 67	126 86	*Janu. complete com. year*
Febr. dies 18 compl.	17 74	0 60	1 50	9 43	28 84	73 66	*Febru. 18 dayes complete*
Summa	602 54	194 81	415 42	393 58	87 52	588 31	*Summe*
Circuli subtrah.	360		360	360	360	360	*Circles subtracted*
Anomaliæ æquales	242 54	194 81	55 24	33 58	227 52	228 31	*The equal Anomalyes*
Planetarum Long.	♍ 11 30′	♋ 8 20′	♐ 9 50′	♎ 21 20′	♒ 18 00′	♓ 20 20′	*The Planets longitudes*
Inclinatio		*aust.* 0 22′	*bor.* 0 49′	*bor* 1 25′	*bor.* 3 20′	*bor.* 6 45′	*Inclination*
Distantiæ à Sole	67	77¾	91¼	55⅔	49⅓	23¾	*Distance from the Sunne*
Distantiæ à Terra		73	90⅔	27	20¾	45¼	*Distance from the Earth*
Planetarum Latitud.		*Aust.* 0 24′	*Bor.* 0 50′	*Bor.* 3 00′	*Bor.* 7 45′	*Bor.* 3 34′	*The Planets Latitudes*

6 *Quot Semidiametris terræ Planeta quispiam distabit à Sole, vel Terra dignoscere.*

MEnsuratis prius distantiis Planetæ à Terrâ, & Sole in Scalis propriis ut ante præceptum est

Pro	♄	Duc distantias in	400	Factum erit inter vallum quæsitum in Semidiametris Terræ.
	♃		200	
	♂		100	
	♀		50	
	☿		50	

In acquirendâ distantia Terræ & Sole majori opus est cautelâ: attamen eodem pariter modo investigatur.

Theoricæ huic rei magis idoneæ sunt istæ *Veneris*, *Mercurii*, aut *Martis*, si distantia Terræ à Sole mensuretur in Theorica *Veneris*, aut *Mercurii*, numerus inventus per Scalam istius laminis ducendus est in 50 numerum (scil.) *Veneris*, & *Mercurii*, sin vero in Theorica *Martis* ducatur in 100 *Marti* propriam.

Sic

6 To know how many Semidiameters of the Earth any Planet at any time is distant from the Earth, or from the Sun.

H*Aving measured the distances of the Planet from the Earth and from the Sun, upon it's proper Scale, as was shewed before; Then*

For	♄	*Multiply the said distances by*	400	*And the product will be the required interval in Semidiameters of the Earth.*
	♃		200	
	♂		100	
	♀		50	
	☿		50	

The Earths distance also from the Sun may be had in the same manner; but with a little more caution. For the fittest Theorics for this work are those of Venus, *and* Mercury, *or else* Mars. *If you take the Earths distance from the Sun upon the plate of* Venus, *and* Mercury, *then you must multiply the number found by the Scale of that plate, by 50, which is the number given before for* Venus, *and* Mercury. *But if you take it from the Theoric of* Mars, *then you must multiply the number there found, by 100, which is the multiplying number given before for* Mars.

Sic

Sic juxta Exemplum primum hæ invenientur distantiæ.

So according to the first Example these Distances will be found.

		♄	♃	♂	♀	☿	*Earth*		
Distantiæ Planetarum in Scalis propriis à	*Sole*	77 $\frac{1}{2}$	93 $\frac{1}{2}$	50	49 $\frac{1}{2}$	31 $\frac{1}{2}$	68	The Plan. dist. in their proper Scales, from the	Sunne
		duc in mult. by 400	*in* 200	*in* 100	*in* 50	*in* 50			
	Terra	74	110	69	62 $\frac{1}{3}$	95 $\frac{1}{2}$			Earth
Distantiæ in Semidiametris Terræ à	*Sole*	31000	18700	5000	2475	1575	3400	Their distances in Semid. of the Earth, from the	Sunne
	Terra	29600	22000	6900	3116	4775			Earth

Juxta secundum Exemplum hæ Semidiametri exurgent.

According to the second Example these numbers of Semidiameters will rise.

		♄	♃	♂	♀	☿	*Earth*		
Distantiæ Planetarum in Scalis propriis à	*Sole*	77 $\frac{3}{4}$	91 $\frac{1}{4}$	55 $\frac{2}{3}$	49 $\frac{1}{3}$	23 $\frac{3}{4}$	67	The Plan. dist. in their proper Scales, from the	Sunne
	Terra	73	90 $\frac{2}{3}$	27	20 $\frac{3}{4}$	45			Earth
Distantiæ in Semidiametris Terræ à	*Sole*	31100	18250	5567	2467	1187	3350	Their distances in Semid. of the Earth, from the	Sunne
	Terra	29200	18133	2700	1037	2250			Earth

7 Ex *Planetæ Longitudine & Latitudine datis rectam ascensionem & declinationem invenire.*

COmmodissimè hæc fiunt per Astrolabia, aut instrumenta istiusmodi Spherica. Ad supplendnm autem hunc defectum Scalas addidi quibus licet majori cum molestiâ, ista perficiantur. Huic rei delineationes in Theoricis *Saturni* & *Jovis* bis repetitæ inserviunt, ut unaquæque lamina suam habeat Scalam istis Theoricis quæ super illâ ducuntur paratam.

¶ Primo, igitur inquirenda est ascensio recta istius puncti Eclip-

7 By the Longitude & Latitude of a Planet being known, how to find the right ascension & declination thereto belonging.

THis work is most proper for Astrolabes, and other such Spherical instruments. Yet because these Theorics should not be altogether defective herein, I have added such Scales as will perform these things, though it be with more trouble. For this purpose those Delineations upon the two Theorics of Saturn *&* Jupiter *are added; both which are the same thing done twice over, that each plate may have one ready at hand, for those Planets which are drawn upon it.*

¶ *The first thing to be done is, to get the right ascension of the*

Eclipticæ quod longitudini Planetæ respondet, quasi Latitudis esset expers. Quod perficitur in scala ascensionum rectarum partium Eclipticæ. Quæ ex inspectione tituli dignosci potest.

the meer longitude of the Planet, as if it were without all Latitude, or in that very point of the Ecliptic which answers to the Longitude. And this is performed upon that Systeme of Scales which is made for the finding out of the right ascensions of the parts of the Ecliptic, as in the title thereof is expressed, by which title it may also be known.

Numera igitur in Zodiaco Elliptico Planetæ Longitudinem, id est, signum & gradum ubi per quartum præcedens inventus fuerit, & ibi applicato filo centrali observa ubi arcum secuerit notatum 1,2,3. Qui in gradibus graduumque partibus æstimatus ostendit differentiam Longitudinis ab ascensione recta, & proinde appellari potest *Longitudinis æquatio.* Hæc æquatio Longitudini antea inventæ vel addenda est, vel subtrahenda prout filum ostenderit cadens in titulos Additivos, vel Subtractivos pone hunc differentialem arcum scriptos. Hoc cite facto prout oportet, summa vel differentia inventa erit ascensio recta meræ Longitudinis Planetæ. Quod primum erat requisitum.

Count therefore upon the Elliptical Zodiac, the Planets Longitude, that is, the signe & degree, in which you found it by the 4th precedent: and thereto applying the Center threed, observe where the same threed cuts the ark noted with 1, 2, 3, the same ark being estimated in degrees & minutes, is that which shews how much the Longitude differs from the right ascension, which may be called, the longitude Equation. *This Equation or difference must either be added to, or subtracted from, the Longit. before found, according as the threed will intimate by falling upon the directions for addition or subtraction, written closely behind this differential ark. And this being accordingly done the sum or difference so found, shall be the right ascension of the Planets meer Longitude, which was the first thing required.*

Hoc modo absque ulteriori

And thus much alone doth get

ri labore acquiruntur aſcenſiones rectæ vel Solis, vel Terræ, quia latitud. expertes ſemper verſentur in plano Eclipticæ.

Secundo hæc aſcenſio recta corrigenda eſt juxta Latitudinem Planetæ ab Ecliptica modo aliquam (quod frequentiſſime accidit) habuerit. Et huic rei maxima pars alterius Syſtematis Scalarum inſervit. Hoc modo.

Super duodecim ſignis juxta ordinem quo in Ellipſi inſcribuntur (quæ ſignis in exteriori Zodiaco reſpondent licet characteres aliter ſignentur) & ſuper gradus exterioris Zodiaci (cujus gr. 30 antedictis ſignis per integram Scalam reſpondent) numera Planetæ Longitudinem, & filum applica. Deinde in Scalâ lineæ mediæ quæ Centrum petit, Planetæ latitudinemnu mera. A quo puncto ad filum cape per circinum minimam diſtantiam; hæc minima diſtantia applicata Scalæ lineæ mediæ a Centro exteriùs, æquationem exhibebit in gradibus & minutis. Sit hæc *Latitudinis æquatio.* Quæ aſcenſioni prius inventæ addi vel ab eadem ſubtrahi debet juxta titulos in Ellipſi notatos Hæc ſumma aut differentia ſic ultimo inventa erit exacta aſcenſio recta

get the true right aſcenſion for the Earth or Sun, becauſe they lye in the plaine of the Ecliptic& have no latitude from it.

The ſecond thing to be done, is to correct this foregoing right aſcenſion, which correction muſt alwayes be made when the Planet hath any Latitude from the Ecliptic, as moſt commonly it hath. And for the effecting of this, The greateſt part of the other Syſteme of Scales is to be uſed, and in this manner.

Upon the 12 ſignes as they are ordered and inſcribed into the Ellipſis (which ſignes do anſwer to thoſe in the exteriour Zodiac, though the charactering of them be different) and upon the degrees of the exteriour Zodiac (30 of which deg. quite through that Scale do anſwer to theſe forementioned ſignes) count the Planets Longitude, and thereto apply the threed. Then again, upon the Scale of the middle line that goes to the Center, count the Planets Latitude; & from that point to the threed, take the leaſt diſtance with your Compaſſes. This leaſt diſtance applyed to the ſame Scale of the middle line, from the Center outwards, will give the equation in degr. and min. This may be the latitude equation. *And it muſt be either added or ſubtracted from that right aſcenſion*

recta Planetæ pro Longitudine, & Latitudine datis.

☾ Ad declinationem Planetæ acquirendam Zodiaco tantum utimur exteriori cum arcu circulari utrinque ad 25 gr. numerato. Hoc modo.

Numera Planetæ latitudinem in arcu 25 grad. latitudini Planetæ pro eo tempore quoad plagam congruo, & illuc filum porrige. Deinde in Zodiaco exteriori (juxta ordinem signorum & graduum illic numeratorum) numera longitudinem Planetæ: in quo puncto fige circini pedem alterum; altero vero cape minimam distantiam a filo: illud observans utrum in hâc operatione circinus supra vel infra filum steterit. Minima hæc distantia applicetur lineæ rectæ 35 partium ab initio Scalæ procedendo & ostendet declinationem quæsitam. Plagam vero Septent. vel Austral. situs circini infra vel supra filum ostendet. Nam superior situs Borealem inferior plagam Meridionalem denotat. Et ut hæc directio semper

sion that was found before, according as the Directions that are written upon the Ellipsis shall prescribe.

By which meanes, the last sum or difference thus found, shall be the perfect right ascension of the Planet, agreeable to the Longit. and Latit. given. This for the rightascension.

☾ *For the Planets declination, you are to make use onely of the exteriour Zodiac, and the circular ark numbred both wayes to 25 d. The way is this. Count the latitude of the Planet upon one of the arks of 25 deg. namely that wch is noted with the same kind of latitude that the Planet at that time hath, & thereto apply the threed. Then upon the exteriour Zodiac (according to the order of the signes and degr. as they are there set on) rekon the Planets longitude; & setting one foot of your compasses in that point, with the other foot take the least distance to the threed, observing whether your compasses in this work do stand above or below the threed. This least distance being so takē must be applyed to the right line of 35 parts, from the beginning forwards upon the Scale, where it will shew you the quantity of the Planets declinatiō. And for the coast of this Declination; whether it be North or South, the former observation of the stand-*

per presto sit utrisque exterioris Zodiaci terminis inscribitur.

standing of the compasses, either above or below the threed, will resolve. For if the compasses do stand above the threed, then the declination is North: if they stand below, then the declination is South. And this directiõ also, that it might be alwayes neer at hand, is written at both ends of the exteriour Zodiac.

Terræ sive Solis declinatio nullâ molestiâ invenitur applicando Scalæ 35 longitudini ab *Ariete* vel *Libra* in exteriori Zodiaco recto.

The Earth or Suns declin. is had, by taking the length from Aries *or* Libra *in the exteriour streight Zodiac, and applying it to the Scale of* 35, *for it will there give the declination without more adoe.*

Sequitur Exemplum Ascensionis rectæ, & Declinationis Terræ reliquorumque Planetarum juxta Longitudines Latitudinesque in prioribus Exemplis inventas, & ad Meridiem quarti diei Octobris 1649 computatum.

Here follows an Example *of the right ascensions & declinations of the Earth and the other 5 planets, according to the Long. & Latit. of them, found in the first of the two former* Examples *computed for the fourth day of* October *at* Noon, 1649.

Ascensiones Rectæ, & Declinationes Planetarum juxta Longit & Latit. Exempli primi.

The Right ascens. and declin. of the Planets according to their Long. & Lat. in the 1 Example.

	Earth	♄	♃	♂	♀	☿	
Longit. solut. in gr. & m.	21 45′	91 20′	200 20′	244 00′	157 15′	212 00′	*Long. resol. into d. & m*
Long. æquat. cum titulis Addit. & Subtractivis.	1 37 *subtr.*	0 07 *adde*	1 34 *subtr.*	2 00 *subtr.*	1 45 *adde*	2 12 *subtr.*	*Longitudes æquat. with titles Ad. Subt.*
Asc. R. simplicis Longit.	20 08	91 27	198 46	242 00	159 00	209 48	*R. Asc. of meer Long.*
Latitudinis æquatio cum titulis Add. Subtract.		0 04 *subtr.*	0 32 *adde*	0 15 *subtr.*	0 15 *adde*	0 12 *subtr.*	*Latitudes equat. with titles of Ad. Subtr.*
Ascens. R. absolut.	20 08	91 23	199 18	241 45	159 15	209 36	*Right ascens. absolute*
Declinationes	*Bor* 8 15	B. 22 00	*A.* 6 45.	*A* 21 45	B. 9 30	*A.* 12 20	*Declination.*

8 *Invenire locum Solis vel Terræ in Ecliptica.*

HOc faciliùs fit pro Terra quàm pro reliquis 5 Planetis, quia Terra & Latitudinis & commutationis est expers, & ad inveniendum verum locum Terræ in Ecliptica commodius utemur majori Theorica : illâ (sc.) quæ comprehendit *Venerum* & *Mercurium* unâ parte, vel illâ alterâ quæ comprehenditur à *Marte* ex altera instrumenti facie.

In Orbitâ Terræ numera Anomaliam ad datum tempus inventam, & ad hunc terminum filum extende quod in exteriori Zodiaco locum terræ designabit, cujus oppositum est locus Solis.

Sic habes in duobus prioribus exemplis locum Terræ ad datum tempus, *viz. Aries* 21 gr. 45 m. & *Virgo* 11 gr. 30 m. quorum oppositam sunt 5 loca Solis *viz. Libra* 21 gr. 45 m. & *Pisces* 11 gr. 30. m.

8 How to find the place of the Earth or Sun in the Ecliptic.

T*His is much more easie to be done for the Earth then it was for the other 5 Planets; because the earths place is free both from commutatiō & Latit. And for the finding of the true place in the Ecliptic, it will be best to use the earths largest Theorics : namely, either that which comprehends* Venus *&* Mercury *upon one Table, or else that which is comprehended by* Mars *upon the other Table.*

Having therefore found the earths Anomaly for the assigned time, Count the same upon the Orbit of the earth, and thereto lay the center-threed, which being so laid, will give the place of the earth, in the degrees of the exteriour Zodiac. And the opposite thereto, is the place of the Sun.

In the two former examples you have the earths places (for those assigned times) expressed by the signe and degree, wherein it then shall be : namely Aries 21 *d.* 45 *m. and* Virgo 11 *d.* 30 *m. And the opposites to these are the places of the Sun at those times : that is,* Libra 21 *d.* 45 *min. and* Pisces 11 *d.* 30 *m.*

9 *De præcipuis nonnullis Planetarum passionibus.*

PRincipium harum Theoricarum officium est ut per illas inveniantur loca Planetarum quoad longitudinem & latitudinem: quod quia jam antea tractavimus operæ prætium erit de præcipuis eorum passionibus pauca addere. Quarum tria præcipuè sunt capita.

1 Planetæ (ob motum longitudinis quem faciunt in Ecliptica) nonnunquam videntur secundum seriem signorum procedere (hoc est) 1 Directi sunt in Motu. Aliquando videntur retrocedere (i.e.) sunt 2 Retrogradi. Et in illorum transmutationibus inter utrunque horum motuum necessario videbuntur stare hoc est sunt 3 Stationarii.

2 Loca Planetarum considerantur vel quoad distantiam à Sole, vel ab invicem; unde varios habent aspectus. Quorum 1 conjunctio dicitur quando duo quilibet Planetæ sunt in eodem gradu longitudinis. 2 Opposite quando sunt in opposita longitudine. 3 Trinus quando $\frac{1}{3}$ circuli vel

9 Concerning some of the principal passions of the Planets.

THe finding out of the places of the 5 Planets in respect of Longit. and Latit. is the thing principally intended in these Theorics. Now this having been already declared, it shall not be amisse to adde somewhat of the principal passions belonging unto them: of which there are these 3 chief heads.

1 *At some times these 5 Planets (in respect of that motion which they make according to the longit. of the Ecliptic) doe appeare to goe forward, agreeably to the order & succession of the signes, that is, they appeare to be* 1 *Direct in motion. Sometimes again the seeme to goe backward in motion, or to be* 2 *Retrograde. And in their changes from the one of these motions to the other, they must necessarily appeare to be standing still, or to be* 3 *Stationary.*

2 *Their places being compared in respect of distance from the Sun, or one from the other, the Planets may have several aspects: as* 1 *Conjunction, when they are (any two of them) in one place of longit.* 2 *Opposition, when they are in opposite longit.* 3 *Trine, when they are* $\frac{1}{3}$ *part of a circle or* 4 *signes distant*

vel quatuor signis, 4 Quartilis quando 3 signis vel circuli quadrante, 5 Sextilis quando sextâ parte circuli vel duobus signis ab invicem distabunt. *Venus*, & *Mercurius* nunquam hos aspectus præter conjunctionem habent ad Solem nec inter se invicem ullum faciunt præter sextilem quo sæpius distant.

3 Locis eorum ad Solem comparatis, vel sunt sub radiis, & dicuntur combusti: Vel post ortum Solis interdiu oriuntur, & vocantur Orientales: aut post Solis occasum seu noctu occidunt, & sunt Occidentales: vel Soli sunt oppositi, & dicuntur Acronychi. *Venus* & *Mercurius* nunquam sunt Acronychi, quia *Venus* nunquam à Sole ultrà 48 gr. *Mercurius* ultrà 29 gr. recedit.

distant from each other: 4 *Quartile, when they are three signes or a quadrant of a circle distant:* 5 *Sextile, when they are* $\frac{1}{6}$ *part of a circle or two signes distant.* Venus *and* Mercury *cannot make any of these Aspects with the Sun. And one of them with the other can make none but the Sextile; which often they doe.*

3 *Their places being compared with the Suns place, they are either under the Sun beames & are thē said to be* 1 *Combust: or else they rise after the Sun, rising when the Sun is up, and are called* 2 *Oriental: or they set after the Sun, while the Sun is down, and are called* 3 *Occidētal: or are opposite to the Sun; and are called* 4 *Acronychal.* Venus *and* Mercury *can never be Acronychal, because they never goe farre enough from the Sun:* Venus *onely* 48 *d.* Mercurius *onely* 29 *degrees.*

10 *De Directione, Retrogradatione, & Statione.*

CUm inventio justi temporis harum mutationum in Planetarum cursibus res fit per se difficilis; per has Theoricas vix accuratè detegentur. Modus optimus est (cognitis prius locis ad diem certum) pro 5 aut decimo post die eorum lon-

10 Of Direction, Retrogradation, and Station.

THese things will not well be discovered by these Theorics, it being a difficult business to set the just times of these changes in their courses. If you desire to know in which of these motions any Planet is, the best way will be (when you have found

longitudines inquirere. Præsertim in *Saturno Jove* & *Marte* quia verò motus *Veneris* & *Mercurii* velociores sunt sufficiet eorum longitudines ad secundum aut quartum post diem investigare. Quo pacto exploratis eorum longitudinibus ad duo tempora diversa quem curiam teneant ratione progressioni, regressionis, aut stationis facile perceperis.

Sic si ad priùs Exemplum loca ad aliquot sequentes diei examinaveris, erunt omnium motus juxta seriem signorum directi, in posteriori omnes excepto *Jove* retrogradi, cujus etiam locus invenietur parum distans à priori in præcedentia tunc primam intracturus stationem.

Nam illud semper est notandum quod si Planeta directio transiverit ad stationem ista dicitur prima statio: quando vero à retrogrado motu, ista statio secunda nuncupatur.

11 *De*

found their places for any one day) to enquire their longitudes about 5 or 10 dayes after in Saturn, Jupiter *and* Mars, *or about 2 or 4 dayes after for* Venus *and* Mercurius, *because the motions of these are much swifter then of the other. And so having found their places of longitude at two several times, you shall perceive what course they hold in respect of progresse or regresse of standing still.*

So *if in the first Example the places were again examined for some other dayes after, they would all be found direct in their motions according to the succession of the 12 signes. But in the second Example, they would all be found Retrograde except* Jupiter: *which Planet also will be found to be very neer to his former place, yet a little more forward, and consequently neer to his first station, then going to enter into it.*

For it must alwayes be noted, that, if a Planet passe from direct motion to station, then that standing is the first station. But if it passe from retrograde motion, then is the station following to be taken for the second station.

11 *Of*

11 *De Latitudine ascendente & descendente.*

INventis sic prius latitudinibus ad rectum tempus examinentur de novo ad 2, 3, 5, vel 10 diem sequentem, & utrum sint ascendentes, vel descendentes dignosces. Hoc modo.

Si post secundam inquisitionem inventi fuerint in eâdem plagâ (*viz.* vel Septentrionali vel Meridionali) quâ antea, tum si sit cujusque latitudo ad utrumque tempus, vel Meridionalis decrescens, vel à Meridie ad Boream mutata, & crescens, dicuntur ascendentes.

Sin verò ad utrunque tempus latitudo fuerit Septentrionalis decrescens, vel mutata à Boreâ ad Meridiem, & tum crescens, vocantur descendentes.

Denique si ad utrumque tempus consistant : sunt in puncto variationis. *viz.* si in Boreâ latitudine constiterint ab ascendente vergunt ad descendentem; si in Meridionali à descendente ad ascendentem.

11 Of latitudes ascendent or descendent

After the latitudes of the Planets are found for any assigned time, if they be again examined for 2, 3, 5, or 10 dayes after, you may know whether they be ascendent, or descendent, in this manner.

If in the second enquiry they be found still in the same coast or denomination (of North or South latitude) that they were before, then

If the latitude at both times be either South and decreasing, or else changed from South to North, and then increasing, they are then said to be ascendent. But

If their latitude at both times of enquiry be either North decreasing, or else change from North to South and then increasing afterwards, they are then said to be descent.

If at these two times of enquiry they be found consistent, then are they upon their change, namely, if consistent and in North latitude, they are changing from ascendent to descendent : but if consistent and in South latitude, then are they changing from descendent to ascendent.

12 *De Planetarum Aspectibus.*

COmpara duorum quorumlibet loca ad datum tempus & deprehendes Aspectus juxta regulas noni præcepti.

Exempli gratiâ in primo præcedentium Exemplorum *Sol* & *Jupiter* sunt propemodum in conjunctione. *Sol* & *Saturnus* prope Trinum. *Saturnus* & *Jupiter* non procul à Trino. *Saturnus* & *Mercurius* prope Trinum. *Venus* & *Mercurius* non procul à Sextilo. Et pariter de reliquis.

Attamen illud obiter notandum, quod licet *Jupiter* & *Sol* tendant ad conjunctionem, & nobis terricolis revera appareant conjuncti, tamen per sextam præcedens distant ab invicem 18700 semidiametris Terræ.

12 Of the Planets Aspects.

COmpare the places of any two of the Planets together, & you shall have their Aspects for the time assigned, according to the former rules in the ninth precept.

Thus (rudely) in the first of the former Examples. The Sun *and* Jupiter *are neer in Conjunction. The* Sun *and* Saturn *not farre from a Trine.* Saturn *&* Jupiter *not farre from a Trine.* Saturn *and* Mercury *neer to a Trine.* Venus *and* Mercury *not farre from a Sextile. In the same manner you may deale with the rest.*

But by the way note this, that though Jupiter *and the* Sun *are neer to a conjunction, and to us that are upon the earth doe appear as if they were really together, yet by the precedent sixth Proposition, they are distant from each other 18700 semidiameters of the* Earth.

13 *Vtrum Planetæ sunt Combusti, Acronychi, Orientales, vel Occidentales.*

PLanetæ dicuntur Orientales quorum loca distabunt à terra minus semicirculo juxta seriem signorum numerato. Occidentales è contra. Si sint in

13 Whether the Planets be combust Acronychal, Oriental, or Occidental.

T*Hose Planets are Orientall whose places being reckoned from the place of the* Earth, *according to the succession of the 12 signes, are distant from it lesse*

in loco Terræ sunt Acronychi, sin loco Terræ oppositi vocantur combusti.

Sic in præcedentium exemplorum primo *Saturnus* erit Orientalis quia à 21 *Arietis* ad primum *Cancri* juxta f. f. non completur semicirculus *Jupiter* combustus, *Mars* Occidentalis, quia à 21 *Arietis* loco (sc.) Terræ ad quartum *Sagittarii* locum *Martis* intercipiuntur plus 180 gradibus. *Venus* Orientalis, *Mercurius* Occidentalis. Nullus hic Acronychus quia eorum loca multum distant à terra.

lesse then a semicircle, or 6 signes. And they again are Occidental whose places so counted, are distant from the Earths place more then a semicircle. If their places be the same with the Earths place, they are Acronychal, if opposite, they are Combust.

Thus in the first of the two former Examples; Saturn *is Oriental, because from the* 21 *deg. of* Aries *to the* 1 *deg. of* Cancer *(which is according to the order of the signes) is lesse then a semicircle.* Jupiter *is combust.* Mars *is Occidental, because from the Earths place which is* Aries 21 *deg. to the place of* Mars *which is* Sagittarius 4 *deg. is more then a semicircle or* 6 *signes.* Venus *is Oriental.* Mercury *is Occidental. None of them are Acronychal, because their places are not neer to the place of the Earth, but much differing from it.*

14 *De Ortu & Occasu Poëtico.*

APud Poëtas dicuntur Planetæ oriri, & occidere Cosmicè, Acronycè, & Heliacè; harum passionum detectio (utpote etiam occultationum, & emersionum) in his Theoricis expectari non debet. Res est

14 Of the Poetical risings and settings.

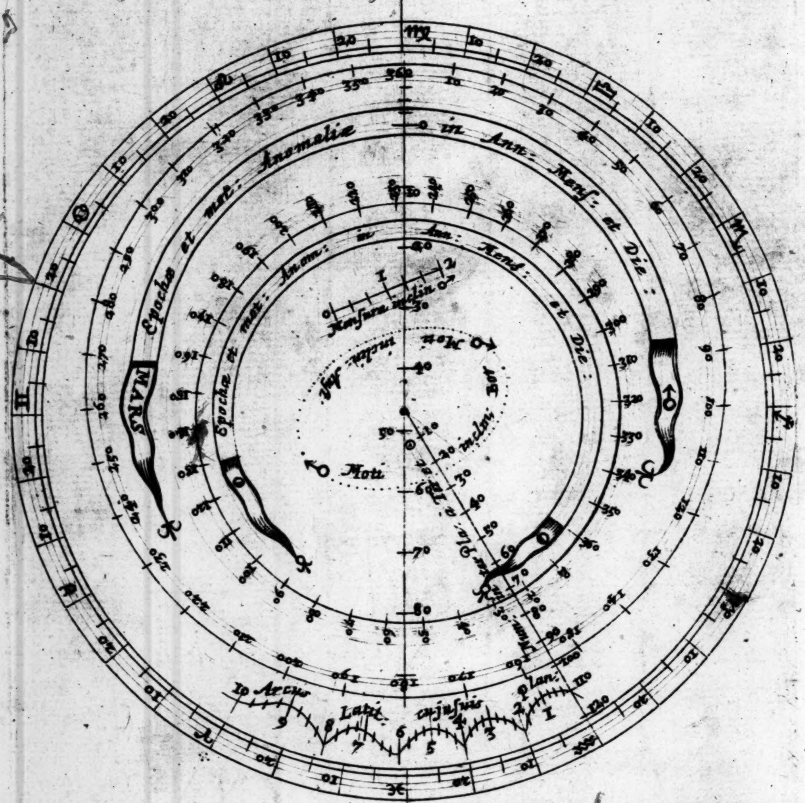
T*He Poëtical kindes of rising and setting are called Cosmical, Acronychal, and Heliacal. These and some other passions of the Planets (such as are the Emersions and Occultations) are not to be expected*

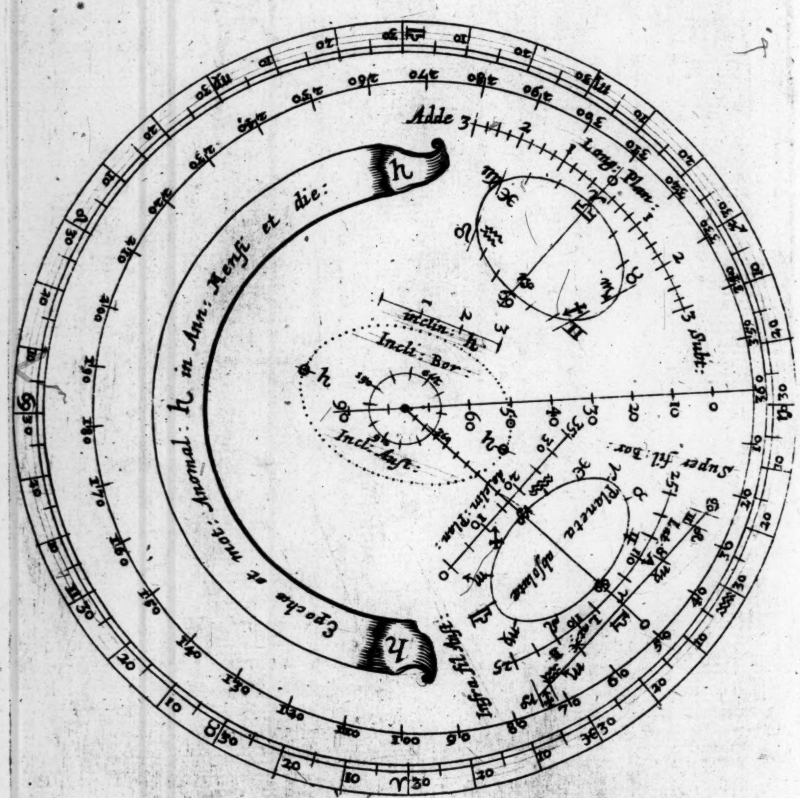
eſt per ſe ardua præſertim in Planetis ob eorum continuum motum & tum Longitudinis, tum Latitudinis variationem. Præterea ad elevationes Poli, & Horizontes particulares referuntur; quapropter Aſtrolabiis, atque iſtiuſmodi projectionibus Spheræ, non Theoricis conveniunt. Exactè ex Tabulis Aſtronomicis, & Calculo Trigonometrico deducuntur. Qui curioſiùs in hæc inquirunt exinde ſatisfactionem petant. Hæc quæ ſcripſimus pro introductione inſerviant ad magis præciſas operationes, vel ſaltem ad ſupplendos eorum defectus quorum peritiâ, vel deſiderium euſque non attingit, & quorum gratia hæc præcipuè intendimus.

expected from theſe Theorics. They are difficult to be found, eſpecially for the Planets, which are alwayes in motion, not reſiding any long time in one Longitude and Latitude. Beſides, the ſame things have relation to the clevations of the Pole above ſeveral Horizons, which kind of concluſions are not proper for Theorics, but muſt be referred to Aſtrolabes and other Spherical Inſtruments. The moſt exact practice this way is to be had in the Aſtronomical Tables, and Trigonometrical Spheric works to be conjoyned therewith for ſuch purpoſes. They therefore that would have more, muſt there ſeek help and wayes to ſatisfie themſelves. This that is here done, may ſerve for an introduction to more exact workings: at leaſt it may ſupply the wants of ſuch, whoſe skill and deſires reach not ſo farre; for whoſe ſakes it was principally intended.

FINIS.

DE





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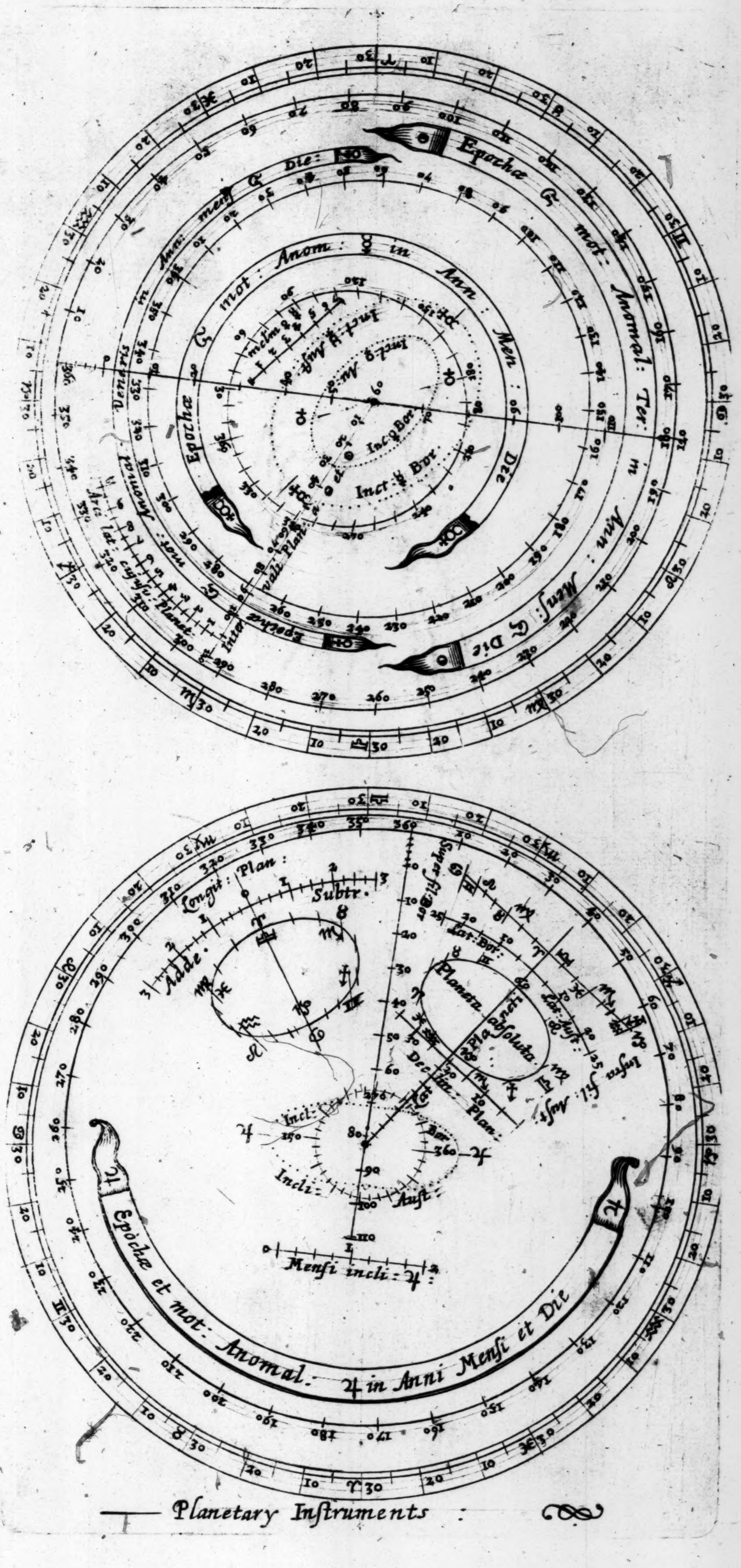


Planetary Instruments

De harum *Theoricarum* Fabricâ.

How these *Theorics* of the Planets are made.

1 *Quomodo quævis Theorica commodissimè disponatur.*

Optimè describuntur super duas laminas ut cujusvis Planetæ orbita, seu Eccentricus majoris sit Diametri.

Methodus quâ incedo, in genere, concordat cum Systemate mundi *Copernicano*, in specie cum istâ ejusdem dispositione quàm introduxit *Keplerus* in suis Tabulis *Rudolphinis* cùm hâc tantùm differentiâ. *Keplerus* orbitas Planetarum facit Ellipses, quòd verò proprius, Ego perfectos Circulos facilitatis gratiâ facio. Defectus ex hoc discrimine procedens non erit magni momenti in Instrumentis non nimium magis amplis.

Ad majorem concinnitatem *Saturnum* & *Martem* in oppositis

1 How every particular Theoric is to be disposed for best convenience.

It is best to make them upon two plates, that each Planets Orbit or Eccentric may be of the larger extent.

The way that I goe is (in general) agreeable to Copernicus *his frame of the World; and in particular, to that which* Kepler *useth in his* Rudolphin *Tables. Onely this difference there is :* Kepler *makes the Orbits of the Planets to be Ellipses, which is the better way; and I here doe make them perfect Circles, which is the easier way. And though it be defective yet it makes no great difference in these small Instruments.*

For most convenience I have put Saturn *and* Mars *upon one Table,*

sitis faciebus ejusdem laminæ disposui. In alterius laminæ facie è quidem altera *Jovem* alterâ terram cum *Venere* & *Mercurio:* interiùs comprehensis, locavi. Scalas etiam aliàs vacuis locis ad alios usus addidi. Insuper, necessitate id requirente, orbita terræ quater repetitur, *viz.* in utrâque laminâ utrinque cum proportione ad exigentiam cujusque Planetæ requisitâ.

Table, each of them taking up one side. Upon the other Table, on one side is set Jupiter, *and upon the other side is the earth at large, with* Venus *and* Mercury *comprehended within it. Other Scales there are added (in spare places) for other uses. Likewise the orbit of the earth is placed upon each side of the two plates, that is, it is four times repeated, need requiring it should be so often iterated. It is also proportioned for the quantity of it, according to the exigence of each several Planet.*

2 *De Planetarum & Terræ eccentricis.*

PRimò in singulis laminarum faciebus describatur Circulus qui priùs in 360 gr. divisus, ulterius in duodecem partes cum 12 Zodiaci signis notatas distinguatur. Numeretur quodlibet signum 10,20, 30. Itaque hi Circuli Zodiacum ad colligendas Planetarum Longitudines necessarium designabunt. In Centro pingatur Solaris effigies monstrans Solem in Centro Mundi locum habere.

2 Concerning the Eccentrics of the Planets and the Earth.

FIrst you are to make 4 limbes upon the 4 sides of your two plates, dividing each of them into 360 deg. and distinguishing the whole Circle into 12 signes, unto which their 12 names, or 12 characters, or both, must be annexed. Each signe is to be numbered by 10,20, 30 deg. and so these Circles will (each of them) represent the Zodiac, in which the Long. & of the Planets must be found. In the Center you may draw the effigies of the Sun, signifying thereby, that the middle or Center of the World is his proper place.

2 Hoc

2 *Then*

2 Hoc facto, sic perge (sit pro exemplo *Saturnus.*) Ex Tabula C, excerpe Aphelium in columna directè sub *Saturni* charactere (nempe, *Sagittarius* 27 gr. 30 m.) A Centro ad 27 gr. 30 min. *Sagittarii* in Zodiaco, duc Semidiametrum, in quâ paululum distans à limbo versus Centrum assume punctum, quòd pro *Saturni* Aphelio habeatur. Distantia verò abindè ad Centrum, dividi concipiatur in 100000 partes æquales quæ instar Scalæ decimalis ad reliquum opus peragendum inserviat.

In hâc Scalâ 100000 sumatur *Saturni* eccentricitas, ex Tabulâ A, nempe 05387 & super eadem lineâ à Centro Solis versus punctum Aphelium transferatur. Istud intervallum vocetur *Saturni* eccentriticas, vel si malueris cape numerum 94631 ex eadem Tabula A, qui super Scalâ eadem, â puncto Aphelio versus Solem translatus, dabit idem eccentricitatis punctum, quod ita inventum erit Centrum orbitæ *Saturni.*

Si

2 *Then for the other work (for instance suppose the Planet* Saturn) *you are first out of the Table* C, *to look where the place of his Aphelium is (which is shewed by the first number in the Table under the character of* Saturn) *namely* Sagittarius 27 *gr.* 30 *m. Wherefore from the center of the* Sun, *to the* 27½ *g. of* Sagittarius *in the Zodiac, draw a Semidiameter : in which, a little within the Zodiac towards the Center, assume any point, which you must suppose to be the Aphelial point of* Saturn : *and the distance from that Aphelial point to the Center, must be supposed to be divided into* 100000 *equal parts, which must serve as a decimal Scale for the rest of the work.*

Out of that Scale of 100000, *take* Saturns *eccentricity, according to the quantity of it set down in the Table* A, *namely,* 05387, *and set it off upon the same line, from the Center of the Sun towards the Aphelial point. This distance is called* Saturns *eccentricity. Or you may take the number* 94613 *(which is also in the same Table* A) *out of the equal Scale, and set that distance from the Aphelial point towards the Center of the Sun, and it will give the same point of eccentricity. This point thus found, is the Center of* Saturns *orbit.*

And

Si igitur, ab hoc Centro ad punctum Aphelii, ut Semidiametro describatur circulus orbitum *Saturni* descripseris.

3 Denuo regulâ ad Centrum Solis applicatâ juxta signa & numeros in Tabula C sub charactre *Saturni* notatos, decimum quemque Anomaliæ sive divisionis orbitæ *Saturni* gradum transferas ; & tandem sub divisis his partibus majoribus in decem minores æquales (nam æquales sufficient licet rigidè sumptæ inæquales esse debent) habebis 360 gradus Anomalos pro *Saturni* orbitâ. Hi â puncto Aphelio per 10,20,30, ad 360 & secundum seriem singulorum numerentur.

4 Orbita terræ circa Solem ad orbitam *Saturni* justè proportionata nunc venit inserenda. Ad quod faciendum inspiciatur secundo Tabula C cujus numerus primus sub signo terræ. Ostendit Aphelium terræ in *Capricorni* 7 gr. 00 m. applicatâ igitur regulâ à centro ad septimum *Capricorni* gr. ducatur linea delebilis quæ lineam terræ Apheliam representabit.

Deinde

And therefore, if you set one foot of your compasses upon that Center, opening the other to the Aphelial point, & describe a Circle to that extent, and upon that Center, you shall then describe the orbit of Saturn.

3 *After this, By laying a ruler to the Center of the Sun, and by the numbers & signes in the Table* C *under the character of* Saturn, *you may inscribe each* 10th *deg. of the Anomaly or division of* Saturns *Orb. And again, dividing each of those large parts into ten lesser equal parts (for, equal will well serve though in rigour they ought to be otherwise) you shall have the* 360 *Anomalar deg. of* Saturns *Orbit. These are to be numbred from the Aphelial point, by* 10, 20, 30, *to* 360, *ending in the same point : and the order of numeration must be according to the series of the* 12 *signes in the Zodiac.*

4 *The next thing to be done, is the setting in of the earth course about the sun, proportioned justly to this orbit of* Saturn. *And for this, look again in the Table* C, *the first number whereof under* Earth *shewes where the Aphelium of the* Earth *lyes, viz. in* Capricorn 7. *d.* 00 *m. Therefore laying a ruler from the center of the Sun to the* 7th *deg. of* Capricorn, *draw an obscure line, which will be the* Earths *Aphelial line.*

Then

Deinde consule Tabulam A, ubi deprehendes punctum Aphelium Terræ à centro Solis distare 10128 partibus prioris Scalæ lineæ sc. *Saturni* in 100000 partes divisæ. Per has partes ex scalâ desumptas punctum terræ Aphelium in debitâ distantia transferas. Consulo rursus prædictam tabula A. Et videbis terræ eccentricitatem esse 00179 partium prioris scalæ decimalis quæ ex scalâ prædictâ desumptæ in lineam terræ Apheliam à centro Solis transferendæ sunt. Punctum translatum erit Eccentrici terræ centrum. Vel si distantia ista sit nimis brevis in eâdem tabulâ invenias distantiam Aphelii terræ à centro Eccentrici ejusdem esse 09949 partium quæ ex priori scalâ decerptæ & à puncto Aphelii terræ super lineâ terræ Apheliâ versus Solis centrum transmissæ centrum eccentrici terræ monstrabunt. Super hoc centro ad intervallum puncti terræ Aphelii scribe circulum qui orbitam terræ repræsentabit ad magnum Saturni orbem justè proportionatam.

5 Minor hic circulus seu terræ

Then look into the Table A, *where you shall find the Earths Aphelial point to be distant from the center of the* Sun 10128 *parts of the former decimal scale or* 100000 *equal parts of* Saturns *line. By which parts taken from that scale, you may set off the Earths Aphelial point in a true distance. Again, look into the* Table A, *and you shall there see the Earths eccentricity to be* 00179, *of the same parts of the former decimal scale, which you are to take and set from the center of the* Sun, *upon the earths Aphelial line, and that point shall be the Center of the earths eccentric. Or if that be too short a distance, you may in the same* Table *find the distance of the Aphelium (or Aphelial point) of the earth from the center of the Earths orbit or eccentric to be* 09949 : *& this number taken out of the former decimal scale, & one foot of it set in the Aphelial point of the earth, the other upon the Aphelial line of the Earth, towards the center of the* Sun, *will shew the same center of the earths eccentric. Upon this center therefore, and to the extent of the Aphelial point of the earth from it, describe a little circle, which is to resemble the earths orbit, being justly proportioned to the great orb of* Saturn.

5 *This little orbit or circle of the*

terræ orbita in debitas partes anomalias dividenda eſt, quarum decima quælibet numeris Tabularibus ſub charactere Terræ in tabula A inſcribi poteſt: regulâ (ſcilicet) ad centrum Solis fixâ, & ad gradus & ſignorum Zodiaci minuta in prædictâ Tabulâ datis applicatâ. Hæ partes denuo biſecentur ut quælibet pars quinque gradus ſigniſicet, vel in Inſtrumentis majoribus in quinque partes æquales poſſint dividi quarum quælibet duos gradus Anomaliæ denotabit. Hæ partes à puncto terræ Aphelio per 10, 20, 30, &c. ad 360 numerandæ ſunt. Atque hoc modo Eccentrici *Saturni* & *Terræ* debite proportionati diſponuntur, & dividuntur.

the Earth, is to be divided into its juſt Anomalar parts. Each tenth of which may be inſcribed by the numbers of the Table C, *which are placed under the word of* Earth, *by a ruler laid to the Center of the Sun, and to ſuch degrees and minutes of the ſignes in the Zodiac, as ſhall be given out of the forementioned Table. And theſe* 10ths *may be biſected, & ſo each diviſion may ſignifie 5 deg. Or elſe each of them may be divided into 5 equal parts, every one of them ſignifying 2 deg. of Anomaly: this is to be done in larger Theorics. Theſe Anomalor parts of the Earth are to be numbred from their Aphelial point, by 10, 20, 30, and to 360. Thus are the Eccentrics of* Saturn *and the* Earth *to be proportioned, placed, and divided.*

Eodem pariter modo in Theoricis *Martis* & *Jovis* operandum eſt, uſurpando columnas *Marti* & *Jovi* deſtinatas in Tab. A, unâ cum columnâ terræ & quales numeri pro *Saturno* ex Tabula A tales pro *Marte* & *Jove* ex Tabula E & D deſumendi ſunt.

In the ſame manner you are to work for the Theorics of Mars *and* Jupiter, *if you uſe the columnes of* Mars *and* Jupiter *in the Table* C, *together with the columne of the* Earth: *and what numbers were taken for* Saturn *out of the Table* A, *the like numbers muſt be taken out if the Tables* E *and* D *for* Mars *aud* Jupiter.

Similiter per *Terrâ*, *Marte*, & *Mercurio:* qui tres ex una laminarum facie collocandi ſunt. Linea terræ Aphelia à centro Solis

So alſo for the Earth, Venus, *and* Mercury. *Theſe three are to be placed together upon one ſide of one of the plates. The*

Solis ad punctum *Terræ* aphelium extensa & in 100000 divisa inservit pro decimali scalâ ad inserendos omnes numeros eccentricos horum trium Planetarum. Ex hâc scalâ numeri proportionandis eccentricis *Terræ*, *Veneris* & *Mercurii* in tabulis B, F & G, desumantur. Quorum lineæ Apheliæ & divisiones graduum Anomalorum disponuntur, & determinantur per columnas tabulæ C, istis Planetis respondentibus: regulâ ut antea ad centrum fixâ, & ad signa, & gradus Zodiaci super has Theoricas ducendos applicata.

The decimal scale for all the numbers of eccentricity for these 3 Planets, is the Aphelial line of the Earth, *reaching from the Center of the Sun to the Aphelial point of the* Earth, *divided into* 100000 *equal parts. And out of that scale the numbers of the* Earth, Venus *and* Mercury *in the Tables* B, F *and* G, *must be taken for the proportioning of their eccentrics. And the right placing of their Aphelial lines, with the divisions of their Anomalar degrees, must be limited by the columns of the Table* C, *which answer to those Planets: a ruler being laid from the Center of the Sun to the signes and degrees of the Zodiacal limbes drawn upon the Theorical plates.*

Minores istæ Tabulæ numerales pro colligendis Anomaliis Terræ reliquorumque Planetarum eodem modo cuique orbitæ inscribantur, prout in scematibus appareat. Et iidem sunt numeri posteà in Anomaliarum Tabulis transcripti.

The little numeral Tables, for gathering the Anomalyes of the Earth and any Planet, may be written to each orbit, in such fashion as my draughts of these Theorics doe shew: & are the same numbers that are set down in the Tables of Anomalyes hereafter specifyed.

Tabulæ numerales pro Terra bis repetuntur in utrâque laminâ semel. *viz.* in Theorica *Martis*, & in illis *Veneris* & *Mercurii* eo fine ut utraque lamina cursum terræ teneret absque alterius ope. Et istic loci disponuntur quia non datur alius magis conveniens.

The numeral Tables for the earth are twice written, upon each plate once; namely, in the Theoric of Mars, *and in that of* Venus *and* Mercurie; *to the end that each table might have the earths motions upon it, without being beholden to the other. And they are there set,*

Circuli

because

Circuli enim terræ in Theoricis *Saturni* & *Jovis* nimis sunt parvi ad eas commodè tenendas.

because in those two places onely is convenient roome for them. For, the Circles of the earth upon the Theorics of Saturn *and* Jupiter, *are two little to hold them.*

3 *De scalis Distantiarum.*

IN singulis Instrumenti faciebus scalæ partium æqualium describuntur ad metiendas distantias Planetæ tam à Sole quàm à Terrâ inscribuntur in lineis Apheliis exterioris Planetæ, *viz.* in Apheliis *Saturni*, *Jovis*, *Martis* & *Terræ* Determinantur ex tabulâ H, & ratio hujus limitationis est ut ejusdem proximè essent ad invicem magnitudinis, & interim numeros admitterent ad semidiametros sine magno labore reducibiles.

Modus conficiendi videatur in exemplo *Saturni*. Numerus *Saturni* in tabulâ H est $85\frac{63}{100}$ si igitur (ope *Sectoris* aut aliter) hujus Planetæ lineam Apheliam (ex Theoricâ) à Solis centro ad *Saturni* Aphelium sumpseris, & *Sectoris* crura ad hanc longitudinem in terminis $85\frac{63}{100}$ in lineâ partium æqua-

3 Concerning the scales of distance.

U*Pon every side of the two Plates, there are scales of equal parts to measure the distances of the Planet from the Sun and from the Earth. They are inscribed upon the Aphelial lines of the exteriour Planet: namely, upon the Aphelial lines* Saturn, Mars, Jupiter, *and* Earth. *The limiting of them is taken from the table* H: *and the reason of this limitation is , because they should be of somewhat neer an equal bigness one to another, and yet also that they might be of some such numbers that may be reduced to semidiameters without any great trouble.*

The manner of making them, may be seen in the example of Saturn. *The number for* Saturn (*in the table* H) *is*, $85\frac{63}{100}$ *If therefore (by help of the Sector*, or *otherwise) you take the Aphelial line of this Planet (out of the Theoric) from the center of the Sun to the Aphelial point of* Saturn, *and open the*

æqualium aperueris habebis numeros quos volueris rotundos utpote 80, 70, &c. pro hujus scalæ divisionibus. Qui à sectore ad lineam Apheliam à puncto *Saturni* Aphelio translati dabunt longitudinem 80, 70, &c. partium in scalâ æqualium quas denuo dividas &prout in schemate continues in *Saturno*, & *Marte*, ad 100 in *Jove* et *Terra* ad 120. Integra scala non necessario dividitur in plures 10 partibus largioribus quarum supremæ in 10 minores subdivisæ (prout moris est) numeri apponantur ut in schematibus videre est.

the Sector to that extent, in the number $85\frac{63}{100}$ *in the line of equal parts, you shall then have any even number or divisiō from the same scale of equal parts, as of* 80, *or* 70, *&c. which being taken from the Sector, and transferred to the Aphelial line, and being set thereon, from the Aphelial point of* Saturn, *you shall have the length of* 80 *or* 70 *of those equal parts. These you may divide and continue as farre as they are in my Theorics: namely, in* Saturn, *and* Mars, *to* 100, *in* Jupiter *and the* Earth *to* 120. *You need not divide the whole scale any more then into* 10 *large parts, and the uppermost of them alone may be sub-divided into* 10 *lesser equal parts. After which they are to be numbred in such manner as is usual in such decimal scales, and as in those Theorics is to be seen.*

Sic in *Jove* dividendum est spatium ab Aphelio ad Solis centrum in $92\frac{87}{100}$ & ita de reliquis juxta numeros Tabulæ H.

So for Jupiter, *you are to divide the space from his Aphelial to the center of the Sun, into* $92\frac{87}{100}$, *and so all the rest accordingly as their numbers, in the Table* H, *do require.*

4 *De Nodis & scalis inclinationum.*

4 Of the Nodes and scales of inclination.

Usus Tabulæ M est ad inserviendos nodos quinque Planetarum nam Terra nullum habet

The Table M *serves to put in the Ascendent* Nodes *of the* 5 *Planets; for the Earth hath*

habet. Methodus videatur in exemplo *Saturni*. Nodus *Saturni* aſcendens eſt 22 grad. 27 min. *Cancri*. Poſitâ igitur regulâ à centro Solis ad 22 gr. 27 min. *Cancri* : in limbo delebilem ducas lineam quæ erit communis ſectio plani eccentrici Planetæ, & Eclipticæ. In hâc lineâ duo quælibet puncta oppoſita æqualis utrinque à centro diſtantiæ aſſumas ut in ſchemate ad characteres ♄ ♄, ob planum in quo curſus *Saturni* deſcribitur. Per hæc duo puncta ducitur ellipſis punctis diſterminata (vel aliâ circularis quælibet ad libitum figura) in cujus altera medietate (iſta ſcilicet) quæ à 22 grad. $\frac{1}{2}$ *Cancri*, juxta ſeriem ſignorum procedit) ſcribatur *SATURNI Inclinatio Borea*. In reliquâ *SATURNI Inclinatio Auſtrina*.

Minor ſcala ad metiendas *Saturni* inclinationes terminos habet et ſuos limites in hunc modum. Inſpicè Tabulam N, ubi invenies maximam *Saturni* inclinationem 2 gr. 32. m. Cape igitur diſtantiam alterutrius

hath none. The manner of it may be ſeen in the example of Saturn. Saturns *Aſcendent Node is in the 22 deg. 27 min. of* Cancer. *Therefore laying a ruler from the Center of the Sun to the 22 deg. 27 min. of* Cancer *in the limbe, you may draw an obſcure line at length : this line is the common ſection of the plain Planets eccentric with the plain of the Ecliptic. In this obſcure line you may aſſume any 2 points, oppoſite one to the other , and of equal diſtance from the Suns Center on both ſides, as is done in my Theorics at the characters of* ♄ ♄, *for the plain on which the courſe of* Saturn *is drawn. Through which two points is drawn a prickt ovall (which might have been of any other compaſſing form, as a Circle, or the like) in the one half of which (namely, that which goes from the* 22$\frac{1}{2}$ *deg. of* Cancer , *according to the ſeries of the 12 ſignes) is written* SATURNI Inclinatio Borea ; *and on the other half is written* SATURNI Inclinatio Auſtrina. *So this particular is done.*

Then for the little-ſcale, which is to be the meaſure of Saturns *inclinations , that is thus to be limited. Look in the Table* N , *where you ſhall ſee the greateſt inclination of* Saturn *to be 2 deg. 32. min. Take then*

utrius puncti (notati ♄, ♄) à centro Solis, & ad hanc distantiam aperiantur crura sectoris in lineâ partium æqualium à terminis $2\frac{32}{60}$.

Ex sectore sic aperto capias distantiam in terminis 3, 3, in lineâ partium sectoris æqualium tres partes ex quæ longitudinem dabit scalæ notatæ 1, 2, 3, ad mensurandas *Saturni* inclinationes. Quæ in tres partes, significantes tres gradus, quarum singula in quatuor aliàs æquales dividatur. Hoc modo opus harum linearum in Theoricis *Saturni* peragitur.

Similiter faciendum est pro reliquis Planetis usurpando numeros illis pertinentes & in Tabulis M & N expressos. Ampliore igitur non opus erit directione.

then the length or distance of either of the fore-named two points (noted with ♄ ♄) from the Center of the Sun, and with that distance, open the sector in the line of equal parts from $2\frac{32}{60}$.

When the sector is so opened, you may take off 3 in the line of equal parts, and that shall give the length of that Scale which is to measure the inclinations of Saturn, *noted with* 1, 2, 3. *This scale may be divided into 3 equal parts: first, which are to signifie 3 degrees: and these again may be quartered. This is the work to be done for these lines upon the* Theoric *of* Saturn.

The like must be done for every other Planet, by making use of the nnmbers belonging to each of them, expressed in the Tables M *and* N. *There will therefore here need no more direction.*

5 *De Scalis Latitudinum.*

IN utrâque laminâ, & super istam faciem ubi Theoricæ *Martis* & *Veneris* ducuntur una istiusmodi scala describitur, ut neutra alterius indigeat. Linea à Solis Centro ducta est partium 120 æqualium. Arcus seu scala curvilinea super

5 Concerning those Scales that are to find the Latitudes.

T*Here is upon each of the two plates one of this sort of scales, that so one plate may have no need to seek help from the other. They are drawn upon those sides on which* Mars *and* Venus *are placed. The line drawn from the Center of the* Sun,

ſuper priorem pendens in 10 grad. diſpeſcitur *Martis* Tabula Q, *Veneris* Tabulâ notatâ R, quod varietatis tantùm cauſâ fit nam aliter Tabula Q ſola utrique ſatisfeciſſet. Sed hæc cautio obſervata digna eſt, quod ſcilicet recta à Centro Solis ad peripheriam tendens, juſtum aliquem Zodiaci gradum ſecet. Quia gradus iſti Tabulares (per quos inæquales ſcalarum partes expenduntur) ex limbi gradibus ſumi debent, & propterea commodiùs, & ad faciliorem numerationem lineâ prædictâ in æqualem gradum cadat.

Atque hoc modo Theoricæ ſcalis ſatis commodis ad inveniendas tam Longitudines quàm Latitudines quinque Planetarum inſtruuntur. Reliquæ de quibus dicendum reſtat accomodantur ad convertendas Longitudines, & Latitudines in Declinationes, & Aſcenſiones Rectas.

6 *De Scalis Aſcenſionum Rectarum.*

SCalæ Aſcenſionum Rectarum, & Declinationum in Planis *Saturni* & *Jovis* deſcribantur, quia magis amplum eſt

Sun is an equal ſcale divided into 120 parts. The arke or curved ſcale which hangeth upon the former, is divided into 10 degrees; that upon Mars, *by the Table noted with* Q: *that upon* Venus, *by the Table* R. *They might have been done both by one Table (as by that with* Q*) but onely for variety. This caution alone is here to be obſerved, namely, that the ſtreight line comming from the Center be made to but upon ſome juſt degree of the Zodiac or limbe: becauſe thoſe degrees in the forementioned Tables (by which the un-equal parts of the annexed ſcales are limited out) are to be taken in the limbe. And therefore it will be moſt expedient for eaſe in account to let the line point upon ſome even degree.*

Thus theſe Theorics are fitted with ſcales ſufficient for the finding out of the Longitudes and Latitudes of the 5 Planets. The other ſcales that yet remain to be ſpoken of, are fitted to turn the Longitudes and Latitudes into Right Aſcenſions and Declinations.

6 Concerning the Scales for Right Aſcenſion.

THeſe *ſcales for Right Aſcenſions with thoſe of Declinations, are ſet upon the planes of* Saturn *and* Jupiter, *becauſe*

est in illis spatium ad eas commodè tenendas.

1 (In loco convenienti) ducenda est lineâ rectâ, & à Centro Solis arcus describendus commodæ attamen arbitrariæ distantiæ cum numeris 1, 2, 3, ex utrâque parte lineæ rectæ adfixis. Gradus isti 1, 2, 3, sunt etiam arbitrarii, interim quantitatis aptæ recipiendis Ellipticæ figuræ divisionibus adeò amplis ut distinctè in quatuor equales partes possint dividi.

2 Ex utrâque parte lineæ rectæ mediæ in scalâ Circulari sic divisâ numera 2 gr. 29 min. per quorum terminos à Centro Solis duc duas lineas delebiles.

3 Intra lineas obscuras duc cujusvis formæ Ellipsim ita tamen ut ejus extremitates justè tangant prædictas lineas delebiles per grad. 2. 29 min. ductas.

4 Huic figuræ ovali inscribantur graduationes ope Tabellæ W, quintus aut decimus quilibet gradus inseri potest reliquis tantum æqualiter divisis. Ordo characterum, numerationis, & divisionis modus videatur in schematibus. Atque hæc pro ratione conficiendi has scalas.

because their is most roome to hold them.

1 *There is first a right line drawn (in some convenient place) without any divisions upon it, and upon the Center of the Sun and ark described at any fit distance, numbred with* 1, 2, 3, *on both sides the right line. The degrees* 1, 2, 3, *are of any arbitrary length, so large that the oval figure may be of some quantity to receive a fit number of divisions, and that the same divisions may receive sub-divisions into large quarters. This is the first work.*

2 *Upon the Circular scale so divided, count* 2 *deg.* 29 *m. on both sides the middle right line, and through these limits draw two obscure right lines from the Center of the Sun.*

3 *Within these two obscure lines, draw an oval figure of any forme, but so, as that the two extreme parts of it may justly touch the two former obscure lines drawn through* 2 *d.* 29 *minutes.*

4 *After this oval figure is drawn, it is also to be graduated by help of the Table* W; *you may put in onely every* 5th & 10th *d.* & *whē they are put in, the rest of the lesser parts may be inserted by equal subdivisions. The order of their characters, and numeration, and the manner of their division, may best be seen in my Theorics.*

Theorics. This will serve for direction to make these scales.

7 *De scalis Declinationum.*

HÆ super iisdem Theoricarum planis quibus scalæ A rectarum insistunt.

1 A Centro Solis ducatur rectâ lineâ. Cujus extremitas Soli proximâ dividatur in 10 partes æquales, quarum quælibet quadri secetur [sin ulterius procedere in animo sit inæqualiter instar tangentium dividenda est] hæc scalâ etiam est arbitrariæ modo, recipiendis minoribus divisionibus, commodæ sit longitudinis.

2 A Centro Solis & super istâ lineâ describitur arcus Circuli continentis ex utrâque parte lineæ rectæ 25 gr. istiusmodi quales integer Circulus contineret 360 numeris utrinque ad fixis 00, 5, 10, 15, 20, 25, &c.

3 Ultra hunc arcum Circuli, ducitur lineâ rectâ infinite protensa quæ priori ductæ insistit ad rectos, & posteà terminatur regulâ à Centro Solis utrinque per gradus Circuli

7 Concerning the scales for Declinations.

T*Hese stand upon the same plaines of the Theorics, with the other scales of right ascension.*

1 *Here is first drawn a streight line from the Center of the Sun. That part which is neerest to the Center is divided into* 10 *equal parts [but if they should goe further then* 10, *they must then be unequal as Tangents are] standing for degrees : and each of them is cut into quarters. This scale of* 10 *degr. is not limited, but may be of any fit length for the subdivisions.*

2 *From the Center of the Sun and upon this line, is described an ark of a Circle, which contains upon it (on each side of the streight line formerly protracted)* 25 *true degrees (such as the whole circle should contain* 360) *which are accordingly numbred on both sides, from* 00, *to* 5, 10, 15, 20, 25.

3 *Without this Circular ark is set a line perpendicular to that first drawn, and extended at length on both sides, but afterwards it is to be limited, by laying a ruler from the*

Circuli 23 grad. ½ dimissâ: Atque ita lineæ ductæ per 23 grad. ½ ad *Cancrem* & *Capricornum* justos hujus perpendiculi limites distinguent. Dividitur verò hæc linea utrinque per Canonem sinuum: quilibet quintus decimusque gradus à cæteris distinguitur, & trigesimus quisque duplici charactere signi alicujus insignitur, prout in schemate videre licet.

the Center of the Sun to 23 ½ d. counted upon the Circular ark both wayes: so shall lines drawn through these 23 ½ deg. give just limits to this perpendicular line, at Cancer *and* Capricorn. *The divisions of this line are nothing but a double scale of sines. Every 10th and 5th degree is to be distinguished from the rest, and every 30th degree is to be double charactered with some or other of the 12 signes, as is to be seen in my Theorics.*

4 Quartò, In loco commodo describenda est altera figura ad libitum Elliptica. At eâ conditione, ut ejus extremitates directè tangant delebiles istas lineas prius per gradus arcus circularis 23 ½ ductas.

4 *Again, there must an oval be here described, it may be of any fashion, but must be set in place convenient, and in such manner, that it may lye justly between the two former obscure lines drawn through 23 ½ degrees touching them with its extremities.*

Divisiones imponuntur ope Zodiaci recti linei prius descripti applicando regulam ad initium cujusque signi, & in hanc ovalem transferendo. Inscriptio initiorum sufficiet, nam gradus ex Zodiaco rectilineo desumendi sunt. Et ista ovalis divisio non fit alio fine nisi ad commodius transferendos gradus Zodiaci prioris, nam in hoc novo signa contrario stant ordine quam in priori *Cancro* cum *Capricorn* in medio *Aries* & *Libra* ad extremitates.

The divisions of it are to be taken from the former streight charactered Zodiac, by laying a ruler from the Center, to the beginning of each of those signes, and so transferring them into this oval. This inscription of the onely beginnings of the 12 signes into the oval is sufficient: for the degrees of these 12 signes must be taken out of the former streight Zodiac; this new division being onely added for conveniency of new charactering the degrees of the old Zodiac. For in this new one you

5 Remanet adhuc Scala altera ſinuum rectorum ad gradus circiter 35, ubicunque volueris inſerenda quæ ſic determinabitur. Cape longitudinem Zodiaci rectilinei ab *Ariete* ad *Canceri* vel *Capricorni*, ad quam aperiatur Sector (commodiſſimè enim perficitur per illud inſtrumentum) in lineis ſinuum & in terminis 23½. Deindè transferantur ſinus 35 grad. in hanc lineam rectam & ſic in partes debitas dividetur. Exemplar omnium videas in ſchematibus.

Hucuſque progreſſus ſum in declaratione Methodi quâ hæ Theoricæ cum omni earum apparatu, conſtruendæ ſunt ſequuntur Tabulæ anteà ſæpiùs nominatæ, ad plurima tam inſerenda quàm determinanda neceſſariæ.

Cancer *and* Capricorn *to ſtand in the middle*, *and* Aries *and* Libra *in the two extream places*, *contrary to what they did in the former* Zodiac.

5 *One Scale yet more remaines*, *containing the right ſines of* 35 *degrees. It may ſtand any where*, *and is thus to be limited. Take the length from* Aries *to* Cancer *or* Capricorn, *in the ſtreight Zodiac*, *and with that length open the* Sector (*for it is ſooneſt done by that inſtrument*) *in the line of ſines from* 23½ *degrees thereon. Then from the* Sector *ſo opened, take the ſeveral ſines of* 35 *degrees, and inſert them into this line, ſo it ſhall be divided into its requiſite parts. The pattern of theſe things may be ſeen in my Theorics.*

Thus farre I have gone in declaring the manner how theſe Theorics are made in all their particulars. There now follow the Tables that are mentioned before, by which many things are to be divided and limited.

	Saturni	*Jovis*	*Martis*
Sit diſtantia Aphelii à centro	1000000	100000	100000
Erit Eccentricitas.	053870	04600	08479
Ab Aphelio ad centrum Eccentrici	946130	95400	91521
Diſtantia Aphelii Terræ à centro	101279	18676	61154
Eccentricitas Terræ	001791	00330	01081
Ab Aphel. Terræ ad centr. Eccentr. Terræ	099488	18346	60073
	A	D	E

	Terra	*Veneris*	*Mercurii*
Si diſtantia Aphelii Terræ à centro Solis ſit, 100000;			
Erit diſtantia Aphelii	100000	71625	46126
Eccentricitas	01768	00491	08006
Ab Aphelio, ad centr. Eccentrici	98232	71134	38120
	B	F	G

C C C

Anom. med.	Earth	♄	♃	♂	♀	☿
360	♑ 7 00′	♐ 27 30′	♎ 7 49′	♍ 0 21′	♒ 2 49′	♐ 14 57′
10	16 39	♑ 6 26	16 55	8 42	12 41	21 38
20	26 19	15 24	26 02	17 05	22 33	28 22
30	♒ 5 59	24 26	♏ 5 12	25 32	♓ 2 25	♑ 5 12
40	15 42	♒ 3 31	14 25	♎ 4 06	12 19	12 11
50	25 27	12 43	23 45	12 48	22 13	19 23
60	♓ 5 14	22 03	♐ 3 11	21 41	♈ 2 08	26 50
70	15 05	♓ 1 21	12 44	♏ 0 48	12 04	♒ 4 38
80	24 59	11 10	22 26	10 09	22 02	12 51
90	♈ 4 56	20 59	♑ 2 18	19 48	♉ 2 01	21 33
100	14 58	♈ 1 00	12 20	29 45	12 02	♓ 0 51
110	25 03	11 13	22 31	♐ 10 01	22 04	10 51
120	♉ 5 12	21 39	♒ 2 53	20 38	♊ 2 08	21 39
130	15 24	♉ 2 16	13 25	♑ 1 36	12 12	♈ 3 21
140	25 39	13 04	24 06	12 53	22 18	16 00
150	♊ 5 57	24 01	♓ 4 54	24 28	♋ 2 25	29 37
160	16 17	♊ 5 07	15 49	♒ 6 17	12 33	♉ 14 08
170	26 38	16 17	26 48	18 16	22 41	29 21
180	♋ 7 00	27 30	♈ 7 49	♓ 0 21	♌ 2 49	♊ 14 57
190	17 22	♋ 8 43	18 50	12 26	12 57	♋ 0 33
200	27 43	19 53	29 49	24 25	23 05	15 46
210	♌ 8 03	♌ 0 59	♉ 10 44	♈ 6 14	♍ 3 13	♌ 0 17
220	18 21	11 56	21 32	17 40	13 20	13 54
230	28 36	22 44	♊ 2 13	29 06	23 26	26 33
240	♍ 8 48	♍ 3 21	12 45	♉ 10 04	♎ 3 30	♍ 8 15
250	18 57	13 47	23 07	20 41	13 34	19 03
260	29 02	24 00	♋ 3 18	♊ 0 57	23 36	29 03
270	♎ 9 04	♎ 4 01	13 20	10 54	♏ 3 37	♎ 8 21
280	19 01	13 50	23 12	20 33	13 36	17 03
290	28 55	23 29	♌ 2 54	29 54	23 34	25 16
300	♏ 8 46	♏ 2 57	12 27	♋ 9 01	♐ 3 30	♏ 3 04
310	18 33	12 17	21 53	17 54	13 25	10 31
320	28 18	21 29	♍ 1 13	26 36	23 19	17 43
330	♐ 8 01	♐ 0 34	10 26	♌ 5 10	♑ 3 13	24 42
340	17 41	9 36	19 36	13 37	13 05	♐ 1 32
350	27 21	18 34	28 43	22 00	22 57	8 16

Quomodo Tabula præcedens tépori futuro accommodetur.

IN 100 annis Aphelia & Nodi Planetarum progrediuntur, ut in adjunctâ Tabellâ.

How to make the præcedent Table ſerve for times to come.

IN 100 *years, the Aphelia and Nodes of the Planets move forward thus much,*

	Aphelia	Nodis
Earth	1, 712	
Saturn	2, 102	1, 985
Jupiter	1, 311	0, 097
Mars	1, 860	1, 104
Veuns	2, 168	1, 306
Mercur.	2, 912	2, 368

K

F Per

Per hos numeros Tabulæ præcedentes (ad annum 1673 completum constructæ) ad alium quemlibet adaptari possunt. Tabulæ istæ notatæ C (quas solummodò intelligo) prout nunc sunt ad annum 1700 inservient. Post periodum istam adimpletam ad annum 1730 ad 30 (scilicet) annos sequentes accommodari possunt, & tunc ad 1760 fœliciter inservient. Nam in 30 annis Nodi progressum faciunt adjunctæ tabulæ, qui in eruendis Latitudinibus non causabit errorem plus $\frac{1}{8}$ gr. in ipsis *Marte* & *Venere* ubi error erit maximus.

♄	36
♃	02
♂	23
♀	23
☿	43

Repeto igitur has Tabulas notatas C, factas esse ad 1763 completum quas si desideras rectificare ad annum 1730 completum. Primo sume differentiam horum annorum (sc.) 57, & in hunc numerum duc progressus Aphelios Tabulæ K. Abscissis quinque dextimis figuris residuum erit gradus. Fractio decimales graduum partes, quæ in sexagesimas facile converti possunt. Et deinde numeri sic inventi addendi sunt numeris Planetarum respectivis in Tabula C, atque ita ad annum 1730 rectificantur.

Eodem

And by these numbers, the Tables precedent (which are made to the year 1673 complet) may be fitted to any year to come. For these said Tables (those noted with C, *I onely speak of) as they now are, will serve till the year 1700. And afterwards they may be fitted to 1730; that is, for 30 years to come, after that period of time, and so they will serve in use till 1760 very well. For in 30 years the Nodes make this progresse onely, which in their latitudes will not erre above $\frac{1}{8}$ of a degree, no not in* Mars *and* Venus, *in which two Planets this errour must be greatest.*

♄	36
♃	02
♂	23
♀	23
☿	43

I say these tables noted with C, *are made for the year 1673 complete. And if you would rectifie them to the year 1730 complete, you are first to take the difference of these two years, 1673 and 1730, which will be 57: and by 57 multiply the Aphelial numbers or progresses at* K, *and from the product cut off the 5 last figures; the remainder shall be the degrees, and the fraction shall be the decimal parts of degrees, which will easily be turned into sexagesimal parts. And then the number so found out for each Planet, must be added respectively to every number of his proper Planet in the precedent Table* C:

Eodem modo rectificabis Nodorum loca multiplicando per 57 motum eorum in Tabula K, ut antè correctio deindè cuique Planetæ reſpective eſt addenda juxta motum in Tabulâ M expreſſum.

M

		d.	′	
Aphelia Planetarum ad An. 1673.	*Earth*	6	59	*Cancer*
	Saturn	27	30	*Sagit.*
	Jupiter	7	49	*Libra*
The Aphelia of the Planets ſtand thus in 1673.	*Mars*	0	21	*Virgo*
	Venus	2	49	*Aqua.*
	Mercury	14	57	*Sagit.*

Aphelia, & Nodii (rigidè ſumpti) non ſunt fixi ſed continuo moventur minimò ſpatio. Interim quia motus eſt tardiſſimus (quòd ad hoc Inſtrumentum) abſque notabili errore per aliquot annorum ſpatium fixâ imaginemur.

Error enim oriens ex Nodis fixis in annis 30, non excedit 8 min. ſcrupula prima in ipſis *Marte* & *Venere*, ut anteà monſtratum. Error etiam ex fixis Apheliis in 30 annorum curſu erit circiter 31 min. in Terra vel Sole, 38 min. in *Saturno*, 24 min. in *Jove*, 33 m. in *Marte*, 39 min. in *Venere*, 52 min. in *Mercurio*. Error ſanè in his Inſtrumentis ſatis tolerabilis.

C: *and ſo the numbers of that Table ſhall be rectified for the year* 1730.

In the ſame manner you may rectifie the places of the Nodes by multiplying the former numbers of the Nodes *motion at* K, *into* 57, *&c. as before. Then the corrections muſt be added to each Planet reſpectively according as the places of their Nodes are expreſſed in the Table* M.

M

	d.	′		
Cancer	22	27	*Saturn*	Nodi Plan. Aſcendentes ſic ſtant Anno 1673.
Cancer	5	30	*Jupiter*	
Taurus	17	33	*Mars*	*The Aſcend. Nodes of the Plan. ſtand thus in 1673.*
Gemini	13	58	*Venus*	
Taurus	14	09	*Mercu.*	

The Aphelia, and Nodes ought not to ſtand ſtill (in rigour) but to move continually ſome ſmall quantity. Yet becauſe theſe motions are very ſlow, they may be permitted to ſtand ſtill for ſome number of years without much prejudice to theſe Planetary Inſtruments.

The errour of Latitude *wch ariſeth from the immobility of the Nodes, is in* 30 *years (even in* Mars *and* Venus*) not above* 8 *minutes, as was ſhewed before. And the errour in Longitude, which ariſeth by reaſon of the immobility of the Aphelia, will in* 30 *years time be about* 31 *minutes in the* Earth *or* Sun; 38 *min. in* Saturn; 24 *min. in* Jupiter; 33 *m. in* Mars,

39 *min.*

39 min. in Venus ; *52 min. in* Mercury ; *which may well be endured in these mannuary Theorics.*

			gr.	′		
N	Maximæ Planetarum Inclinationes.	*Saturn*	2	32	*The Planets greatest Inclinations.*	N
		Jupiter	1	19		
		Mars	1	$50\frac{1}{2}$		
		Venus	3	22		
		Mercury	6	54		

Distantia Apheliorum dividendæ sunt per numeros cuique Planetæ in Tabula H adjunctos, ultra Centrum in iisdem partibus quousque opus fuerit continuandæ. Sic distantiam *Solis* à *Terrâ* comparaveris in Semidiametris *Terræ*. Si primò, in propriâ cuique Planetæ scalâ mensuraveris, & secundò, si *Saturni* distantiam multiplicaveris in 400, *Jovis* in 200, *Martis* in 100, *Veneris*, *Mercurii*, & *Terræ* in eâdem, cum illis Tabula per 50 numeros facile ob eorum proportionem subduplam in memoriâ retinueris.

Let the Aphelial distances be divided into these numbers here set to every Planet, and continued in the same parts beyond the Center, so farre as is needfull. So shall their distances from the Earth and the Sun be had in semidiameters of the Earth ; If first they be measured upon their proper scales : and secondly, if Saturns *distance be multiplyed by* 400 ; Jupiters *by* 200, Mars *his distance by* 100; Venus, Mercury *and the* Earth *upon the same side with them by* 50. *Which numbers may be easily remembred, because they goe in a subduple proportion.*

H

Saturn	$85\frac{36}{100}$
Jupiter	$92\frac{87}{100}$
Mars	$56\frac{73}{100}$
The Earth	$69\frac{38}{100}$

R

Fitted to just 40 degrees.

	gr.	′
1	3	39
2	7	19
3	11	01
4	14	46
5	18	36
6	22	32
7	26	35
8	30	49
9	35	16
10	40	00

Q

Fitted to 60 degrees.

	gr.	′		gr.	′
	1	14		26	59
	2	28		28	23
	3	42		29	48
1	4	57	6	31	15
	6	12		32	43
	7	27		34	12
	8	42		35	43
2	9	57	7	37	15
	11	12		38	49
	12	28		40	26
	13	44		42	05
3	15	01	8	43	48
	16	18		45	34
	17	36		47	23
	18	54		49	15
4	20	12	9	51	10
	21	33		53	10
	22	53		55	27
	24	14		57	33
5	25	36	10	60	00

This Table is to devide the Oval in the Theorics, out of the equally divided 3 degrees.

	gr.	′		gr.	′
2	0	10	46	2	29
4	0	20	48	2	28
5	0	25	50	2	27
6	0	30	52	2	26
8	0	39	54	2	23
10	0	49	55	2	22
12	0	58	56	2	20
14	1	07	58	2	16
15	1	12	60	2	12
16	1	16	62	2	06
18	1	24	64	2	00
20	1	32	65	1	57
22	1	40	66	1	54
24	1	48	68	1	47
25	1	51	70	1	39
26	1	54	72	1	31
28	2	00	74	1	22
30	2	06	75	1	17
32	2	21	76	1	13
34	2	16	78	1	03
35	2	14	80	0	53
36	2	20	82	0	43
38	2	23	84	0	32
40	2	25	85	0	27
42	2	27	86	0	22
44	2	28	88	0	11
45	2	28½	90	0	00

W

Maxima obliquitas Eclipticæ.

deg.	′
23	31½
23	31
23	30

Maxima reductio

deg.	′	″
2	29	06
2	28	59
2	28	45

W

Epochæ. ANOMALIÆ *Epochæ.*

Ad Annos	Terræ Epocha	Saturni Epocha	Jovis Epocha	Martis Epocha	Veneris Epocha	Mercur Epocha
1644	194 80	119 90	229 28	299 78	238 78	61 55
52	194 72	217 62	112 08	30 97	240 15	139 27
60	194 64	315 33	354 88	122 15	241 53	216 99
68	194 57	53 04	237 68	213 34	242 91	294 71
76	194 49	150 75	120 48	304 52	244 29	12 42
84	194 41	248 46	3 28	35 71	245 67	90 14
92	194 34	346 17	246 08	126 89	247 04	167 86
100	194 26	83 88	128 88	218 08	248 42	245 58

Ad Meridiem primi diei Januarii, ſub Meridiano LONDINI.

Hæ Epochæ uti nunc ſunt durabunt ad 1700, & ulterius ab 8 in 8 annos continuabuntur hoc modo. Ab ultimâ Terræ Epochâ ſubducatur numerus Terræ affixus in Tabulâ adnexâ, *viz.* 0. 077, in reliquis Planetis ultimis eorum Epochis numeri affixi prout Tabula monſtrabit ſunt addendi Tabulæ motuum ſequentes nullâ indigent correctione, correctis enim Epochis nihil amplius reſtat corrigendum.

Theſe Epochaes do endure till 1700. *If it be required to continue them further for every* 8 *years, then from the laſt Epocha of the Earth muſt be ſubtracted the number here ſtanding by the Earth, namely,* 0. 077; *and in all the other Planets the numbers here ſet down muſt be added to the laſt Epocha of each of them ſtanding in the ſuperiour Table of Epochaes. All the correction that is requiſite is to be done in the Epochaes, in the reſt of the Tables of motions, which now follow, there will be no need of any ſuch things.*

Pro ſingulis annis.	*Earth*	000 . 077	Subtr	*For every 8 years.*
	Saturn	097 . 711	Adde	
	Jupiter	242 . 800	Adde	
	Mars	091 . 186	Adde	
	Venus	001 . 377	Adde	
	Mercury	077 . 719	Adde	

Motus

MOTUS ANOMALIÆ.

In annis	Earth	♄	♃	♂	♀	☿
1	359.74	12.21	30.33	191.27	224.27	53.69
2	359.49	24.41	60.66	22.53	89.54	107.38
3	359.23	36.62	90.99	213.80	314.32	161.08
4	359.69	48.86	121.40	45.59	180.69	218.86
5	359.71	61.06	151.73	236.86	45.46	272.55
6	359.45	73.27	182.06	68.13	270.23	326.24
7	359.19	85.47	212.39	259.39	135.00	19.93

In Mensibus Anni Communis.

	Earth	♄	♃	♂	♀	☿
Janu.	30.55	1.04	2.58	16.24	49.67	126.86
Febr.	58.15	1.97	4.90	30.72	94.52	241.45
Mart.	88.70	3.01	7.48	47.16	144.19	8.31
April.	118.27	4.01	9.97	62.88	192.25	131.08
Maj.	148.03	5.05	12.55	79.13	241.92	257.94
Jun.	178.[illegible]9	6.05	15.04	94.85	289.98	20.71
Jul.	208.95	7.09	17.62	111.09	339.65	147.57
Aug.	239.50	8.13	20.19	127.34	29.31	274.43
Sept.	269.07	9.23	22.68	143.06	77.38	37.20
Octob.	299.62	10.17	25.26	159.30	127.04	164.06
Nov.	329.19	11.17	27.75	175.02	175.11	286.83
Dec.	359.74	12.21	30.33	191.27	224.77	53.69

In Mensibus Anni Bissextilis.

	Earth	♄	♃	♂	♀	☿
Jan.	30.55	1.04	2.58	16.24	49.67	126.86
Febr.	59.14	2.01	4.99	31.44	96.13	245.54
Mart.	89.69	3.04	7.56	47.69	145.79	12.40
Arril.	119.26	4.05	9.95	63.41	193.86	135.17
Maj.	149.81	5.08	12.63	79.65	243.52	262.03
Jun.	179.38	6.09	15.12	95.37	291.58	24.80
Jul.	209.93	7.12	17.70	111.62	341.25	151.66
Aug.	240.49	8.16	20.27	127.86	30.92	278.52
Sept.	270.05	9.16	22.77	143.58	78.98	41.29
Octo.	300.61	10.20	25.34	159.83	128.64	168.15
Nov.	330.18	11.20	27.84	175.55	176.71	290.92
Dec.	360.73	12.24	30.41	191.79	226.37	57.78

MOTUS ANOMALIÆ.

In dieb.	Earth	♄	♃	♂	♀	☿
1	0.99	0.03	0.08	0.52	1.60	4.09
2	1.97	0.07	0.17	1.05	3.20	8.18
3	2.96	0.10	0.25	1.57	4.81	12.28
4	3.94	0.13	0.33	2.10	6.41	16.37
5	4.93	0.17	0.24	2.62	8.01	20.46
6	5.91	0.20	0.50	3.14	9.61	24.55
7	6.90	0.23	0.58	3.67	11.21	28.65
8	7.88	0.27	0.66	4.19	12.82	32.74
9	8.87	0.30	0.75	4.72	14.42	36.83
10	9.86	0.33	0.83	5.24	16.02	40.92
11	10.84	0.37	0.91	5.76	17.62	45.02
12	11.83	0.40	1.00	6.29	19.23	49.11
13	12.81	0.43	1.08	6.81	20.83	53.20
14	13.80	0.47	1.16	7.34	22.43	57.29
15	14.78	0.50	1.25	7.86	24.03	61.38
16	15.77	0.53	1.33	8.38	25.63	65.48
17	16.76	0.57	1.41	8.91	27.24	69.57
18	17.74	0.60	1.50	9.43	28.84	73.66
19	18.73	0.63	1.58	9.96	30.44	77.75
20	19.71	0.67	1.66	10.48	32.04	81.85
21	20.70	0.70	1.75	11.00	33.64	85.94
22	21.68	0.73	1.83	11.53	35.25	90.03
23	22.67	0.77	1.91	12.05	36.85	94.12
24	23.65	0.80	1.99	12.58	38.45	98.22
25	24.64	0.83	2.08	13.10	40.05	102.31
26	25.63	0.87	2.16	13.62	41.66	106.40
27	26.61	0.90	2.24	14.15	43.26	110.49
28	27.60	0.93	2.33	14.67	44.86	114.58
29	28.58	0.97	2.41	15.20	46.45	118.68
30	29.57	1.00	2.49	15.72	48.06	122.77
31	30.55	1.04	2.58	16.24	49.67	126.86

Sic tandem abſolvimus omnes Tabulas his Theoricis neceſſarias ad colligendas æquales ſive medias Anomalias in cujuſque diei Meridie. Quomodo autem concinne inſcribantur in Inſtrumentis, & unaquæque affixa Orbitæ, propriæ Planetæ convenientiſſimè diſponatur ad uſum, abſque reliqui operis impedimento in ſchematibus videre eſt.

Theſe are all the Tables that are to be ſet upon the Theorical plates, whereby the equal or Mean Anomalyes may be gathered to any day at Noon. The manner how they are to ſtand upon the two Plates with ſuch convenience that they may be ready for uſe, annexed each to the proper Orbit of its own Planet, without hindrance of the other work that is there drawn, may beſt be ſeen upon my Theorics.

FINIS.

OBSERVATIONES ECLIPSIUM.

Observatio Eclipsis Lunaris, Anno 1638, *habita ad* New-hous *propè* Coventriam. *Decembris die nono completo horis* 13, 58 *min. post meridiem.*

Presentibus & assistentibus JOHANNE PALMER, *&* JOHANNE TWYSDEN.

Obscuratio Circuli $\frac{1}{2}$ (i.e.) 7$\frac{1}{4}$ dig. *Rigel* alta 24 gr. 58$\frac{1}{2}$ min. Hora noctis 12 45 min. Obscuratio Diametri $\frac{2}{3}$ sive 8 digit, *Rigel* alta 24. 37, Hora noctis 12 gr. 50 m.

Illuminatio diametri $\frac{11}{12}$ sive 11 dig. paulò plus alt. *Arcturi* 31g. 56$\frac{1}{2}$m. Hora noctis 3 gr. 45 m. Versabatur *Rigel*, inter Meridiem & occasum *Arcturus* autem circa plagam Orientis.

Ascensio

The observation of the Moons Eclips, which happened at *New-hous* neer *Coventry*, the ninth day of Decemb. complete in the year 1638, 13 h. 58 m. after noon.

In the presence and with the assistance of JOHN PALMER, and JOHN TWYSDEN.

Half her Circle, that is to say, 7 dig. $\frac{1}{4}$ *were darkned when* Rigel *was high* 24° 58$\frac{1}{2}$ *m. The hour of the night* 12g. 45*m. Two thirds of her diameter, or eight digits, were obscured when* Rigel, *was high* 24 *gr.* 37 *m. the hour of the night* 12 *gr.* 50 *m.*

$\frac{11}{12}$ *Of the diameter enlightned or* 11 *dig. and some what more* Arctur. *high* 31 *gr.* 56$\frac{1}{2}$ *m. the hour* 3 *h.* 45 *m.* Rigel *was between the South & West.* Arcturus *upon the Eastern coast.*

The

Ascensio recta Solis ad mediam noctem post diem decimam Decemb. 268 gr. 49 m. ad horam 4m sequentem 269g.00 m. Ergo hora noctis ad observationem primam erat 12g. 45m. Ad observationem secundam 12 gr. 50 min. Ad observationem tertiam 3 gr. 45 m. Prout calculo accuratissimo patescit. Latitudo enim *Coventriæ* est 52 gr. 29 m. quod sæpè expertus sum. *Rigel* autem declinat 8 gr. 41 m. versus austrum, & ejusdem As. R. 74 gr. 20 min. Nam Longitudinem habet 71 gr. 49 m. latitudinem 31 gr. 11 m. ½ Australem *Arcturus* etiam declinat versus Boream 21 gr. 8 min. & Asc. R. habet 209 gr. 49. m. Nam longitudo stellæ 199 gr. 11 m. ½ latitudo Borea 31 gr. 02 min,

The right Ascen. of the Sun at midnight, after the tenth of December 268 *gr.* 49 *m. four hours after* 269 *g.* 00 *m. Therefore the hour of the night was at the first observation* 12 *gr.* 45 *m. At the second* 12 *gr.* 50 *m. At the third* 3 *gr.* 45 *m. as by an exact calculation it appeareth. For the latitude of* Coventry *is* 52 gr. 29 min. *as I have often made trial.* Rigel *declines* 8 *gr.* 41 *m. towards the South, and hath right Ascens.* 74 *gr.* 20 *m. For its longitude is* 71 *gr.* 49 *m. with Southern latitude* 31 *gr.* 11 *min.* ½ Arctur. *declines toward the* North 21 *gr.* 8 *m. and hath right Ascens.* 209 *g.* 49 *m. For its longitude is* 199 *gr.* 11 ½ *min. with* North *latitude* 31 *gr.* 02 *m.*

II *Observato Eclipsis Lunaris anno* 1641 *habita* Londini *in* Turri *ad clivum St.* Mariæ, *octavo die Octobris circa horam octavam post meridiem.*

II The observation of the Eclips of the Moon, made upon St. *Mary-hill* neer the *Tower* in London, the eighth day of October about 8 at uight,

INitium non visum densis nubibus impeditum.

Quadrans peripheriæ obscuratus quando horologium ostendit minuta 3 post quintam horam. Hora noctis 6 gr. 09 min.

Altitudo *Arcturi* 17 gr. 31 m. unde hora noctis 6 gr. 17. min. horolo-

CLouds hindred the sight of the beginning of it.

A quarter of the Moons periphery was obscured at three minutes past five by the clock.

The true hour of the night was then 6. 09 *m.*

The altitude of Arcturus 17 *gr* 31 *m. whence the hour of*

horologium autem ostendit 5 horam 10 m $\frac{1}{2}$

Circumferentiæ Lunæ $\frac{2}{3}$ obscuratæ indicante horologio 5 horam 32m. $\frac{1}{2}$ Inclinatio 90 gr. a Zenith.

Altitudo Arcturi 12 gr. 18 m. hora igitur noctis 6. 52 min. Indicante horologio 5. 43 m. *Arcturus* versabatur ad Occidentem.

Altitudo *Capellæ* 21 g. 03m. hora igitur noctis 7 gr. 26 m. indicante horologio 6, 14 $\frac{1}{2}$, inclinatio 53 $\frac{2}{3}$ a Zenith. Hæc observatio fuit accurata. *Capella* inter Septentrionem, & ortum sita.

Altitudo *Capellæ* 23 g. 35 m. Hora igitur noctis 7. 49 $\frac{2}{3}$, per horologium 6. 35 $\frac{1}{2}$. Circumferentiæ, & diametri Lunæ pars tertia obscurata. Inclinatio à Zenith versus austrum 45 gr. $\frac{5}{6}$. Si observatio istæc fuerit justa ita ut $\frac{1}{3}$ circumferentiæ Lunæ, & $\frac{1}{3}$ diametri fuerint eodem momento obscuratæ sequitur diametrum umbræ ad diametrum Lunæ 2 $\frac{1}{3}$ plani, vel prout 7 ad 3 *Copernicus* statuit 2 $\frac{2}{3}$ ferè. Ut 403 ad 150.

Altitudo *Capellæ* 25 g. 03 m. Hor. igitur noctis 8 hora 08 m. obser-

of the night was 6. 17 m. the clock shewed 5 hours 10 m. $\frac{1}{2}$.

$\frac{2}{3}$ *Of the Moons circumference were obscured when the clock shewed 5 hours 32 m.* $\frac{1}{2}$, *her Inclination from the Zenith was 90 gr.*

The altitude of Arcturus 12 g. 18 m. The hour of the night 6. 52 m. The hour of the clock 5. 43 m. Arctur. *was in the West quarter.*

The altitude of Capella was 21 gr. 03 m. The hour of the night 7, 26 m. The hour of the clock 6, 14 m. $\frac{1}{2}$. *The Inclination from the Zenith was* 53 $\frac{2}{3}$. *This observation was very exact.* Capella *was between the North and East.*

The Altitude of Capella *23 gr. 35 m. The hour of the night 7, 49* $\frac{2}{3}$ *m. The clock 6, 35* $\frac{1}{2}$. *A third part of the diameter of the Moon, and likewise of her circumference were obscured. The Inclination from the Zenith toward the South 45 gr.* $\frac{5}{6}$, *If this observation were true, so that a third part of the Moons diameter and periphery were both obscured at the same time, it followeth that the diameter of the shadow is to the diameter of the Moon 2* $\frac{1}{3}$ *of her plain, or as 7 to 3* Copernicus *makes it 2* $\frac{2}{3}$. *As 403, to 150.*

Capella *was high 25 g. 03 m. The hour of the night 8 g. 08 m.* This

observatio dubia. Per horologium 6 gr. 47 m. inclinatio $33\frac{5}{6}$. a Zenith.

Quadrans circumferentiæ Lunæ obscuratus indicante horologio 6 gr. 51 min. hora igitur noctis 7 h. 02 m.

Finis Eclipseos præcisus indicante horologio 7 h. 15 m. altitudo *Capellæ* 28 gr. 43 m. hora igitur noctis 8 h. 34 m.

Inclinatio 66 gr. 34 m. a Zenith. Altitudo *Arcturi* 8 h. 34 m. Hora noctis 7 h. 17 m. $\frac{1}{2}$

This observation is uncertain. The clock shewed 6 h. 47 m. Inclination from the Zenith $33\frac{5}{6}$.

A quarter of the Moons circumference obscured when the clock shewed 6 deg. 51 m. The hour of the night 7, 02 m.

The precise end of the Eclips at 7, 15 m. by the clock, the altitude of Capella *28 deg. 43 m. The hour of the night 8, 34.*

The inclination 66 deg. 34 m. from the Zenith, Arctur. *high 8 deg. 34 m. The hour of the night 7, 17$\frac{1}{2}$.*

Obscuratio maxima non ultra 6 $\frac{1}{2}$ Digit.

The greatest obscuration did not exceed 6 Digit $\frac{1}{2}$.

	Asc. R.	Decl. Bo.				
Arctur.	209. 52	21° 8′.	Locus Soils ♎ 25° 40′ Kepler.		Locus Sol. ♎ 25. 36	
Capellæ	72. 35		A. R. Solis 203. 47		Asc. Rect. Solis 203. 43	

Horæ noctis	6. 17.	6. 52.	7. 26.	7. 50.	8. 08.	8. 34
Horæ horologii	5. 10.	5. 43.	6. 14.	6. 35.	6. 47.	7. 15
Differentiæ	1. 07.	1. 09.	1. 12.	1. 15.	1. 19.	1. 19

Correctæ per horologium juxta hanc Proportionem.
Ut 2^h 05 m. ad 2^h 17 min. : : Ita &c.

Horæ noctis	6. 17.	6. 53.	7. 27.	7. 50.	8. 03.	8. 34
Horæ horologii	5. 10.	5. 43.	6. 14.	6. 35.	6. 47.	7. 15
Differentiæ æquabiliores.	1. 07.	1. 10.	1. 13.	1. 15.	1. 16.	1. 19

Minuta horologii automati $54\frac{3}{4}$ constituunt horam integram. Tota Eclipseos observatio, quoad quantitatem per conjecturam. Altitudines captæ per amplum quadrantem trium pedum in Semidiametro. Auto-

$54\frac{3}{4}$ In the clock made up a whole hour. The quantities of the parts Eclipsed were all estimated by guesse. The altitudes were all observed by a large quadrant of three feet in Semidiameter. The Clock was ex-

Automatum sanum & optimi artificii.

excellent good work.

Horæ Automati		Horæ respondentes veræ.	
h.	′	h.	′
5.	10	6.	17
5.	01	6.	07
5.	03	6.	09
5.	32½	6.	42
6.	51	7.	02

3 *Eclipsis Lunæ observata* Aubreæ, *in Agro* Somersetensi. *Latitudo Villulæ est* 51 *gr.* 10 *m. quod ex crebris ad Solem observationibus mihi innotuit.*

3 The observation of the Moons Eclips, as it happened at *Aubrey*, in *Somersetshire.* The latitude of that Village is 51 degr. 10 m. as it hath several times been observed by me.

Dubhe in Ursa *Majori.*

Longitudo ♌ 10 gr. 09 m. As. R. 160 gr. 17 m.
Latit. Borealis 49 gr. 40 m. Decl. B. 63 gr. 41 m.
Locus Solis ♎ 14 gr. 41 m. A.R. Sol. 193 gr. 29 m.

Anno Dom. 1642 Septemb. die 27, & nocte in sequente, vel nocte post diem *Martis*; & diem *Mercurij* publici jejunij proximè antecedente 1 Digit. diametri Lunæ obscuratus. Altit. Dubhe 34 gr. 30 m. inter Orientem, & Septentrionem. Hora igitur noctis erat h. 1, 54′ post med. noctem.

Immersio totalis Alt. Dubhe 39 gr. 15 m. inter Septent. & Ortum. Hora igitur noctis, h. 2, 48 m. ante meridiem diei 28.

In the yeer 1642 *Septem.* 27 *at night, or on* Tuesday *night preceding* Wednesday *a day of publicke fasting.*

1 *Dig. of the Moons diameter obscured. Dubhe high* 34 *deg.* 30 *m. between the East and North. The hour therefore of the night* 1, 54 *m. after midnight.*

The total immersion. Altit. of Dubhe 39 *deg.* 15 *m. between North and East. The hour of the night* 2, 48 *m. before the noon, of* 28 *day.*

11 Dig. Lunæ 54′ horæ quæ sunt differentia inter horas 1, 54′ & 2, 48′
12 Dig. Lunæ 58′ 55″ horæ. Quibus ex 2, 48′ subductis restant horæ 1, 49′; hâc igitur hora cæpit Eclipsis fere.

11 Dig. Moon 7 *54′ of an hour, which are the difference between 1, 45′ and 2, 48′*
12 Dig. Moon *58′ 55″ which being subducted out of 2, 48′ there remain 1 h. 49′*
the beginning of the Eclips, very neer.

4 *Eclipsis Lunæ observata* Londini *Anno* 1643 *Septem. die* 17, *inter horas Vespertinas* 7 & 8.

Latitudo 51 *gr.* 30 *m.*

ARcturi in Occidenti altitudo 19 grad. 30 min. Quando $\frac{1}{4}$ diametri Lunæ erat obscurata hora Vespertina 7 h. 23 min.

Emersio totalis. Inter observandum altitudinem stellæ filum quadrantis effractum est ita ut non potuit ullo modo emendari. Tempus inter eundem à loco observationis ad cubiculum (passibus & pulsibus æstimatum) erat circiter 8 horæ quo tempore pedis Australis *Andromedæ* altitudo in Oriente erat 34 gr. 38. Hora igitur post merid. fuit 7 h. 55 m. unde sublatis 8 m. restat tempus emersionis justæ hor. 7, 45 m. Tempus totius restitutionis.

4 The Eclips of the Moon observed at *London* Anno 1643 Septem. 17. Between 7 and 8 after-noon.

Latitude 51 deg. 30 m.

T*He altitude of* Arctur. *in the West* 19 *degr.* 30 *m.*

When $\frac{1}{4}$ *of the Moons diameter was obscured. The hour of the night was* 7 *deg.* 23 *m.*

The time of the total emersion, or of the full restitution of her light was 7*h.* 45.

	Arcturi	Australis ped. Androm.	
R. Ascen.	209 g. 53 m.	R. A. 25 g. 53. m.	Loc. Sol. ♎ 4 g. 20 m.
Decl. Bor.	21 g. 06 m.	Decl. B. 40 g. 36 m.	Asc. R. 183 g. 58 m.

5 *Eclipsis Lunæ observata* Londini *Anno* 1645 *Janua.* 31 *die* Veneris *inter horas Vespertinas* 7 & 9.

Latit. 51 *deg.* 30. *m.*

5 The Eclips of the Moon observed at *London* An. 1645 Janu. 31 upon *Friday*, between the hours of 7 and 9, at night.

The latit. 51 deg. 30 m.

Erat

Erat *Cor Leonis* ſub altitudine 27 grad. 35 min. Quando $\frac{1}{4}$ diametri Lunæ erat obſcurata. Hora Veſpertina 7, 58 $\frac{1}{2}$.

Emerſio totalis contigit quando *Cor Leonis* erat ſub altitudine 32 gr. 15 m. Hora igitur pomeridiana fuit 8, 25 $\frac{4}{5}$.

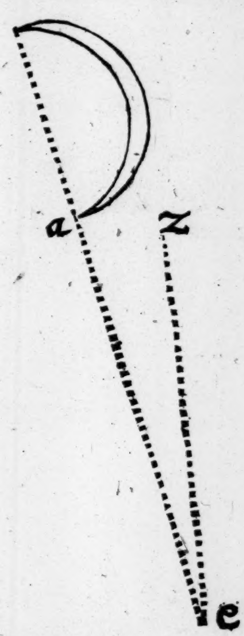
A Quarter of the Moons diameter obſcured when Cor Leon. *was* 27 *deg.* 35 *min. high. The hour of the night was therefore* 7 *h.* 58 $\frac{1}{2}$.

Total emerſion happened when Cor Leon. *was* 32 *gr.* 15 *min. height, therefore the hour of the night was* 8 *h.* 29 $\frac{4}{5}$.

	Baſiliſcus		
Aſc. R.	147 g. 22 m.	Loc. Sol. Jan. 31, hora 9. p. m. ♒	22 g. 44 m.
Dec. B.	13 g. 40 m.	Aſcenſion recta	325 g. 07 m.

6. Eclipſis Lunæ obſervata *Londini* 1649, Maij 15.

Dig. obſcur.	Horæ noctis. h	′
2.	1.	15
4.	1.	22
6.	1.	33
7.	1.	38
8. $\frac{1}{2}$	1.	46
9. $\frac{1}{2}$	1.	52
11.	1.	58
12.	2.	02



Inclinatio cornuum Lunæ a e z 5$\frac{3}{4}$ *gr.*

Luna humilis & tempus nebuloſum non permiſerunt obſervationes fieri ad votum. Attamen magna adhibita erat diligentia tam in inveſtigandis horis quam in phaſibus judicandis.

Eclip-

The Moon was low, and the time cloudy, affoorded not ſo punctual obſervations as was deſired. Yet great care was taken both for enquiring the true hours, and judging the parts Eclipſed.

An

7 *Eclipsis Lunæ observata in Collegio* Greshamensi.

COepit obscuratio hora 8 00 min. p. mer. 10 diei Januarii, Anno 1647.

Desiit obscuratio hora 10, 20 min.

Digiti abscissi 4 ½, non plus in obscuratione maximâ.

7 An Eclips of the Moon observed at *Gresham* Colledge.

THe obscuration began 8*h.* 00 *min. afternoon January* 10, 1647.

The darkness ended 10 *h.* 20 *min.*

Digits obscured were but 4 ½ *in the greatest obscuration.*

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8 *Eclipsis Lunæ observata* Estonæ *in agro* Northamptoniensi, *Anno Domini* 1652 *Martij* 14, *hora tertia p.med. noctem.*

8 The Moons Eclips observed as it happened at *Easton* in *Northamptonshire*, March the 14, 1652 about three of the clock at night.

Latitudo loci 52 *gr.* 15 *m.*

A JOH. TWYSDEN.
& JOH. PALMER.

DIg. obscuratus 1. Quando Aquila distabat à meridie 79 gr. 44 min. in plaga Orientali, ergo hora noctis 2 h. 30 m.

Dig. obscuratus 1 ½. Cauda Cigni alta 52 gr. 30 m. in plaga Orientali.

Digit. obscurati 6 fere. Horologium Solare monstravit horam tertiam juste. *Spica Virginis* distabat à Meridie versus Occidentem 36 gr. 42 m. Ergo hora noctis 3 h. 6 m.

Digiti tandem obscurati erant circiter 10, sed tempus nebulosum erat, ut reliquas phases, nec finem potuimus observare.

ONe Dig. obscured when Aquila *was distant from the Meridian in Azimuth* 79 *gr.* 44 *m. in the East quarter. Therefore the hour of the night was* 2 *h.* 30 *m.*

Dig. obscured 1 ½, *when* Cauda Cigni *was* 52 *gr.* 30 *m. high in the East quarter.*

Dig. obscured 6, *when the Moon shewed just three of the clock upon the Sun Dial,* Spica Virginis *had then Azimuth from the South Westward* 36 *gr.* 42 *m. Therefore the hour of the night was* 3 *h.* 6 *m.*

There were at last about ten Digits Eclipsed, but the skie became cloudy, so that we could make no farther observations.

1 Ob-

4 *Observationes habitæ* Londini *in vico appellato* (Old Bayly) *ad Eclipsim Solis Anno* 1639, Maij 22. *Post meridiem.*

1 The Eclipse of the Sunne which happened M*ay* 22. P. M. 1639, observed in *Old Bayly* at *London.*

A SAMUELE FOSTER, & JOH. TWYSDEN.

Latitudo *Londini* hic statuitur 51 gr. 32 m. tantum enim alii antehac, & nos dicidum deprehendimus. Altitudo solis observata ante inceptam Eclipsim 37 gr. 36 m. Unde deducitur hora diei hor. 3 gr. 47 m. 20″. Locus enim Solis erat ♊ 10 gr. 50 min. & declinatio competens 22 gr. 8 min. Post Eclipsim finitam altitudo Solis observata 15 gr. 00 min. per parallaxim, & refractionem correcta erat 14 gr. 54 min. Ex illâ altitudine eiicitur, hora 6 h. 15′ 00″. Differentia temporum observatorum H. 3.47′ 20″ & H. 6.15 m. 00″ est H. 2 h. 27 m. 40″. At motus Automati, per quod phases observavimus, ad tempus istius observationis erat tantum Hor. 2 h. 26 min. 30″, tardior justo 1 min. 10″, vel 70″, correcta evadit ut infra.

The Latitude of London *is* 51 *deg.* 32 *min. for so others and we have also observed it. The Suns altitude before the Eclipse began was* 37 *deg.* 36 *min. the hour* 3 *h.* 47 *m.* 20″. *At the end of the Eclipse the altitude of the Sun corrected was* 14 *deg.* 54 *min, the hour* 6 *h.* 15 *min.* 00″. *The difference between* H. 3.47′ 20″, *and* H. 6.15 *m.* 00″, *is* 2 *h.* 27 *m.* 40″. *But in that time the observatory Watch had gone but* 2 *h.* 26 *m.* 30″, *too slow by* 1 *m.* 10″, *or* 70″. *The time of the Watch corrected, is as below.*

Digiti in disco Solis obscurati.	*Horæ correctæ* h. ′ ″	*Dig. in disco Sol. obscurati.*	*Horæ correctæ* h. ′ ″
0.00	4.01.46	8.00	4.51.50
1.00	4.07.44	8.44	4.57.53
2.27	4.13.17	9.17½	5.04.26
3.00	4.22.16	9.24	*Max. obscur. medium Eclipsis.*
4.00	4.25.18	7.17	5.29.38
5.00	4.31.41	6.44	5.34.10
6.00	4.37.58	5.17⅓	5.40.43
7.00	4.45.47	3.40	5.51.18
		1.00	6.05.40
Finis totius Eclipseos.			6.10.27

2 *Observatio Eclipsis Solaris* Londini *Augusti* 11, 1645.

Initium Obscurationis accurate observata
The beginning of the obscuration was carefully observed } h. 9. 53

Obscurat. Dig.	$1\frac{1}{2}$	hor. 10. 07
Digit.	$3.\frac{1}{5}$	hor. 10. $23\frac{1}{3}$
Digit.	4.00	hor. 10. $32\frac{2}{3}$
Digit.	$4.\frac{2}{5}$	hor. 10. $37\frac{1}{3}$
Digit.	5.00	hor. 10. 49
Et postea	$5.\frac{4}{10}$ dig. Observavimus.	

Hucusque tantum duravit observatio reliqua (nimirum durationem, & quantitatem maximæ observationis &c.) nobis inviderunt nubes. Cœpit etiam circa punctum 25 gr. descendens à supremo disco solari versus occasum.

Observavimus etiam tria puncta quæ discus Lunæ in margine, & diameter disci Solis pertransiit. Nimirum 334 g. & 85 g. (in circulo disci à supremo puncto s. s. signorum) & $5\frac{2}{10}$ digit. diametri.

Jam vero Solis erat circiter 52 gr. $\frac{1}{2}$ ab Apogæo. Et Luna 96 gr. $\frac{1}{2}$ ab Apogæo. Et juxta *Lansbergium* Diameter Solis ad istam Anomaliam est 34 m. diameter Lunæ 32 gr. $\frac{3}{4}$. At istæ diametri non consentiunt cum observatione. Nam Solis discum in 12 dig. vel 120 partes distri-

2 The Eclips of the Sun observed at *London August* 11, 1645.

The digits Eclipsed were at last $5\frac{4}{10}$ *observed.*

The Clouds now hindred any farther observation.

It began at 25 deg. descending from the supreme diske of the Sun towards the West.

We observed likewise three points made by the diske of the Moon in the limb, and the diske of the Sun, to wit 334 *deg. and* 85 *m. (in the circle of the disk from the highest point according to the series of the signes) and* $5\frac{2}{10}$ *dig. of the diameter.*

The Sun *at this time was about* 52 *deg.* $\frac{1}{2}$ *from the Apogæum. And the Moon* 96 *deg.* $\frac{1}{2}$ *from her Apogæum.* Lansbergius *makes the diameter of the Sun at that Anomaly* 34. *min. and of the Moon* 32 *deg.* $\frac{1}{4}$. *But these diameters agree not with observation. For we divided the* Suns

distribuimus quarum discus Lunæ occupavit saltem 119. Ut vero 120, 119 : : 34′ ad 33′ 43″ Minor igitur est diameter lunæ *Lansbergiana* quam apparuit è Cœlis 58″ id est minuto ferè solido.

Juxta *Keplerum* Diameter Solis ad istam Anomaliam est 30′ 10″. Diameter Lunæ 31′, 41, adeoque diameter lunæ major est diametro Solis at observavimus minorem. Nempè Solis diametrum 120 lunæ 119 partium. Oportuit igitur diameter lunæ fuisse 29′ 55″, non autem 31′ 41″. Lunæ igitur diameter est (juxta *Keplerum*) 1′ 46″ major justo, id est duobus ferè minutis.

Suns diske into 12 dig. or 120 parts, of which the Moon filled at least 119. But as 120, 119 : 34′, 33′ 34″ Therefore Lansberg *Diameter is lesse then it appeared in the Heavens by* 58″, *that is almost a full minute.*

Kepler *makes the diameter of the Sun at that Anomaly 30′ 10″. The diameter of the Moon 31′ 41″, so the diameter of the Moon is greater then that of the Sun, but the observation makes it lesse. To wit, the Suns diameter 120, the Moons 119 parts. The diameter therefore of the Moon ought to have been 29′ 55″ not 31′ 41″. So that* Kepler *makes the Moons diameter 1′ 46″ too great, almost 2 minutes.*

3 *Eclipsis Solis 1649, Octob. 25 p. m. observata in Collegio* Greshamensi *Londini.*

3 The Eclips of the Sun 1649 October 25 afternoon, observed at *Gresham* Colledge in London.

Tempora accuratissima.

The accurate times.

Horæ	*Minuta.*		
12. 41$\frac{1}{2}$	*Justum Initium.*		
12. 53$\frac{2}{3}$	1 Dig.	1. 59′	4 Dig.
1. 02	2 Dig.	2. 14$\frac{1}{3}$	2$\frac{4}{5}$ Dig.
1. 12	3 Dig.	2. 18	2$\frac{2}{5}$ Dig.
1. 26	4 Dig.	2. 21$\frac{1}{3}$	2 Dig.
Max. obscur.	4$\frac{1}{2}$ Dig.	2. 29$\frac{1}{2}$	1 Dig.

2 h. 36 m. $\frac{1}{2}$ *Justus finis.*

Eclipsis

4 *Eclipsis Solis* 1652, *Martij* 29, *ante meridiem, observata* Londini.

Tempora veræ.	*Digiti Ecliptici.*
H. M.	
9.46	4.2
9.52	5.2
9.58	6.2
10.04½	7.35
10.98½	8.1
10.30	10.8
19.31½	11
10.38	10.8
10.51	9
10.56	8.1
10.57½	8
11.07	6.4
11.11	5.8
11.18	4.5
11.22	4
11.28	3
11.34	2
11.40	1
11.46	0 *Justus finis.*

Maxima obscuratio 11 Dig. exactè,

IN initio nubes obstiterunt quò minus cerneretur. Posteà verò, sequentia observavimus.

Justum initium colligi poterit proportionaliter, si inter se comparentur observationes tres primæ. Nam per eas, colligimus 1 digitum absolvi in 6 minutis horarijs; adeoque 4.2 digitos peragi in 25 minutis. Sublatis 25′, ex 9 h.46 m. Restat 9 h.21 m. pro horâ initij justi.

Duratio erat hor. 2, & 25 m.

Medium Eclipsis erat, hor. 10 32 min. si comparentur quinta & decima observationes. At si comparentur observationes sexta & octava incidet medium tempus maximæ obscurationis, in hor. 10, 34′ sit sanè obscurationis maximæ momentum, hor. 10.33

In margine, 11 digiti affiguntur horæ 10,31 ½ min. Lubricitati maximæ subjicitur observatio momenti maximæ obscurationis. Et 31 ½ differt tantum 1 ½ m. à superiori tempore. Ergo obscur. max. tutò capi

4 The Suns Eclips, observed at *London* 1652, March 29, before noon.

THe beginning could not be seen by reason of the clouds.

The just beginning may be collected by proportionality if the three first observations be compared together. For from them we may gather that one digit was absolved in 6 minutes of an hour, and therefore 4.2 in 25′. Take 25′ out of 9 h. 46′ there rest, 9 h. 21′ the very beginning.

It continued about 2 hours, 25 minutes.

The middle was at 10 hours 32 min. if you compare the fift and tenth observation. But by the sixth and eight it will be at 10 h. 34′. Let us allow it therefore to be at 10 h. 33.

In the margine 11 dig. are assigned to 10 h. 31 ½. But the observation of the moment of the greatest obscuration is subject to much uncertainty. The difference is but 1 min. ½ from the time above specified. Therefore

64

capi poterit in hora 10, 33 m. ante meridiem *Londini*.

Ex tribus punctis disci Lunaris in disco Solis observatis, collegimus, diametrum Solis, ad diametrum Lunæ esse ut 12 ad 12.24.

Juxta *Keplerum*, Diameter apparens Solis erat (*Martij* 29) 30.40. Ergo apparens diameter Lunæ fuit 31.008.

At vero ex Tabb. *Kepleri* diameter Lunæ est 32.466. Error est 1′.458 nimis.

Juxta *Lansbergium* apparens Solis diameter erat 34.53. Ergo apparens diameter Lunæ fuit 35.008. Et ita equidem ex *Lansbergij* Tabb. exerpitur, nempè 35 minutorum exactè.

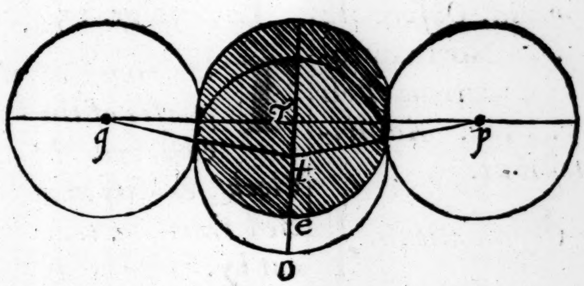
fore we may safely allow it to be at London *before noon, at* 10.*h.* 33 *min.*

From the three points of the Moons discus observed in the Suns, we gathered the diameter of the Sun to be to the diater of the Moon as 12 *to* 12.24.

According to Kepler *the apparent diameter of the Sun was* (March 29) 30.40. *Therefore the apparent diameter of the Moon will be made* 31.008.

But by Kelplers *Tables the diameter of the Moon will be found* 32.466. *The error is* 1′. 458 *too much.*

According to Lansberg, *the Suns apparent Diameter was* 34.53. *Therefore the apparent diameter of the Moon will be made* 35.008. *And accordingly it is taken out of* Lansbergs *Tables to wit,* 35 *m. exactly.*



Semidiameter Solis to = 6.0000 Digit. Solis.

Semidiameter Lunæ τe = 6.1200 Digit. Solis.

Summa Semidiametrorum tp, vel tq, = 12.12.

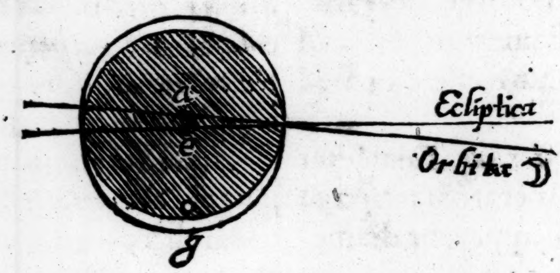
$t\tau$ = 1.12 Dig. Solis, $\overline{t\tau}|$ = 1.2544. $\overline{tp}|$ = 146.8944.

$\overline{tp}| - \overline{t\tau}|$ = 145.64 = $\overline{\tau p}|$.

Hujus $\surd q$ = τp = 12.06814 $\begin{cases}\tau p\\ \tau q\end{cases}$

 Ergo

Ergo $p\,q$ = 24.13628. Dig. Sol. Tota duratio erat 2 h. 25 m.
Ut 2 h. 25 m. . 1 hor. : : 24.13628 . 9.9874 Dig. Solarium.
Et tantus erat motus horarius visibilis Lunæ à Sole, nempè 9 deg. 59 min. Dig. Sol.



$a\,o = 6.\ 12$
$a\,g = 7.\ 12.\ \ o\,g = 1$ *dig.*
$e\,g = 6.00$
$a\,e = 1.\ 12$ Digit. Solarium.

Et tanta erat visibilis latitudo Lunæ, in medio Eclipsis, sept. Ascend.

And so much was the Moons visible latitude in the middle of the Eclips Septent. Ascendent.

5 *Eclipsis Solis habita Anno Domini* 1652, 28 *die* Martij, *horis p. mer.* $21\frac{1}{4}$, *hoc est* 29 Martij *currente, horis circiter tribus ante merid. Observata* Estonæ *in agro* Northamtoniensi, *sub elevatione Poli Borea.* 52 *gr.* 15 *m. Adhibitis idoneis testibus.*

Methodus Observationis.

HOra aut circiter unâ ante Eclipsis initium composui horologium ambulatorium optimi artificij, minuta prima accurate indicans, ad horam proximè veram eodem tempore altitudinem Solis observavi per quadrantem. Erat autem

5 The Suns Eclips observed at *Easton* in *Northamptonshire* 1652, *March* 29 current, about 9 in the morning Lat. 52 gr. 15 m.

THe times of the several phases of the Eclips were observed by a minute Clock, exactly made and corrected from the true hour found out by the Suns Azimuth, often observed during the Eclips, which I judged the better way in this Eclips, because the end of it falling neer noon, a little error in the altitude would have caused a considerable difference in the time, which by this way is avoided.

Digit.

autem Solis altitudo observata 19 gr. 13 m. sed per refractionem, & parallaxim *Lansbergianam* correcta 19 gr. 10 m. unde hora ex calculo erat 7 h 26 m. 48″ horologium monstravit 7 h. 21′ 00″

Paulo post finitam Eclipsim observavi denuo Solis altitudinem 45 gr. 20 m. Parallaxis addenda 1′ 37′ 20″. Ergo vera altitudo erat 45 gr. 21′ 37″ Automaton indicavit 11 h. 49′.

Locus Solis ad tempus maximæ obscurationis est.

♈ 19 gr. 16 m. 48″ ex Tabb. *Vinc. Wing.*

Declin. Solis 7 gr. 33 min. Anguli horarij supputati, sunt ex observata Solis Azimutha ad diversas Eclipseos phases, unde horæ automati correctæ sunt.

Azimutha Solis ad horam 7 h. 26′ 48″ erat 77 gr. 11 m. 30″ à merid.

Digiti obscurat.	*Horæ Automati.*	*Horæ correctæ.*	*Azimutha Solis.*	*Angul. horarij.*
00.00	9.15	9.19		
1.15	9.24	9.27		
2.30	9.32	9.35	46.32	36.09
3.00	9.35	9.38	45.56	35.37
4.30	9.44	9.47		
5.00	9.46	9.49		
5.30	9.52 1/8	9.56	40.50	31.05
8.10	10.05	10.08		
8.30	10.08 1/2	11.11 1/2		
9.00	10.11	10.14		
9.30	10.15	10.18		
10.00	10.18	10.21		
10.30	10.23	10.26		
11.00	10.26	10.29		
11. 1/4	10.28	10.31		
10.45	10.32 5/6	10.35 1/6		
10.00	10.34	10.37		
9.00	10.46	10.49		
8.45	10.50 1/2	10.53		
8.00	10.52 2/3	10.55 2/3		
7.00	10.59	11.02		
6.00	11.04	11.07	18.29	13.20
4.00	11.17 1/2			
3.15	11.25 1/4			
1.05	11.31 1/2			
1.15	11.35			
0.45	11.38 1/2			
00.00	11.41 1/2	11.42.24	6,11	4.24

6 *Motus nuperi Cometæ Observatus* Estonæ *in agro* Northamptoniensi, *sub elevatione poli* 52 *gr.* 15 *m. Anno* 1652.

Die *Martis* 16 Decembris quando media in sectione *Tauri* erat in Meridie.

Cometa distabat à pede *Heniochi* dext. 21.00

Distabat ab Oculo *Tauri*, *Albdebaran.* 12.10

6 The motion of the late Comet as it was observed at *Easton* in *Northampton-shire.* Anno 1652, Lat. 52 d. 15 m.

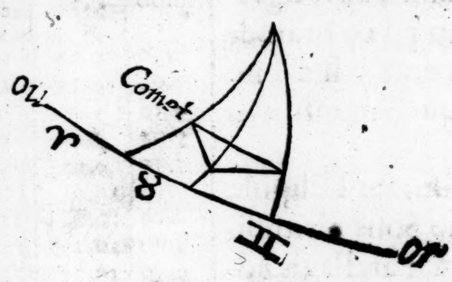
Tuesday *Decem.* 14, *when the middle starre in the section of* Taurus *was South.*

The Comet was distant from the right foot of Heniochus 21.00

From the Buls *eye* 12.00

Cometa erat reliquis Occidentalior ut in adjunctâ figurâ.

And was West from the rest as in the figure.



Invenietur ex Calculo.

Longitudo Cometæ ad hanc observationem *Tauri* 26.45

Latitudo Bor. 3.30

Die *Mercurij* 15 Decemb.

Distantia à pede *Heniochi* dextro 22 d. 17 m.

Ab *Hirco* 25 00

Hinc longit. ♉ 25 17

Latitudo Borea 9 00

Australis in *Cauda Ceti* in meridie.

His longitude at this observation will be found Taurus, 26.45

With North Latitude 3.30

Wednesday *the* 15 *Decemb.*

Distance from the right foot of Heniochus 22 *g.* 17 *m.*

From the bright star called Hircus. 25.00

Therefore his longitude was Taurus 25.17

North latitude 0.09.00

The Southernmost in the Whales-tail *was in the Meridian.*

Die *Jovis* 16 Decembris. Linea recta extensa per Centra duarum in pede sinistro *Persei*, attigit limbum Cometæ inferiorem, & ab invicem æquè distabant.

Thursday 16 *December. A right line extended through the Centers of the two starres in the left foot of* Perseus, *touched the lower limb of the Comet, and they were at an equal distance, one from the other.*

Die *Veneris* 17. Tempus nebulosum.

Friday 17. *Was cloudy.*

Die *Sabbati* 18. Formabat triangulum quasi æquilaterum cum duabus lucidioribus capitis *Medusæ*.

Saturday 18. *It made very neer an equilateral Triangle with the two bright starres in* Medusas *head.*

Die *Solis* 19. Tempus nebulosum semel tamen adspexi lumine

Sunday 19. *was cloudy, but once I saw in the light of it much*

lumine valde diminutum, & ab oculo *Gorgonis*, duobus circiter gradibus distantem & pene in lineâ rectâ cum oculo *Tauri*.

Die *Mercurij* 22 Decemb. Intercipiebatur a lineâ rectâ transeunte per oculum *Gorgonis*, & obscuram stellulam in sinistro humero *Persei*. Distantia Cometæ ab oculo *Gorgonis* erat 4 g. 40 m. versus Occidentem, paulò superior ad Boream, & inter utrasque stellas pene medius videbatur Cometa.

Prima in *Capite Ceti* in Meridie.

Die *Jovis* 23 Decem. horâ noctis paulò supra undecimam. Cometa distabat ab oculo *Gorgonis* 5 g. 25 m. superior versus Boream, & Occidentalior, & in rectâ lineâ cum sinistro humero *Persei*. Lux valdè debilis.

Die *Veneris* 24 Decembri. Distabat ab oculo *Gorgonis* 6 gr. 23 min. & in rectâ lineâ cum humero *Persei* sinistro. Lux adeo debilis ut visum ferè effugit. Ad verticem nostrum videbatur tendere.

much decayed about two degrees distant from Gorgons *eye, and very neer in a streight line with the* Buls *eye.*

VVednesday *the* 22. *The Comet was intercepted by a right line that passed through the* Gorgons *eye, and the obscure starre in the left shoulder of* Perseus. *It was distant from the* Gorgons *eye Westward* 4 *d.* 40 *m. It was a little above those starres toward the* North, *and in the middle between them very neer.*

The first in the VVhales *head was in the Meridian.*

Thursday 23, *a little past eleven. The Comet was distant from the* Gorgons *eye* 5 *d.* 25 *m. Westward, yet above it toward the North, and in a right line with the left shoulder of* Perseus. *Its light was very dimme.*

Friday *the* 24. *It was distant from the* Gorgons *eye* 6 *d.* 23 *m. and in a right line with the left shoulder of* Perseus. *The light was so dimme, I could hardly see it. It seemed to tend to our Zenith.*

DIe *Mercurij* secundo Julij stilo veteri An. *Dom.* 1651 *Estonæ* in agro *Northamptoniensi*, sub elevatione poli Borealis 52 gr. 15 min. ad horam circiter octavam pomeridianam Sole tunc tendente ad occasum. Per Tellescopium optimum

UPon Tuesday *the second of* July *in the year* 1651, *about eight of the clock at night, at* Easton *in* Northampton-shire, *under the elevation of the North pole* 52 *d.* 15 *min. I saw in the body of the Sun (through an excellent Tellescope whose Glasses* were

mum cujus vitra erant probe abstersa intuebar in disco Solari maculam rotundam nigerrimam, cujus diameter erat 12, aut circiter diametri Solis pars, & licet tenues nubeculæ hic illic volitantes frequenter corpus Solis visui eripuerunt; attamen redeante lumine ad horæ minuta 9, aut decem quoad visum apparuit loco immota. Sinistra Solis margo cernebatur instar serræ dentata, ut in subjecta figura.

Credo fuisse unam ex maculis quas *Galileus*, *Scheinerus*, *Hevelius*, & alij observarunt. Nam *Mercurium* istic loci suspicari non possum ob latitudinem quatuor, aut 5 graduum quam recentiores Tabulæ Astronomicæ ei tribuunt licet in longitudine non multum distabat a Sole. Stupenda esset ista refractio quæ Planetam in ipso forsan Horizonte elevaret ad Solem, tunc altum gr. 1 $\frac{1}{2}$ aut circiter, & propterea tantæ refractioni non subjectum. Æqualem tamen, aut certe paulò minorem aliquando prodiderunt historiæ. *Thomas Jacobius* (vulgò *James*) Navarchus *Bristolliensis*, dum hybernaret in Insulâ quâdam Americana in longitudine ferè 305 grad. lat. Borea 52 gr. An. Dom. 1632 mense *Februarij*, comperit ortum Solis apparentem citiorem ortu verò 20 minutis temporis, sicut

were very clean) a very dark round spot in diameter about the 12 part of the Suns diamter, which to my sight appeared still in the same place for a matter of 9 or 10 min. though thin clouds often interposed, and hindred me from the sight of the Sun for a short time. The left margine of the Sun was very uneven, and tooth'd in the manner of a saw, as in the adjoyning Scheme.

I conceive it was one of those spots which Galileus, Scheinerus, Hevelius, *and others have observed. For I cannot suspect* Mercury *in that place, by reason the latest Tables, give him neer 5 deg. South latitude, though in longitude he be not far distant from the Sun. It must be a strange refraction that could lift him up (perhaps in the very Horizon) to the Sun then high* 1 $\frac{1}{2}$ *deg. and therefore not subject to so great a refraction as was* Mercury. *Yet have we relations in History of some refractions either equal, or not much inferior.* Captain James *of* Bristol, *wintering in an* Island *of* America *in the longitude of* 305 *deg. almost, & North latitude* 52 *d. in the year of our Lord* 1632, *in the moneth of* February, *found the Suns apparent rising* 20 *m. of time before the true ought to have been, as he saith in his voyage, printed*

ſicut ipſe teſtatur itinerarij ſui Anglice conſcripti pag. 64 unde ſequitur refractionem Solis fuiſſe fere trium *gr.*

Hollandi quoque poſt *Tartariam* Hybernantes notarunt in Sole Oriente refractionem aliquot graduum referente *Keplero* Epit. Aſtr. lib. 1. pag. 60. *W. Lantgravius Haſſiæ* obſervavit Venerem duobus gr. ſupra Horizontem quaſi per horæ quadrantem, & prorſus evaneſcere. *Hevelius Selenograp.* pag. 197. Quicquid certe fuerit mihi maculum iſtiuſmodi, nec marginem Solis ita dentatam poſtea, licet ſæpius tentavi, non contigit videre.

Calculum loci *Mercurij* ex recentioribus Tabb. ſupputati hic infra ſubjectum habes.

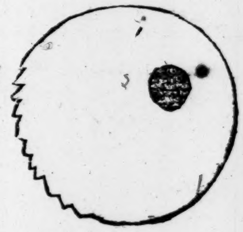
ted in the English *tongue* p. 64. *Whence it follows that the Suns refraction was neer* 3 *deg.*

The Hollanders *wintering behind* Tartaria, *obſerved in the Sun riſing a refraction of ſome degrees.* Kepler Epit. Aſtr. Cop. lib. 1. p. 60. W. Lantgrave *of* Heſſen, *obſerved* Venus 2 *d. above the Horizon to ſtand ſtill there about* ¼ *of an hour, and then ſuddenly to vaniſh* Hevel. pag. 195 Selenog. *What ever it was of this, I am certain it was never my fortune ſince to ſee the like ſpot, nor the margine of the Sun ſo uneven, though I have often tried.*

I have added the calculation of the place of Mercury *out of the lateſt Tables.*

Ephemerides Origani *exhibent locum* Mercurij.

		gr. ′	
In Meridie quoad longitud.	♋	8. 58	☉ ♋ 19 d. 19′ 16′
Latitudinem		4. 10	
Argoli Ephem. faciuut long.	♋	9. 41	☉ ♋ 19 d. 42. 27
Latitudinem		3. 17	
Ecſtadii Ephemer. Longit.	♋	19. 08	☉ ♋ 19 d. 45. 10
Latitudinem		4. 05	



Locus Mercurij *juxta Tab.* Lansbergij.

	S	d	′	″
Æqual. motus orbis ☿	3	17	20	33
Æq. motus Solis	1	50	18	40
Æq. motus Apog. ☿	4	00	03	19
Anomalia Centri	3	50	15	21
Proſthaphær. centri additiv.	0	02	27	30
Longit. Mercurij Centrica	1	52	46	10
Anomalia Orbis vera	3	14	53	03
Proſthap. orb. abſolut. ſubtr.	0	7	07	43
Verus locus ☿ ab Æq. medio	1	45	38	27
Proſthaphær. Æquinoctiorũ	0	00	10	13
Ergo verus *Mercurij* locus	1	45	48	40
Longitudo ☿ ♋		15 d.	48′	40″

Tempus in ſexage. 2‴ 47″ 27′ 24d. 20′

	S	d	′	″
Diſtan. ☿ à nodo Auſtrino	4	8	22	25
Ergo latitudo declinationis auſtrina correcta	0	3	30	11
Latit. reflexionis auſtrina correcta	0	0	19	3
Latitudo auſtrina *Mercur.*		3 d.	49′	14″
Longitudo ☿ ♋		15 d.	48′	40″
Latitude auſtrina		3	49	14
Locus Solis ♋		20	04	03
Pro Meridiano *Gœrano*				

Locus

Locus Mercurij *ex Tabulis* Britannicis.

Locus Solis ♋ 20 d. 03′ 47″
Logarithmus 500744

Tempora completa	☿ Long. S	d	′	″	Aphel. S	d	′	″	Nodus S	d	′	″
1640	3	6	43	23	8	12	47	17	1	13	34	25
10	6	5	23	44	0	00	17	22	0	00	15	56
Junius	0	20	43	19	0	00	00	52	0	00	00	47
Dies 1	0	04	05	32								
Horæ 8	0	01	21	51								
	10	08	17	49	8	13	05	31	1	13	51	08
	08	13	05	31								

	S	d	′	″
Anom. media	1	25	12	18
Æq. correct. su.		16	34	55
Locus ☿ Centr.	9	21	42	54
Logarithmus		464633		
Nodus subtrah.	1	13	51	08
Argum. Latit.	8	7	51	46
Reduct. subtr.			8	39
Eccentr. reductus	9	21	34	15
Curtat ex log. sub.			270	
Logarithm. curtat.		464363		
Locus Solis	3	20	03	47
Eccentr. reduct.	9	21	34	15
Anomal. orbis	5	28	29	32
Dimid. Anom.	2	29	14	46
Logar. curtatus radio addito.	1,464363			
Logar. Solis subtr.	500744			
	9.63619			
Tang. 23 *d.* 25′ 33″				
Adde 45				

Sum. 68 25 33 Co-tan. 959706
Tang. dim. Ano. 89.14 46 1188303
Tang. 88 06′ 00″ 1148009

Elong. à Solis 1 *d.* 8′ 46″
Locus Solis ♋ 20 3 47
♋ 18 55 01

Longitudo ☿ ♋ 18 *d.* 55′ 01″

Tang. maximæ inclinat. ☿	90828
Sinus elongationis à Sole	83005
Sinus argum. latitud.	99667
Compl. Arithm anom. Orbis	15797
Tang. 4 d. 51′ latit. ☿ Aust.	2.89297

Longitudo ☿ ♋ 18 d. 55′ 01″
Latit. Austr. 4 51
Ad Meridiem Londini.

Locus Mercurij *ex Tabulis* Vincentij Wing.

Locus Solis ♋ 20 d. 04′ 16″
Distantia Solis à Terra 101748

Tempora	Longitudo S	d	′	″	Aphel. S	d	′	″	Nodus S	d	′	″
1601	2	2	49	43	8	11	07	15	1	12	30	00
40	0	29	46	02	0	01	08	10	0	01	30	00
10	6	05	23	44	0	00	17	03	0	00	15	00
Julius	0	20	43	19	0	00	00	51	0	00	00	44
Dies 2	0	08	11	05								
Horæ 8	0	01	21	51								
Long. med. ✱	10	08	15	44	8	12	33	19	1	14	15	44
Aphel. subtr.	8	12	33	19								

	S	d	′	″
Anomal. med.	1	25	42	25
Æqual. sub. ✱		16	55	40
Locus eccent. ∴	9	21	20	04
Nodus subd.	1	14	15	44
Argum. latit.	8	07	04	20
Reduct. sub. ∴				858
Locus eccent. reduc.	9	21	11	06
Locus ☉ subd.	3	20	04	16
Anom. commut.	6	01	06	50
Dimid. Anom.	3	00	33	25

Inclin. ☿	6 *d.* 21′ 18″
Reduct.	6′ 58″
Curtat.	234
Distant. ☿ à Sole	44223
	234
Distant. curtat	44009
Summa	145757
Different.	57739
Logar. summæ abscissa figura ult. ad dextr.	4163608
Log. different.	3761401
Tan. 89*d* 26′ 35″	12006483

Summa 15767884
4163608

Tang. 88 35 11604276

Tangens 89 26 35
Angu. parallact. 51 35
Locus Solis 20 04 16
Long. ☿ vera 19 12 41 ♋

Sinus anguli comutationis	8289773
Co-tang. inclinationis	10953566
Summa	19243339
Sin. aug. parallact 51′ 35″ subd.	8174996
Co-tang. latitud.	11068343

Ergo lati Australis est 4 *d.* 53 *m.*

Longitudo ☿ vera ♋ 19 12 41
Latitudo australis 4 d. 53 m.

Ad Meridiem Londini.

FINIS.

Ratio facillima Computandi altitudinem Solis horariam ad quamlibet latitudinem, struendis Tabulis altitudinum commodissima: quam à D. Fostero *olim acceptam, communicavit mihi* D. Palmerus, Ectonensis.

1 Altitudo Solis Meridiana in Borealibus signis colligitur ex elevatione Æquatoris, & declinatione solis simul compositis: in Australibus relinquitur altitudo merid. post subtractionem declinationis solis ex elevatione Æquatoris.

2 Altitudo solis sexti (hoc est, in horâ sextâ costituti) invenitur hac ratione:

Ut radius ad sinum latitudinis, ita sinus declinationis solis ad sin. alt. EP. PS : : EB. BF.

3 Ut radius ad differentiam sinuum altitudinum solis meridiani & sexti; ita cosinus horarum à meridie primæ, secundæ, tertiæ, quartæ, & quintæ, ad rectas primam, secundam, tertiam, quartam, & quintam, quæ per solem prostaphæresin dabunt altitudines horarias solis, per totam paralle-

An easie way to calculate Tables of the Suns Horarie altit. for any latitude: which being communicated to me by Mr. *John Palmer*, of *Ecton*, who received it long since from Mr. *Foster*, I thought worthy to be here inserted.

1 *The suns meridian altitude is had by adding the declination of the sun in North signes, or by subtracting it in South signes, from the elevation of the Equator.*

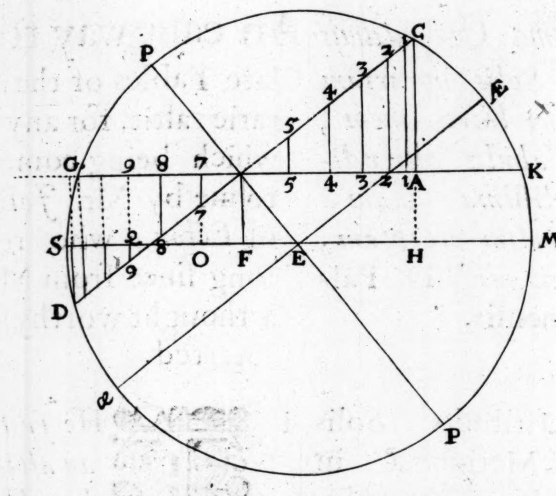
2 *The altitude of the sun at six of the clock, is thus found:*

As the radius to the sine of the latitude: so the sine of the suns declination, to the sine of the altitude EP.PS :: EB.BF.

3 *As the radius to the difference of the sines of the suns altitude at noon and at six; so is the cosine of the first, second, third, fourth, and fifth houre from noon, severally to a first, second, third, fourth, and fifth lines, which by addition or subtraction onely, shall give you the horary altitudes of the sun through*

parallelum, itemque per parallelum positum.

throughout both, the same and the opposite parallel.



Nam recta prima vel secunda cum sinu altitudinis solis sexti composita fit sinus altitudinis solis in horâ primâ vel secundâ; eademque est ratio in cæteris horis, horarumque partibus in superiore Quadrante usque ad hor. sextam.

Infra sextam vero rectæ minores (quæ demi possunt) de sinu altitudinis solis sexti detractæ relinquunt sinus altitudinum horariarum, quamdiu sol versatur supra Horizontem.

Ex rectis vero longioribus sinus altitudinis solis sexti detractus relinquit sinus altitudinum horariarum solis apud *Antipodas* nostros in eodem declinationis parallelo; vel (quod idem est) apud nos in parallelo, infra Æquatorem opposito.

For the first line or number found, being added to the sine of the suns altitude at six a clock, makes the sine of the suns altitude at one of the clock, the second line added as before makes the sine of altitude at two; and so for the other hours above six.

Below six, take the shorter lines out of the sine of the suns altitude at 6, and there shall remain the sine of the suns altitude in the correspondent hours.

And take the sine of the altitude at 6, out of the longer lines, and you shall leave the sines of the altitudes with our Antipodes *in the same parallel, which are equal to the altitudes with us in the opposite parallel.*

DE

DECLARATIO.

In Analemmato opposito est Axis Mundi, itemque circulus horæ sextæ P *p* Tropicus ♋, D C. Horizon S M. parallelus altitudinis solis sexti G K ejusque sinus B F. Dico, Ut radius ad sin. lat. ita sinus declin. ad sinuum altitudini. E *P*. *P* S : : E B. B F. cui æqualis A H. Detracta vero A H de sinu altitudinis meridiana solis C H relinquitur diametra C A cui etiam æqualis est in opposito Quadrant. D G.

Jam, Ut radius B C, ad differentiam C A : ita cosinus horæ quintæ B 5, ad rectam 5 5, quæ cum A H composita fit sinus altitudinis solis in horâ quintâ.

Eademque recta in inferiori Quadrante, vel æqualis ejus 7, 7 dempta de 7 ⊙, vel A H, relinquit ⊙ 7 sin. altit. solis in hora 7 post mer.

Porrò ex recta 3, 3, vel huic æquali 9, 9, si domas sin. altitudo solis sexti, 9 Q vel A H restabit Q 9 sin. altitudinis solis apud *Antipodas* in hora nonâ, eademq; est altitudo solis apud nos ad horam nonam vel tertiam, in opposito Tropico ♑.

Placuit

DECLARATION.

In the Analemma P p *is the Axis of the World, and the hour-circle of 6, D C Tropick of* ♋. *S M Horizon. G K the parallel of the suns altitude at 6. B F the sine thereof. I say, As the radius to the sine of the latitude : so is the sine of the declinat. to the sine of the altitude at 6, E P. P S : : E B. B F. whereto A H is equal. And A H taken out of C H (the sine of the suns meridian altitude) shall leave the difference C A or D G.*

Now, as the radius B C to the difference C A: so the cosine of the hour at 5, B 5, to the line 5, 5, which added to A H, makes the sine of suns altitude at 5.

And the same line or his equal 7, 7 in the lower Quadrant taken out of 7 ⊙ or A H, leaveth 7 ⊙ the sine of the suns altitude at 7 afternoon.

Also the sine of the suns altitude ad 6, 9 Q or A H, being taken out of 3, 3, or 9, 9, leaveth Q 9 the sine of the suns altitude with our Antipodes *at 9 afternoon, and the same altitude hath the sun with us in the Tropick of* ♑ *at 9 and at 3 a clock in the day.*

Here

Placuit hic subjungere Artificium novum ejusdem D. Palmeri *quo solis altitudo horaria pariter, & Azimuthum facillimè computantur, sive per veros sinus & tangentes, sive per eorum Logarithmos.*

Problema I.

Dato circuli horarii angulo cum Horizonte, ejusdemque circuli arcu inter solem & Horizontem comprehenso, altitudinem solis ad quamlibet declinationem & latitudinem invenire.

Ut radius ad sinum anguli inter horarium circulum & Horizontem comprehensi: ita sinus arcus inter solem & Horizontem comprehensi ad sinum altitudinis. Per Axiom. Sphær. 1. *Pitisci.*

Illustratio Arithmetica, per Logarithmos.

Ut Radius B C ad sinum A sive C A 72 gr. 10′) *Log.*	9978.6148
Ita sinus B *c* (57 gr. 21′)	9625.3028
Ad sinum *c a* (53 gr. 17′)	1|9903.9176

Porrò angulus circuli horarii cum Horizonte sic invenitur. In triangulo P S H dantur angulus ad S rectus, item angulus horarius ad P 30 gr. 00′, &

Here also I thought good to annexe a new invention, of the said Mr. *Palmer*, for finding the Suns altitude and Azimuth, at any houre, very speedily; either by true sines and tangents, or their Logarithms.

Problem I.

Having the angle of the hour-circle with the Horizon, and the arch of the said hour-circle comprehended between the sun and the Horizon, to find the suns horary altitude for any declination and latitude.

As the radius to the sine of the angle between the hour-circle and the Horizon: so is the sine of the arch of the hour-circle between the sun and horizon, to the sine of the altitude. By the 1 *Axio. of Spher. Tr. of* Pitiscus.

Illustration Arithmetical, by Logarithms.

As the Radius B C *to the sine of* A. *or of* C A (72 gr. 10′) Log.	9978.6148
So the sine of B C (57 gr. 21′)	9925.3028
To the sine of c a (53 gr. 17′)	1|9903.9176

Now the angle of the hour-circle with the Horizon is thus found. In the triangle P S H, S *the right angle is given*, P 30 *grad.* 00 *min. and* P S *the*

& latus P S arcus latitudinis.

Proindè per Compendium *Neperianum* dixeris.

Ut Rad. ad co-sin. latit. P S	9786.9056
Ita sinus horæ P	9698.9700
ad 1	19485.8756

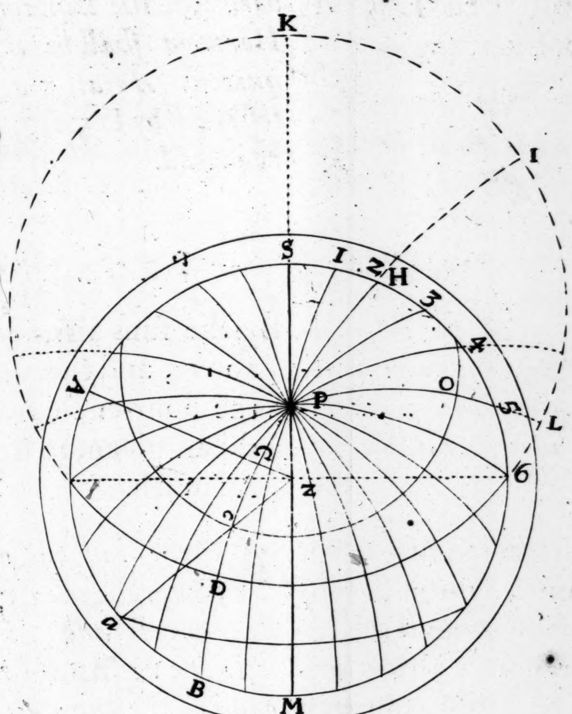
co-sin. anguli S H *P* 72 *gr.* 10′, cui æqualis est ABC, vel C A.

the arch of latitude.

Therefore by my L. Nepairs *Compendium, you may say;*

As the Rad. to the co-s. of the lat. P S	9786.9056
So is the sine of the hour P	9698.9700
to 1	19485.8756

the co-sine of the angle S H P 72 gr. 10 m. whereto A B C is equal, or C A.



Arcum vero inter solem & Horizontem sic inveneris per idem Compendium,

Ut tang. latit. P S, ad Rad.	19937.5306
Ita co-s. hor. P, ad co-t. arcus P H	10111.1004
Vel, quod tantundem est, ad tang. arcus H I,	9826.4302
	33 gr. 51′

cui æqualis est arcus B D. Arcus vero B D auctus arcu decl. solis in signis Borealibus, vel multatus arcu declin. solis in

The arch between the sun & the Horizon is found by the same Compendium, thus:

As the tang. of the lat. P S, *to the Rad.*	19937.5306
So is the cosine of P *(the hour)*	10111.1004
to the cotang. of the arch P H, *or to the tangent of the arch* H I	9826.4302
	33 gr. 51

to which B D is equal. And the arch BD increased by the addition of the suns declinat. in North signes, or diminished as much

in signis Australibus erit arcus postulatus (qualis B *c* 57 gr. 21 min.) pro horis citra sextam. Sed, pro horis ultra sextam, arcus declinationis solis multatus arcu horarii circuli inter Horizontem & Æquatorem comprehenso, est arcus postulatus. Ut inter hora 5, L O multatus arcu L 5 est 5 O arcus postulatus.

Problema II.

Ex inventâ, ut supra, solis altitudine; & datâ circuli horarii amplitudine à meridiano, azimuthum solis invenire.

Ut tangens anguli inter horarium circulum & Horizontem comprehensi ad radium: ita tangens altitudinis ad sinum arcus, qui cum arcu amplitudinis dato compositus mensurat azimuthum solis à meridie. Per Axiom. 2. Sph. *Pitisci.*

Scilicet, Ut C A. 72 gr. 10′, ad A B 90: ita *c a* 57 g. 17 m. ad *a* B 13 gr. 25 m. qui compositus cum arcu B M 24 gr. 32 m. constituit arcum M *a* 37 gr. 57 m. mensuram anguli azimuth M Z *a*.

Ampli-

much as the declinatiō comes to in South signes, shall be the arch required (as B c 57 *gr.* 21 *m.) in the hours from noon to* 6. *But for the hours between* 6 *& mid-night, the arch of the suns declination diminished by subtraction of the arch of the horary circle, comprehended between the Equator and the Horizon shall be the arch required. As at* 5 *a clock* L O *lessened by* L 5 *is* 5 O, *the arch required.*

Problem II.

By the suns altitude found as above, and the amplitude of the hour-circle from the meridian given, to find the suns Azimuth.

As the tangent of the angle comprehended between the hour-circle and the Horizon, is to the radius: so is the tangent of the suns altitude to the sine of an arch, which being added to the arch of the amplitude, makes the arch of the suns azimuth from the meridian. By Pitisc. *Sph. Axio.* 2.

Namely, As C *A* 72 *gr.* 10′, *to A B* 90: *so* c a 53 *gr.* 17 *m. to* a B 13 *gr.* 25 *m. which being added to* BM 24 *gr.* 32 *m. makes* M a 37 *gr.* 57 *m. the measure of the angle* M Z a.

The

Amplitudines verò circulorum horariorum inveniuntur hac ratione.

Ut radius P K, ad tangent. arcus K I (30 gr. 00 m.)

Ita sinus P S (52 gr. 15 m.) ad tang. amplitudinis S H. cui æqualis est M B 24 gr. 32′.

Computa igitur pro tua latitudine quinque angulos ad 1, 2, 3, 4, & 5.

Et quinque arcus circulorum horariorum P 1, P 2, P 3, P 4, & P 5.

Et quinque arcus Horizontis S 1, S 2, S 3, S 4, & S 5.

Quibus semel compertis, facili negotio computaveris Tabulas altitudinum & azimuthorum ad omnes declinationis gradus. Nam angulus ad 6 semper est æqualis latitudini loci: & arcus S 6, & 6 P semper sunt quadrantes.

Expedit autem primo supputare omnes altitudines & azimutha in uno circulo horario per totum annum priusquam procedas ad alium circulum.

En

The amplitudes of the hour circles are thus found,

As the radius P K to the tangent of K I 30 gr. 00 m.

So is the sine of P S 52 gr. 15 m. to the tangent of S H or of M B 24 gr. 32 m. the amplitude required.

Compute therefore once for your latitude the 5 angles at 1, 2, 3, 4, & 5.

And the 5 arches of the hour-circles P 1, P 2, P 3, P 4, and P 5.

And the 5 arches of the Horizon S 1, S 2, S 3, S 4, & S 5.

Which being found once shall for ever serve you in your latitude to make Tables of the suns altitude and azimuth for the whole year, with ease and speed. For the angle at 6 is equal to the latitude of your place. And the angles S 6, and 6 P be evermore quadrants.

And note, that it is your best way to take the hour-circles in order: and compute all the altitudes and azimuths for the whole year in one circle, before you proceed to another.

Here

En hic Tabellam angulorum & arcuum pro latitudine 52 gr. 15 min. requisitoram. Tu Lector similem facile construxeris latitudini suæ conformem.

Here is a Table of the arches and angles requisite for my latitude 52 g. 15 m. The Reader may easily make such another for his own latitude whatsoever it be.

	Hor. 1.		2.		3.		4.		5.	
	deg.	min	deg.	min.	deg.	min	deg.	min	deg.	min
Anguli horar. circ. cum Horiz.	80	53	72	10	64	21	57	59	53	45
Arcus circulorum horariorum	36	48	33	51	28	42	21	10	11	20
Amplitud. circulor. horarior.	11	58	24	32	38	20	53	52	71	17

FINIS.

PROBLEMATA
GEOMETRICA
VARIA.

A SAMUELE FOSTERO
Olim Astronomiæ Professore, in Gollegio *Greshami*, LONDINI.

LONDINI,
Ex Officina LEYBOURNIANA.
M. DC. LIX.

De ratione Diametri ad Peripheriam.

PROPOSITIO I.

Peripheria circuli est minor perimetro ordinati Poligoni circulo circumscripti, major perimetro ordinati Poligoni circulo inscripti.

Fig. 1. *Declarat.* ARcus *c b e* minor est *d b f* major *c g e*(i.e.) minor tangente arcus ejusdem major subtensâ.

Demonstratio I. Major est subtensa quia subtensa *c g e* brevior est quacunque curvâ lineâ inter puncta *c*, *e* ducendâ, & propterea brevior est arcu *c b e* per eosdem terminos ducto, per Definit 4. lib. 1. *Euclid.*

II. Minor est tangente *d f*. Nam per demonstrata ab *Archimede* ad propositionem primam libri de Dimensione Circuli Triangulum, cujus latus sit *a c*, basis verò æqualis arcui *c b c*. erit æquale sectori *c b e a*. Sed triangulum cujus latus sive altitudo erit *a b* æqual. *a c*, & cujus basis est tangens *d b f* excedit prædictum sectorem figurâ exteriori *c b e f b d* attamen altitudo *a b*, vel *a c* in utrisque est æqualis. Unde sequetur tangentem *d b f* esse majorem arcui *c b e*.

PROPOSITIO II.

Si Diameter Circuli sit		10000.00000.00000 *par-*
tium erit Peripheria.	*major quam*	31415.92653.57748.
	minor quam	31415.92653.61440.

SIt arcus *c b*, vel *b e* 1′.

Ergo si radius *a e*, *a b*, *a c* sit 10000.00000.00000.00000

Erit *g e*, vel *g c* sinus 1′ 04848.13681.10764

Erit etiam *b f*, vel *b d* tangens 1′ 04848.13681.11333

Duplicatis

Of the ratio that is between the Diameter of a Circle, and the circumference.

PROPOSITIO I.

The circumference of a circle is lesse then the perimeter of any ordinate Polygon that circumscribes the circle: but greater then the diameter of any ordinate Polygon inscribed within the circle.

Declaration. *THe ark* c b e *is lesse then* d b f, *greater then* c g e: *that is, lesse then the tangent of the ark, greater then the subtense.* Fig. 1.

Demonstrat. I. *Greater then the subtense, because between the two points* c *and* e *the subtense* c g e *is shorter then any other curved line, and therefore then the ark* c b e *drawn through the same terms, by the 4th. Defin.* 1. Euclid.

2 *Lesse then the tangent* d f. *For by that which* Archimedes *proves in his first Proposition (of the Quadrature of a Circle) a triangle whose side is* a c, *whose base is equal to the ark* c b e *will be equal to the sector* c b e a. *But the Triangle whose side, or altitude is* a b *equal to* a c, *and whose base is the tangent* d b f, *is greater then the forenamed sector by the exterior figure* c b e f b d, *and yet the altitude, or radius* a b, *or* a c *is in both equal, therefore it will follow, that* d f *is greater then the ark* c b e.

PROPOSITIO II.

If the diameter of a circle be of		10000.00000.00000.parts,
The periphery shall be	greater	31415.92653.57748
	lesse	31415.92653.61440

LEt the ark c b *or* b e *be* 1′.

Therefore if the radius	a e / a b / a c *be*	10000.00000.00000.00000.
g e, *or* g c *the sine of* 1″		04848.13681.10764
And b f, *or* b c *shall be the tang. of* 1″		04848.13681.11333

Double

Duplicatis vero ſinu iſto,	$c\,e$ *	09696.27362.21527
& tangente erunt	$d\,f$ *	09696.27362.22667

Quoniam igitur $b\,a\,c$ vel $b\,a\,e$ eſt 1″ erit angulus $c\,a\,e$, vel $d\,a\,f$ 2″ ſecundorum, in circulo autem toto ſunt 1296000″ ſecunda, vel 1296000.

Anguli qualem hic ponimus $b\,a\,e$ eſſe 1″. Ergo dimidio tot anguli qualem $d\,a\,f$ hic ponimus (*viz.* 2″) nimirum 648000 anguli tales qualis eſt $d\,a\,f$. Tot etiam latera $c\,e$, Polygoni circulo inſcripti, tot etiam & latera $d\,f$, polygoni circulo circumſcripti. Multiplicatis igitur $c\,e$, & $d\,f$ in 648000 fient perimetri

Polygonorum Circulo	Inſcripti	62831.85307.15497.26946
	Circumſcripti	62831.85307.22881.40408
Diameter etiam erat		20000.00000.00000.00000
Biſect. igit. term. ſi diam. ponat.		10000.00000.00000.00000
Erit perimeter figuræ circulo	Inſcriptæ	31415.92653.57748.63473
	Circumſcriptæ	31415.92653.61440.70204
Et circuli perimeter inter terminos	minorem	31415.92653.57748
	majorem	31415.92653.61440

Sit igitur ratio diametri ad Peripheriam. Ut 100000.00000. ad 314159.26536.

Ludovicus van Cullen, ſic poſuit.

Diam.	100000.00000.00000.00000.00000.00000.000000
Perip.	314159.26535.89793.23846.26433.83279.50288
	314159.26535.89793.23846.26433.83279.50288

Adrianus Metius, pag. 69. Geomet. practicæ, impreſſ. *Franequeræ* 1611 ſic ait. Parens meus Illuſtrium D.D. Ordinum confœdaratarum Belgiæ Provinciarum Geometra, in Libello quem ſcripſit adverſus quadraturam Circuli *Simonis à Quercu* demonſtravit eſſe minorem quam $\frac{377}{120}$, majorem vero $\frac{333}{106}$, quarum proportionum intermedia exiſtit $\frac{355}{113}$. Quæ quidem intermedia proportio aliquantulum exiſtit major quam ea quam invenit Mr. *Ludolph à Collen* cujus diametria eſt minor quam $\frac{1}{1000000}$.

Ut 1.00000.00000, ad 3.14159.26536 :: ita 113. ad 354 $\frac{99997}{100000}$.

Ut 113, ad 355, ita 100000000, ad 314159292, qui eſt major juſto $\frac{27}{1000000}$.

Peripheria

Double the sine, and tangent {c e * 09696.27362.21527 *Fig.* 1.
they shall be {d f * 09696.27362.22667

Because therefore b a c, *or* b a e, *is one second, the anlge* c a e, *or* d a f *shall be* 2″, *but in the whole circle there are* 129600″ *or* 1296000 *such angles, as we suppose* b a e *to be* 1″. *Therefore there shall be half so many angles as we suppose* d a f 2″, *to wit* 648000 *such angles as* d a f. *There shall be also so many sides of* c e *the polygon inscribed in the circle: and as many sides of* d f *the polygon circumscribed about the circle. Multiply* c e, *and* d f *into* 648000, *the product shall be the perimeters*

of the Polygons,	*Inscribed*	62831.85307.15497.26946
	Circumscribed	62831.85307.22881.40408
The diameter also shall be		20000.00000.00000.00000
Bis. the term. & the dia. being put.		10000.00000.00000.00000
The perimeter of the figure	*Inscribed is*	31415.92653.57748.63473
	Circumscribed	31415.92653.61440.70204
And the periphery between these terms	*lesser*	31415.92653.57748
	greater	31415.92653.61440

Therefore the proportion of the diameter to the periphery may be: As 100000.00000, *to* 314159.26536.

Ludovicus van Ceulen, *put it so.*

Diam. 100000.00000.00000.00000.00000.00000.000000

Perip. {314159.26535.89793.23846.26433.83279.50288
{314159.26535.89793.23846.26433.83279.50288

Adrianus Metius *in his practical Geometry, printed at* Franequer 1611, *saith thus. My father Geometrician to the Illustrious Lords the States of the United Provinces in the Low-Countreys, hath in a Book of his written against the quadrature of a Circle, put out by* Simon à Quercu, *demonstrated it to be lesse then* $\frac{377}{120}$, *greater then* $\frac{333}{106}$, *the intermediate proportion of them is* $\frac{355}{113}$, *which middle proportion is a little greater then what* Mr. Ludolp of Ceulen *found, whose difference is lesse then* $\frac{1}{1000000}$.

As 1.00000.00000, *to* 3.14159.26536, *so* 113, *to* 354 $\frac{99997}{100000}$.

As 113, *to* 355, *so* 100000000, *to* 314159292 *to great by* $\frac{27}{1000000}$.

Fig. 1. Peripheria ad diametrum, ex *Archimedis* ſententia eſt, ut 22, ad 7 ferè.

Radius circuli eſt æqualis 57 gr. 17′ 44″ 48‴ = 57 gr. 17 m. $\frac{3}{4}$ ferè.

De areâ Circuli.

Quadratum Radii eſt, ad aream Circuli, ut Diameter, ad peripheriam.

Quadratum peripheriæ eſt, ad aream circuli,

Ut $\begin{cases} 12.56637.06145 \\ 01.00000.00000 \end{cases}$ vel ut $\begin{cases} 10.00000.00000.0 \text{ — } 88 \\ 00.79577.47154.5 \text{ — } 7 \end{cases}$

Quadratum Diametri, ad aream Circuli,

Ut $\begin{cases} 1.27323.95477 \\ 0.28209.47918 \end{cases}$ vel, ut $\begin{cases} 1.00000.00000 \text{ — } 14 \\ 0.78593.81634 \text{ — } 11 \end{cases}$

Peripheria eſt, ad latus Quadrati circulo æquali;

Ut $\begin{cases} 1.00000.00000 \\ 0.28209.47918 \end{cases}$ vel, ut $\begin{cases} 3.54490.77018 \\ 1.00000.00000 \end{cases}$

Diameter eſt, ad latus Quadrati circulo æqual.

Ut $\begin{cases} 1.00000.00000 \\ 0.88622.69244 \end{cases}$ vel, ut $\begin{cases} 1.12837.91671 \\ 1.00000.00000 \end{cases}$

Diamet. 113 — 7 Quadrat. Peripher. ad aream circuli, Ut 1420 — 88
Peripher. 355 — 22 113 — 7

Quadratum diametri ad aream circuli, Ut 452 — 14
355 — 11

Cubus diam. ad Sphær. Ut 678 — 21 / 355 — 11 vel, 10.00000.00000.00000.00000
vel, 5.23598.77559.82988.73076

1 . 113 . 355 :: Diam. periph. :: Q. Diametri Sphæricum.
4 . 452 . 355 :: Q. Diamet. area circuli.
6 . 678 . 355 :: Cubus Diamet. Sphæra.

Nota. Si ponas diametrum 7, & peripheriam 22, ſive accuratius diametrum 113, peripheriam 355, & hinc facias Sphæram, & cylindrum Sphæræ circumſcriptum. Erunt accurate Sphæra & cylindrus, ut 2 & 3. Idem etiam eveniet ex aliis quibuſlibet numeris illo modo tractatis licet (ſi proportionem

Archimedes *makes it, as* 22, *to* 7 *almost.* *Fig.* 1.

Radius of the circle is equal to 57 *deg.* 17′ 44″ 48‴, that is = 57 *deg.* 17 *m.* $\frac{3}{4}$ *almost.*

Of the area of a Circle.

THe *square of the Radius, is to the area of the Circle, as the Diameter to the periphery.*

The square of the periphery, is to the area of the circle,

As $\begin{cases}12.56637.06145\\01.00000.00000\end{cases}$ *or as* $\begin{cases}10.00000.00000.0 & 88\\00.79577.47154.5 & 7\end{cases}$

The square of the diameter, is to the area of the circle,

As $\begin{cases}1.27323.95477\\0.28209.47918\end{cases}$ *or, as* $\begin{cases}1.00000.00000 & 14\\0.78593.81634 & 11\end{cases}$

The periphery is to the side of a square equal to a circle,

As $\begin{cases}1.00000.00000\\0.28209.47918\end{cases}$ *or, as* $\begin{cases}3.54490.77018\\1.00000.00000\end{cases}$

The diameter is to the side of a square, equal to a circle,

As $\begin{cases}1.00000.00000\\0.88622.69244\end{cases}$ *or, as* $\begin{cases}1.12837.91671\\1.00000.00000\end{cases}$

Diameter 113 — 7 *Square of the periphery to* *As* 1420 — 88
Periphery 355 — 22 *the area of the Circle,* 113 — 7

Square of the diameter, to the area of the circle, as $\begin{cases}455 & 14\\355 & 11\end{cases}$

Cube of the dia. *As* 678 — 21 *or,* 10.00000.00000.00000.00000
is to the Sphere, 355 — 11 *or,* 5.23598.77559.82988.73076

Note. *Whether you put the Diameter* 7, *and the periphery* 22, *or more accurately the diameter* 113, *and the circumference* 355, *and from these numbers make a Sphere and a cylinder circumscribed about it. The Sphere to the cylinder, shall be exactly, as* 2 *to* 3. *The same thing will also fall out from*

any

Fig. 1. tionem diametri ad peripheriam spectes) sint falsissimi. Exempli gratia: Sit diameter 18, peripheria 32, his inter se ductis fiet 576 pro Sphærico; cujus $\frac{1}{4}$ sc. 144 est area circuli. Area autem in diametrum faciet 2592, cylindrum Sphæræ suprapositum. Sphæricum 576 in $\frac{1}{6}$ diametri, id est 3, ductum dabit 1728 Sphæram. Jam vero quis non videt 1728 ad 2592 esse, ut 2 ad 3, cum tamen proportio diametri 18, & peripheriæ 32, longissime à veritate discedit.

Ratio vero hæc est. Quia $\frac{1}{4}$ Sphærici, id est, area circuli multiplicata in totam diametrum, & totum Sphæricum multiplicatum in $\frac{1}{4}$, diametri eundem numerum producunt, id est, cylindrum. At vero totum Sphæricum ductum in $\frac{1}{6}$ diametri producet numerum ad illum qui ex $\frac{1}{4}$ diametri factus erat in ratione ea qua est 4 ad 6, hoc est 2 ad 3. Vel sic,

Semidiameter 9 ducta in $\frac{1}{2}$ peripheriam 16 est, 144 area circuli quæ ducta in diametrum 18 facit cylindrum.

Semidiameter 9 ducta in $\frac{1}{2}$ peripheriam 16, est 144 area circuli quæ multiplicata semper per 4, & productum etiam per $\frac{1}{6}$, diametri hoc est semper per $\frac{4}{6}$ diametri, vel $\frac{2}{3}$ diametri necessario, producet numerum Sphæræ $\frac{2}{3}$ tantum ejus numeri quem tota diameter antea produxerat, hoc est $\frac{2}{3}$ tantum cylindri circumvestientis.

Ratio igitur Sphæræ & cylindri non pendet ex ratione diametri ad Peripheriam. Quod observata dignum est.

De *Sphæra*, *Sphæroide*, & *Cylindro*.

I. UT $\frac{1}{6}$ Sphærici, ad basin cylindri: ita Sphæra ad cylindrum æquè altum.

Demonstratio. Q. $\frac{1}{6}$ Sphærici, & diameter efficiunt Sphæram, & basis cum altitudine (id est eadem diametro) efficiunt cylindrum.

Fig. 2. II. Ut circulus Sphæræ *a c d b*; ad circulum Sphæroideos *c d*: ita Sphæra ad sphæroides, id est, in duplicata ratione diametrorum breviorum.

Demon.

any numbers so handled although (if you consider them in proportio of the diameter to the circumference) they are most false. As for example: Let the diameter be 18, *the circumference* 32; *these multiplyed produce* 576 *for a spherick,* $\frac{1}{4}$ *of which, to wit* 144, *is the area of the circle. The area multiplyed into the diameter shall make* 2592 *a cylinder put upon the sphere. The spherick* 576 *multiplyed by* $\frac{1}{6}$ *of the diameter* (viz.) 3, *shall produce* 1728 *the sphere. Now who knows not, that* 1728, *is to* 2592, *as* 2 *to* 3, *notwithstanding the proportion of the diameter* 18, *to the circumference* 32, *are very far from truth.* Fig. 1.

The reason is this. *Because* $\frac{1}{4}$ *of the spherick, that is to say, the area of the circle, multiplyed into the whole diameter, and the whole spherick Multiplyed by* $\frac{1}{4}$ *of the diameter, shall produce the same number, that is to say, a cylinder. But the whole spherick drawn into* $\frac{1}{6}$ *of the diameter, shall produce a number in proportion to that which was from* $\frac{1}{4}$ *of the diameter, that* 4 *is to* 6, *or* 2 *to* 3. *Or thus.*

The semidiameter 9 *drawn into* $\frac{1}{2}$, *the periphery* 16 *is* 144, *the area of the circle, which drawn into the diameter* 18, *makes a cylinder.*

The semidiameter 9 *drawn into* $\frac{1}{2}$ *the circumference* 16, *is* 144, *the area of a circle, which still multiplyed by* 4, *and the product by* $\frac{1}{6}$ *of the diameter, that is alwayes by* $\frac{4}{6}$, *or* $\frac{2}{3}$ *of the diameter, shall necessarily produce the number of the sphere, only* $\frac{2}{3}$ *of that number which the whole diameter had before produced that is* $\frac{2}{3}$ *only of a cylinder that circumscribes.*

The Ratio therefore of a sphere to a cylinder depends not upon the Ratio of the diameter to the circumference. Which is worth noting.

Of a *Sphere*, *Spheroides*, and a *Cylinder*.

I. As $\frac{1}{6}$ *of a spherick, is to the base of a cylinder. So is the sphere to a cylinder, of equal heighth with it.* Fig. 2.

Demonst. *Because* $\frac{1}{6}$ *of a spherick, and the diameter, make the sphere; and the base with the altitude (that is, the same diameter) make the cylinder.*

II. *As the circle of the sphere* a c d b, *is to the circle of the spheroides* c d: *So is the sphere to the spheroides, that is, in a duplicate proportion of the shorter diameters.*

Fig. 2. *Demonstrat.* Nam ubicunque planum *a b* (axi communi *e g* perpendicularè) ſecuerit ſphæram, & ſpheroides, ſectiones efficiet circulos. Et quia hoc obtinet in omni quâque ſectione hujuſmodi, ut & circuli illi quia ſunt ubique inter ſe in eadem ratione, id indicio erit proportionem obtinere. Quemadmodum in ellipſi quæ circumferentia circumveſtitur quoniam ordinatim applicatæ ſunt ſemper in eâdem proportione, ideo eſt quod circulus ſe habeat ad ellipſin: ut diameter circuli, ad diametrum breviorem ellipſeos.

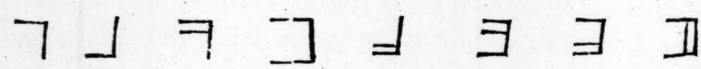
III. Ex conſequenti, & hæc vera ſunt.

1. *Coroll.* Ut $\frac{1}{6}$ ſphærici, ad baſin cylindri, ita ſphæra ad cylindrum.
2 *Coroll.* Ut circulus ſphæræ, ad circulum ſphæroideos, ita ſphæræ ad ſphæroides.

Ergo, Ut $\frac{1}{4}$ ſphærici, ad $\frac{1}{6}$ ſphærici, ita cylindrus ad ſphæroides intra cylindrum. Hoc eſt, Ut $\frac{6}{24}$, ad $\frac{4}{24}$, vel ut 6 ad 4, vel ut 3 ad 2; ita cylindrus ad ſphæroides cylindro incluſum.

De *Parabola.*

Parabola eſt $\frac{2}{3}$ *circumſcripti parallelogrammi.*



q	*c*	*q q*	*q c*	*c c*	*q q c*	*q c c*	*c c c*
2	3	4	5	6	7	8	9

a e.	*p h.*	*c s.*	*u i.*
æ	*f*	*x*	*y*

Fig. 3. NAm A C. A S : : C D. S Q. (R P). At A C. A R : : $\overline{C\,D}|$. $\overline{R\,P}|$ per 20.1. *Apollonii.* Ergo A C. A S. A R ∺ vel, N O. N Q. N P ∺. Deinde $\overline{N\,O}|$ $\overline{N\,Q}|$: : N O. N P. id eſt, Ut cylindrus ad conum, ita parallelogrammum A D, ad trilineam APDM. Ergò ſi MC ſit cylindrus, & AMD conus (uterque ſuper baſi M D) erit circulus N O cylindri, ad circulum N Q coni; ut NO ad NP. At ita quoque eſt recta *N O* parallelogrammi C M ad rectam N P trilinei A P D M A; & hoc ſemper, & ubique obtinet ubicunque ducatur N O, ergo, ut conus A D M eſt $\frac{1}{3}$, cylindri C M: ita trilineum A P D M A eſt $\frac{1}{3}$ parallelogrammi C M. Et ſemiparabola A C D P A eſt $\frac{2}{3}$ parallelogrammi C M quod oportuit.

PRO-

emonstrat. *For wheresoever the plain* a b *(perpendicular to the common axis* e g*) shall cut the sphere, and the spheroides: the sections shall be circles. And because this is so in every such like section, as also because those circles are every where between themselves in the same proportion, that shewes that they have that proportion: As in an* Ellipsis, *which is clothed about with a circumference, because the lines ordinately applyed are still in the same proportion; therefore it is, that a circle, is to an ellipsis, as the diameter of the circle, to the lesser diameter of the ellipsis.* Fig. 2.

III. *Consequently these things are true.*

1 Coroll. *As* $\frac{1}{6}$ *of the spherick, is to the base of the cylinder; So is the sphere to the cylinder.*

2 Coroll. *As the circle of the sphere, is to the circle of the spheroides: So is the sphere, to the spheroides.*

Therefore, as $\frac{1}{4}$ *of the spherick, is to* $\frac{1}{6}$ *of the spherick: So is a cylinder, to a spheroides within it. That is, As* $\frac{6}{24}$, *to* $\frac{4}{24}$, *or, as* 6 *is to* 4: *or, as* 2 *is to* 3; *So is a cylinder, to a sphere included in it.*

Of the *Parabola*.

The Parabola is $\frac{2}{3}$ of the circumscribed parallelogram.

FOr A C. A S :: C D. S Q. (R P). *But* A C. A R :: $\overline{CD}$. $\overline{RP}$| *by* 20.1. *of* Apollon. *Therefore* A C. A S. A R ∻. *Or* N O. N Q. N P :: . *Then* $\overline{NO}$. $\overline{NQ}$ :: N O. N P, *that is, as a cylinder is to a cone :: So is the parallelogram* A D, *to the trilineum* A P D M. *Therefore, if* M C *be a cylinder, and* A M D *a cone (both of them upon the base* M D*) the circle of the cylinder* N O *shall be to the circle of the cone* N Q, *as* N O *to* N P. *But so also is the right line* N O *of the parallelogram* C M; *to the right line* N P *of the trilineum* A P D M A, *and this shall alwayes happen wheresoever the line* N O *is drawn. Therefore, as the Cone* A D M, *is* $\frac{1}{3}$ *of the cylinder* C M; *So the trilineum* A P D M A *is* $\frac{1}{3}$ *of the parallelogram* C M. *And half the Parabola* A C D P A, *is* $\frac{2}{3}$ *of the parallelogram* C M, *as it ought to be.* Fig. 3.

PRO-

PROPOSITIO I.

In Parabola DAF.

Fig. 4. UT NO.OP : : $\overline{FC}|$. FOD : : $\overline{LO}|$. OH. Nam $\overline{DC}|$. $\overline{PR}|$: : AC. AR. Ergo $\overline{DC}|$. $\overline{DC}| - \overline{PR}|$: : AC, AC − AR. Vel $\overline{CH}|$. $\overline{CH}| - \overline{CO}|$: : AC. CR, vel $\overline{CH}|$. $\overline{OH}|$: : NO. OP. Id est, $\overline{CH}|$. FOD : : NO. OP. Ergo $\overline{FC}|$

$$\overline{FC}|\ FOD :: NO.\ OP :: \overline{LO}|\ \overline{OH}|$$
$$\overline{CH}|$$

PROPOSITIO II.

Ut Sphæra ad cylindrum circumscriptum, ita Parabola ad parallelogrammum circumscriptum.

SIt DKF hæmisphærium, & DBKGF cylindrus circumscriptus super basi cujus radius DB, vel FG. Quia NO, OP : : ita $\overline{OL}|$. $\overline{OH}|$ vel ita circulus super radium OL: (qui æquatur basi cylindri) ad circulum super radium OH. Quia inquam hoc semper fit in omni puncto rectæ DC, ergo quanta pars sphæra est cylindri tanta pars erit parabola parallelogrammi.

PROPOSITIO III.

Sphæra est $\frac{2}{3}$ *cylindri circumscripti.*

Fig. 5. SIt hæmispherium DKF, cylindrus circumscriptus DBGF cujus basis sit circulus super diametrum BG; & super eâdem basi, sit conus cylindro æquealtus BCG, dico primo, armillam sive annulum *a e* æquari circulo coni cujus diameter est *m n*. Nam (circulus *c e* vel sic)

c e cir. = *o e* cir. + *o c* cir. Adeoque

c e cir. − *o e* cir. = *o c* cir. = *o n* cir. Vel

o a cir. − *o e* cir. = *o n* cir. Et in eorum quadruples.

a a cir. − *e e* cir. = *m n* cir. Id est, (annulus seu potius) armilla cujus major diameter est *a a*, minor diameter *e e* semper æquatur circulo *m n*. Si igitur *a a* moveretur parallelas à

DF

PROPOSITIO I.

In the Parabola DAF.

AS NO.OP :: $\overline{FC}|$. FOD :: $\overline{LO}|$. OH. *For* $\overline{DC}|$. $\overline{PR}|$ :: AC. AR. *Therefore* $\overline{DC}|$. $\overline{DC}| - \overline{PR}|$ :: AC, AC − AR. *Or*, $\overline{CH}|$. $\overline{CH}| - \overline{CO}|$ :: AC. CR, *or*, $\overline{CH}$. $\overline{OH}|$ :: NO. OP. *That is*, $\overline{CH}|$. FOD :: NO. OP. *Therefore* $\overline{FC}|$ *Fig.* 4.

$$\overline{FC}|\ FOD :: NO.\ OP :: \frac{\overline{LO}|\ \overline{OH}|}{\overline{CH}|}$$

PROPOSITIO II.

As a sphere is to a cylinder circumscribed ; So is a parabola to a parallelogram circumscribed.

L*Et* DKF *be a hemisphere, and* DBKGF *a cylinder circumscribed upon the base , whose radius is* DB, *or* FG. *Because, as* NO, *is to* OP, *so is* $\overline{OL}|$ *to* $\overline{OH}|$, *or so is the circle upon the radius* OL: (*which is equal to the base of the cylinder*) *to a circle upon the radius* OH. *I say, because this comes to passe in every point of the right line* DC, *therefore what part the sphere is of the cylinder, such a part shall the Parabola be of the parallelogram.*

PROPOSITIO III.

A sphere is $\frac{2}{3}$ of a cylinder circumscribed.

L*Et* DKF *be a hemisphere,* DBGF *a cylinder circumscribed, whose base let be the circle upon the diameter* BG; & *upon the same base, let there be a cone* BCG, *equal in height to the cylinder. I say first, that the bracelet, or ring* (ae) *is equal to the circle of the cone whose diameter is* (mn). *For the circle* ce, *Fig.* 5.

or thus ce *cir.* ∞ oe *cir.* + oc *cir.* *Therefore*

ce *cir.* − oe *cir.* ∞ oc *cir.* ∞ on *cir.* *Or*

oa *cir.* − oe *cir.* ∞ on *cir.* *And in their quadrupls*

aa *cir.* − ee *cir.* ∞ mn *cir.* *That is* (*the ring, or rather*) *the bracelet whose greater diamet. is* aa, *and lesser diameter* ee *shall be alwayes equal to the circle* mn. *If therefore* aa

Fig. 5. *D* F ad B G circulus ſupra *m n* (nempe cujus diameter eſt ſemper intra latera coni C B, C G) æquabitur armillæ *a e*. Et hoc obtinebit in omni puncto radij C K. Hinc ergo dictus circulus ita auctus & motus creabit figuram D *e* K *e* F G B D æqualem cono B C G. Quandoquidem vero conus B C G eſt $\frac{1}{3}$ pars cylindri *D* B G F, ideo figura D *e* B K *e* G F, erit etiam $\frac{1}{3}$ pars cylindri D B G F. Pars igitur reliqua D *e* K *e* F D (id eſt hemiſphærium) eſt $\frac{2}{3}$ dicti cylindri.

Corollarium. Cum igitur ſphæra eſt $\frac{2}{3}$ cylindri circumſcripti, per Propoſ. III. erit parabola $\frac{2}{3}$ parallelogrammi circumſcripti.

Delineatio *Paraboles.*

Fig. 6. IN Schem *Fig. 6. No.* 1. Æquales ſunt per ſtructuram *a f*, *o d*, *u w*, *y c*, *l tt*, *m ſt*, *n æ*, quia ſunt omnes parallelæ, parallelis *f æ*, *a n*, terminatæ. Itemque, ſi *k o* ſit 1; erit *t u* 2, *b y* 3, & *l* 4; *ll m* 5, ♉ *n*, 6, & ut *k o*, *t u*, *b y*, &c. ſunt adinvicem: ita *a o*, *a u*, *a y*, &c. inter ſe. At vero plana ex *d o*, *o k*, ex *u w*, *u t*; ex *c y*, *y b*, &c. ſunt æquealta, quorum æquales altitudines ſunt rectæ æquales *o d*, *u w*, *y c*, &c. quare plana eadem ſunt ut baſes *k o*, *t u*, *b y*, & *l*, &c. vel (ut patet ex ſupradictis) ut *a o*, *a u*, *a y*, *a l*, &c. Quandoquidem autem quadrata, *e x*, *o x*, *u z*, *y g*, *l p*, &c. ſunt æqualia planis *d o k*, *w u t*, *c y b*, *tt l* &, &c. [quia quadratorum illorum latera ſunt media proportionalia inter latera planorum:] Ergo quadrata *o x*, *u z*, *y g*, *l p*, &c. ſunt prout rectæ *a o*, *a u*, *a y*, *a l*, &c. in *Fig.* 6. *No.* 1. *No.* 3. prout propoſuit *Archimedes* Propoſ. 3. de Quadratura paraboles.

Ex his fundamentis ope lineæ partium æqualium, & ſuperficierum in circino proportionis, vel ſectore parabola ſic poterit deſcribi. *S*uper A *n*, diametro ſumantur, ex linea partium æqualium, æquales A *o*, A *u*, A *y*, A *l*, A *m*, A *n*, &c. (*Fig.* 6. *No.* 3) & ab his punctis ſuſcitentur perpendiculares *o x*, *y z*, *y g*, *l p*, *m q*, *n h*, &c. In perpendiculari *o x* ſumatur punctum quodlibet *x*, ad diſtantiam vero *o x* (cum ſit prima perpendicularium) aperiatur circinus proportionis in linea ſuperficierum, & in terminis hujus lineæ 1...1, termini 2...2 dabunt *u z*, 3...3 *y g*,

4...4

ſhould be moved parallely from DF *to* B C *the circle upon*(m n) *(to wit whoſe diameter is alwayes within the ſides of the cone* C B, C G *) ſhall be equal to the bracelet* a e, *and this ſhall be ſo in every point of the radius* C K. *From hence therefore, it followes, that the circle ſo increaſed, and moved, ſhall make the figure* D e K e F G B D *equal to the cone* B C G. *But ſince the cone* B C G *is* $\frac{1}{3}$ *of the cylinder* D B G F, *therefore the figure* D e B K e G F *ſhall be alſo* $\frac{1}{3}$ *of the cylinder* D B G F, *therefore the reſidue* D e K e F D *(that is the hemiſpherium) is* $\frac{2}{3}$ *of the ſaid cylinder.* *Fig.* 5.

Corollar. *Since therefore a ſphere is* $\frac{2}{3}$ *of a cylinder circumſcribed by the third Prop. a Parabola ſhall be* $\frac{2}{3}$ *of a parallelogram circumſcribed.*

The delineation of a *Parabola.*

IN *the Scheme(Fig. 6. No.* 1.) a f, o d, u w, y c, l tt, m ſt, n æ, *are equal by ſtructure, becauſe they are all parallel, & terminated by the parallels* f æ, a n. *Hence if* k o *be* 1. t u *ſhall be* 2, b y 3, & l 4, ll m 5, & n 6: *and as* k o, t u, b y, &c. *are one to another; ſo ſhall* a o, a u, a y, &c. *be one to another. But the plains of* d o, o k, *of* w u, u t, *of* c y, y b, &c. *are of equal altitude, whoſe altitudes are the equal right lines* o d, u w, y c, &c. *therefore the ſame plains are as their baſes* k o, t u, b y, & l, &c. *Or (as it appeares by what is aboveſaid) as* a o, a u, a l, a y, &c. *Since therefore the ſquares* e x, o x, u z, y g, l p, &c. *are equal to the plains* d o k, w u t, c y b, tt l &, &c. *[becauſe the ſides of thoſe ſquares are mean proportionals between the ſides of the plains:] therefore the ſquares* o x, u z, y g, l p, &c. *are as the right* a o, a u, a y, a l, &c. *in the Scheme Fig 6. No.* 1. *No.* 3. *as* Archimedes, *his Propoſition is, in his Book* de Quadratura paraboles: Propoſ. 3. *Fig.* 6.

Out of theſe grounds by the line of lines, and ſuperficies in the ſector, a Parabole may be deſcribed thus. Upon A n *as the diameter, prick downe by the line of lines, the equal parts* A o A v, (*Fig.* 6. *No.* 3.) A y, A L, A m, A n, &c. *And from theſe points raiſe the perpendiculars* o x, v z, y g, l p, m q, n h, &c. *And upon the perpendicular* o x *aſſume the point* x, *and open the ſector in the line of ſuperficies, ſo that* o x *(being the firſt perpendicular) may fall in with the points* 1...1 *(the firſt of the line of ſuperficies) then if you take off from the ſame line* 2...2,

you

Fig. 6. 4...4 *l p*, 5...5 *m q*, 6...6 *n b*, &c. Vel incipiendo ab *n b*, cum ſit perpendicularium ſexta aperiatur circinus ad hanc diſtantiam in terminis lineæ ſuperficiei 6...6, & parili quo prius modo reliqua correſpondentia notentur puncta *b*, *q*, *p*, *g*, &c. per quæ æquabili manu ducatur parabola.

Parabolæ infinite variæ poſſunt deſcribi (juxta conos ex quibus ſumantur) quæ tamen ejuſdem erunt longitudinis. Nota.

Coni reſecti imperatam partem abſcindere.

Fig. 7. UT *a o* differentia ſemidiametrorum : ad (*o b*) longitudinem fruſti : ita (*b c*) minor ſemidiameter, ad (*c d*) quæ addita (*e d*) complebit coni integri longitudinem. Vocetur [A].

Deinde, ſi ex baſi majori requiratur ſecare partem *i a*, primo computetur integri coni ſoliditas hoc modo; Ut 10000.2618 : : ita quadratum diametri (*a m*), ad numerum quartum, qui erit $\frac{1}{3}$ areæ baſis (*a m*), vocetur [B] B x A dabit coni totius ſoliditatem vocetur [C].

Dic ſecundo, Ut *C*. *C*—1 : : ita cubus A, ad quartum, vocetur [D], radix cubica [D] dabit longitudinem *d b*: *d b*—*d e* relinquit longitudinem unius unciæ, pedis vel cujuſlibet alius menſuræ juxta quam conus prius fuerit menſuratus.

Secundo. Si requiratur à termino coni minore partem unam abſcindere. Inveniatur primo ſoliditas adjecti coni *d b c*. vocetur [E]. Dic, Ut E. ad E + 1, ita cubus *d c* ad quartum. Vocetur [F], radix cubica F eſt *d x*. *d x* — *d c* eſt *c x* longitudo unius unciæ, pedis vel cujuſlibet menſuræ juxta quam conus prius fuerit menſuratus.

Opera-

you ſhall prick down y z, *and* 3...3 *gives* y g; 4...4 L p, 5...5 m q, 6...6 n h, *&c. Or you may begin from* n h, *which, becauſe it is the ſixt perpendicular, take from* n *to* h *the point aſſumed, and ſet that length in the line of ſuperficies from* 6 *to* 6. *So may you prick down the other points correſpondently. Through theſe points* h, q, p, g, *&c. with an even regular hand, draw the Parabole.* Note, *That Parabolas may be deſcribed of infinite varieties, according to the cones from whence they are taken, yet keeping all one and the ſame length.* *Fig.* 6.

To cut off from a reſected Cone, any part required.

As a o, *the difference of the ſemidiameters, is to* (o b) *the longitude of the fruſtum, or piece: So is.* (b c) *the leſſer ſemidiameter, to* (c d,) *which added to* (e d,) *ſhall complete the entire longitude of the Cone. Call that* [A]. *Fig.* 7.

Then if it be required from the greater Baſe, to cut off the part i a; *firſt let the ſolidity of the whole cone be thus computed. As* 10000, 2618 : : *So is the ſquare of the diameter* a m, *to a fourth number, which ſhall be a third part of the area of the baſe* (a m,) *let it be called* [B], *B* x *A ſhall be the ſolidity of the whole cone. Call it* [C]. *Say in the ſecond place. As C is to C*—1 : : *So is the cube A, unto a fourth, call it* [D]. *The cube root of* [D] *ſhall give the longitude* d h, d h—d e *leaves the longitude of one inch, foot, or whatſoever meaſure the cone was before meaſured by.*

Secondly. If it be required from the leſſer end of the cone, to cut off a part. Firſt, let the ſolidity of the additions cone (d b c) *call it* [E]. *Say, As* E *is to* E + 1, *So is the cube* d c *to a fourth. Call it* [F] *The cube root of F is* d x, d x—d c, *is* c x *the length of one inch, foot, or what other meaſure the cone was before meaſured by.*

Fig. 7.

Operationes prædictæ compendiosius.	
Differ. Semidiam. Longitudo Segm. Major diamet. ad A.	Differ. Semidiam. Longitudo Segm. Minor diamet. ad A.
10000 2618 □ Major diamet. ad B	10000 2618 □ Minor diamet. ad B.
: : : B in A facit C	: : : B in A facit C
Ut C ad C — 1 Ita A A A, ad D.	Ut C ad C + 1 Ita A A A ad D
A — $\surd$ C, D est 1 Si à majori basi secetur.	$\surd$ C D — A est 1 Si à minori basi secetur.

Novembris 19, 1644. *Inter horas* 9 *&* 11 *Cœlo sereno* Londini.

Fig. 8. Apparuit Iris hora $9\frac{1}{3}$
Disparuit hora $10\frac{1}{4}$
Apparuerunt Parhelii hora $9\frac{1}{4}$
Disparuerunt hora $10\frac{1}{2}$

Distabant utrinque à Sole 12 ulnas, & Iris ad duplam distantiam.

Mensuratio areæ Trianguli sphærici.

Lemma I. Superficies lunares hæmisphærici sunt ut earundem superficierum anguli.

Fig. 9. Probandi modus, è multis, hic esto; concipiatur semicirculus meridianus A E D, moveri æqualiter per longitudinem æquatoris B E C, super polos A & D. Erunt ergo anguli deinceps ad A & D (nimirum F & G) ut tempora. Erunt quoque superficies K & M, ut eadem tempora. Ergo F. G : : superf. K. superf. M. Et G. F : : superf. M. superf. K, &c. quomodocunque accipiantur. [Nam quæ conveniunt in tertio conveniunt inter se.]

Corolla.

The foresaid operations more cōpendiously.	
Differ. of Semidiam. *Length of the Segm.* *The greater Diam. to A.*	*Differ. of Semidiam.* *Length of the Segm.* *Lesser diameter to A*
10000 2618 □ *of the greater diameter to B*	10000 2618 □ *Lesser diameter to B*
: : : *B in A makes C*	: : : *B in A makes C*
As C to C — 1 *So A A A, to D.*	*As C to C* + 1 *So A A A, to D.*
A — √ *C, D is* 1 *If it be cut off from the greater base.*	√ *C D* — *A is* 1 *If it be cut off from the lesser base.*

Fig. 7.

November the 19, 1644, between 9 and 11, in a cleare sky at *London.*

There appeared a Rainbow at 9 *h.* $\frac{1}{3}$
It vanished at 10 *h.* $\frac{1}{4}$
There appeared three Parhelii at 9 *h.* $\frac{1}{4}$
They vanished at 10 *h.* $\frac{1}{2}$

Fig. 8.

They were distant from the Sun on each side 12 *ells, and the rainbow about twice asmuch.*

The mensuration of the area of a Spherical Triangle.

Lemma I. *The Lunary superficieses of the hemisphericks are as the angles of the same superficieses.* *Fig.* 9.

The proof of it amongst many other wayes, may be this. Let the Meridian Semicircle A E D be imagined to be equally moved over the longitude of the Equator B E C, upon the Poles A and D. Therefore the angles on the other side at A and D (to wit F and G) shall be as the times. The superficieses also K and M shall be as the same times. Therefore F shall be to G, as the superficies K, to the superficies M; And G unto F : : as the superficies M, to the superficies K, &c. howsoever they can be taken. [*For those things that agree to a third, agree among themselves.*]

Coroll.

Corollarium. Ergo componendo F + G. G :: K + *M*. *M*. vel, Ut F + G. F :: K + *M*. K. hoc est.

Ut, duo recti.G :: $\frac{1}{2}$ sphær. superf. *M*. & 2.recti.F :: $\frac{1}{2}$ sphær. superf. ad K.

Fig. 10. *Lemma* II. Triangulum G æquatur triang. H. Quoniam anguli, & latera unius, æquantur angulis, & lateribus alterius. Nempe A=*D*. B=E. C=F. Irem L=O. *M* = *P*. N=Q. Ergo sunt congrua, & æqualia.

THEOREMA.

Excessus trium angulorum supra duos rectos divisus per 720 ostendit, quanta sit trianguli area respectu totius Sphærici.

Nam per Lemma I. $\begin{cases} 180 . A :: \frac{1}{2} \text{Sphær. } G + R \\ 180 . B :: \frac{1}{2} \text{Sphær. } G + S \\ 180 . C :: \frac{1}{2} \text{Sphær. } G + T = H + T \end{cases}$ (per Lem. secundum.) Ergo 180. A + B + C :: $\frac{1}{2}$ Sphær. 3 G + R + S + T per 24 quinti: Et per divisionem rationis contrarium (cujus meminit *Clavius* ad 15 definit. quinti Elementorum.)

Ut 180. A + B + C − 180 :: $\frac{1}{2}$ Sphær. 3 G + R + S + T − $\frac{1}{2}$ Sphær. At G + R + S + T = $\frac{1}{2}$ Sphær. Ergo 3 G + R + S + T − $\frac{1}{2}$ Sphær. = 2 G. Adeoque, 180. A+B+C − 180 :: $\frac{1}{2}$ Sphær. 2 G. Et quadruplicatis antecedentibus erit 720. A + B + C − 180 :: 2 Sphær. 2 G, & ita Sphær. ad G.

Ergo $\frac{A+B+C-180}{720}$ Ostendit triangulum quota sit pars totius Sphærici.

Hæc vera quoque sunt in Polygonis Sphæricis cujuscunque sint figuræ ordinatæ nimirum, vel inordinatæ modo omnes anguli dentur. Atque hoc ideo quia Polygona omnia in triangula resolvantur. Hæc igitur jam regula in istis multangulis tenebit.

Duc 180 *gr.* in numerum angulorum. Hunc factum subtrahe ex aggregato omnium augulorum aucto 360 *gr.* Residuum divisum per 720 *gr.* dat aream Polygoni.

De

Coroll. *Therefore by composition* $F+G.\ G :: K+M.\ M.$ *Or, As* $F+G.\ F :: K+M.\ K.$ *That is,*

As the two right angles, are unto G :: *So is* $\frac{1}{2}$ *the spherical superficies, to* M. *And as two right angles, are to* F :: *So is* $\frac{1}{2}$ *the spherical superficies, to* K.

Lem. II. *The triangle* G *is equal to the triangle* H, *because the angles, and sides of one, are equal to the angles and sides of the other. To wit,* A ∞ *to* D. B ∞ E. C ∞ F. *Also* L ∞ O. M ∞ P. N ∞ Q. *Therefore they are congruous and equal.* Fig. 10.

THEOREME.

The excesse of the three angles over and above two right ones, divided by 720, shewes what the area of the triangle is in respect of the whole spherick.

For by the 1 *Lemma.* $\begin{cases} 180.\ A :: \frac{1}{2}\ spher.\ to\ G+R \\ 180.\ B :: \frac{1}{2}\ spher.\ to\ G+S \\ 180.\ C :: \frac{1}{2}\ spher.\ to\ G+T ∞ H+T \end{cases}$ *(by the second Lem.) Therefore* $180.\ A+B+C :: \frac{1}{2}$ *spher. is to* $3G+R+S+T$. *by* 24 *of the* 5: *and by contrary division of the ratio (which* Clavius *mentions, upon the* 15 *Definition of the* 5 *of the Elements.)*

As $180.\ A+B+C-180 :: \frac{1}{2}$ *spher. to* $3G+R+S+T-\frac{1}{2}$ *spher. but* $G+R+S+T$ ∞ $\frac{1}{2}$ *spher. Therefore* $3G+R+S+T-\frac{1}{2}$ *spher.* ∞ $2G$. *So that* $180.\ A+B+C-180 :: \frac{1}{2}$ *spher. is to* $2G$. *And the antecedent terms being quadrupled, it shall be* $720.\ A+B+C-180 :: 2$ *spher. to* $2G$. *And so the spher. to* G.

Therefore $\frac{A+B+C-180}{720}$ *shewes what part the triangle is of the whole spherick.*

These things are likewise true in all spherical Polygons of what ordinate figure soever they be, or inordinate so all the angles be given. And the reason is, because all Polygons may be resolved into triangles. Therefore this Rule shall hold in these multangles.

Multiply 180 deg. *by the number of the angles. Subduct the product out of the aggregate of all the angles increased by* 360 gr. *the residue divided by* 720 gr. *gives the area of the Polygon.*

F Of

De completione loci ſolidi.

Hic fructus naſcitur ex priori menſuratione.

Fig. 11. SI radius ſphæræ ſit 100000.00 latus *Icoſaëdri* inſcripti erit 105146.22 = ſubtenſæ 63 *gr.* 26′ 10″. Triangulum ergo planum æquilaterum F E O (in *Icoſaëdro*) reſpondet triangulo æquilatero ſphærico in ſphæra; cujus tres anguli ſphærici connectuntur cum dictis angulis planis in eiſdem punctis F, E, O.

Et latera hujus trianguli ſphærici ſunt ſigillatim 63 *gr.* 26′ 10″, nimirum quia eorum ſubtenſæ F E, E O, O F. in triangulo plano ſunt invicem æquales.

Demittatur jam perpendiculum E P, Erit ergo E P O triangulum ſphæricum rectangulum ubi præter rectum P, dantur E O, & P O = $\frac{1}{2}$ E O. Quare verticalis angulus P E O erit 36 *gr.* præciſe, & totus angulus ad E erit 72 *gr.* & ſumma trium æqualium ad E, F, O, erit 216 *gr.* unde detractis duobus rectis = 180 *gr.* reſtant 36 *gr.* Ergo triangulum F E O, eſt $\frac{36}{720}$ totius ſphærici hoc eſt $\frac{1}{20}$ pars totius ſphærici: & hoc rectiſſimè. Nam 20 pyramides F E O C complent locum ſolidum *Icoſaëdri*: & 20 (adeo) baſes ſphæricæ (baſibus planis triangularibus obductæ) complent totum ſphæricum.

F E O C eſt una è pyramidibns 20 in *Icoſaëdro* triangulum planum B eſt una ex hedris C eſt centum corporis, vel ſphæræ circumſcribentis.

4.77 *Icoſaëdra* 4.24 *Dodecaëdra* 9.244 *Octaëdra* 8.000 *Cubi*	} Complent locum ſolidum. Uti apparebit, ex praxi ſuperiori per triangula, & Polygona ſphærica.

Id eſt, nullum è quinque corporibus regularibus complet locum ſolidum ſolo cubo excepto.

Contra quod *Potamon*, & ex eo *Ramus*, & omnes *Ramum* ſecuri tradidere.

FINIS.

Of the completion of a Solid body.

This fruit ariseth from the precedent mensuration.

If the radius of the Sphere be 100000.00 *the side of an inscribed* Icosaedrum *shall be* 105146.22 ∞ *to the subtense of* 63 d. 26′ 10″. *Therefore the plain equilateral triangle* F E O (*in the* Icosaedrum) *answers to the equilateral spherical triangle in the sphere; whose three spherical angles are connected with the plain angles in the same points* F, E, O. *Fig.* 11.

And the sides of this spherical triangle are separately taken 63 d. 26′ 10′, *to wit, because their subtenses* F E, E O, O F *in the plain triangle are equal to one another.*

Let fall now the perpendicular E P, *the spherical trian.* E P O *shall be rectangled, where over and above the right angle at* P, *are given* E O *and* P O ∞ ½ E O, *wherefore the vertical angle* P E O *shall be* 36 deg. *just, and the whole angle at* E 72 deg. *and the sum of the three equal angles at* E, F, O, *shall be* 216 d. *from whence taking two right angles, equal to* 180, *there remains* 36 d. *therefore the triangle* F E O *is* $\frac{36}{720}$ *of the whole spherick, that is,* $\frac{1}{20}$ *part. And this most truly for* 20 *pyramides* F E O C *fill the solid place of the* Icosaedre. *And so* 20 *spherical bases* (*covered over with* 20 *triangular plain bases*) *complete the whole spherick.*

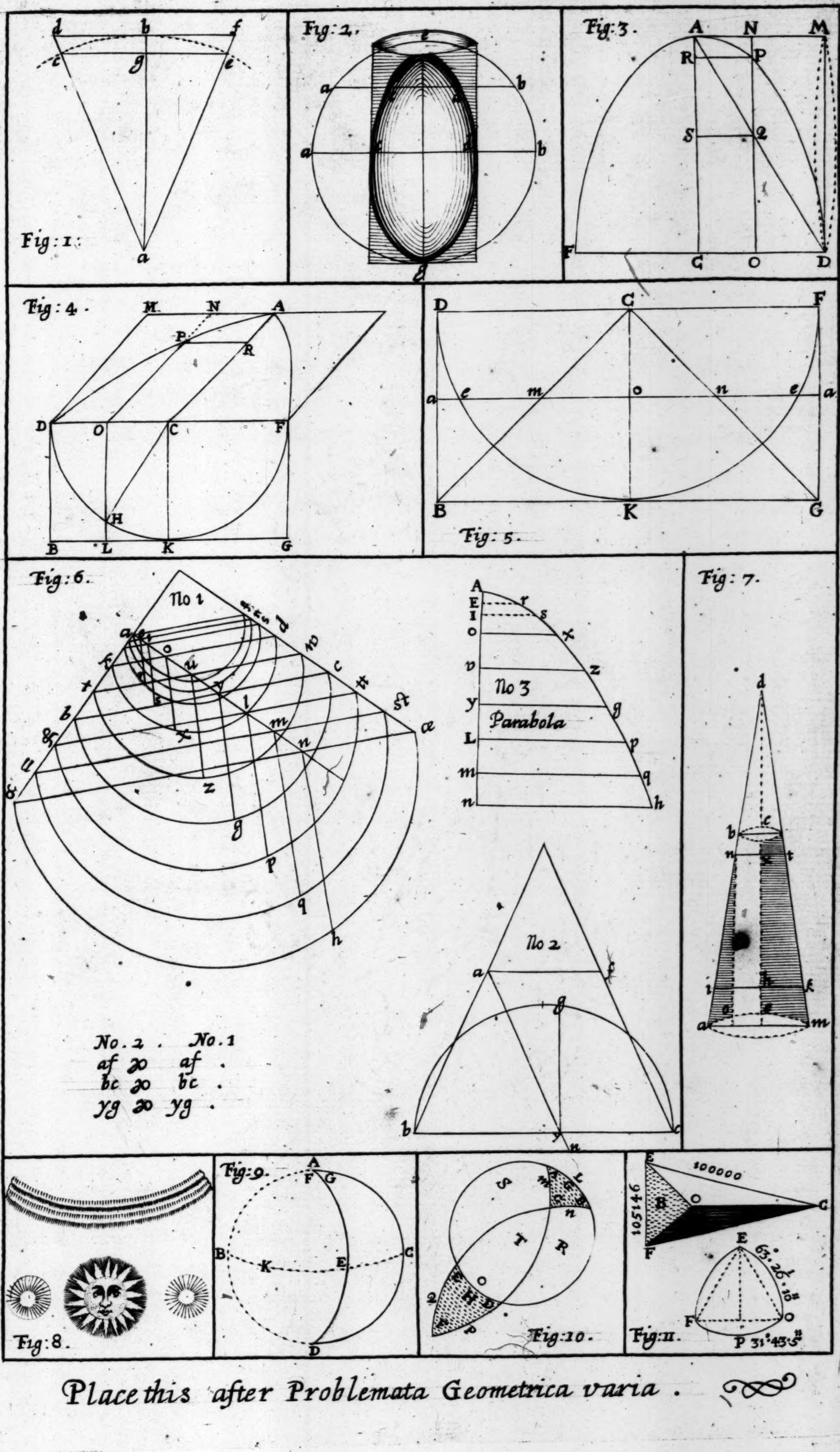
F E O C *is one of the* 20 *pyramids in the* Icosaedre. *The plain triangle* B *is one of the hedræ* C *is the center of the body, or sphere that circumscribes it.*

4.77 Icosaedres 4.24 Dodecaedres 9.244 Octaedres 8.000 Cubes.	*Fill a solid place, as will appear out of precedent practice by triangles, and spherical Polygons.*

That is to say, None of the five regular bodies fill a solid place, the Cube only excepted.

Contrary to what Potamon, *and from him* Ramus, *and all that have followed* Ramus, *to wit,* Snellius, *and others, have delivered.*

The END.



Place this after Problemata Geometrica varia.

PROBLEMATUM QUORUNDAM MATHEMATICORUM,

(De Triangulis tam Rectangulis quam Obliquangulis,)

ANALYTICA SOLUTIO, ET CONSTRUCTIO.

Authore J. TWYSDEN.

CERTAIN MATHEMATICAL PROBLEMS,

(Concerning Triangles as well Oblique as Rectangled,)

ANALYTICALLY RESOLVED, AND EFFECTED,

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PROBLEMA I.	PROBLEM I.
Datis trianguli plani rectanguli summa laterum (c) *&* *basi* (b) *in venire tum cathetum tum pypotenusam.*	In a plain rectangled triangle, (c) the sum of the hypotenuse, & perpendicular being given, together with (b) the base, to find the rest.

Fig. 1.

Puta factum sitque,

(a) Cathetus, erit $c-a$ hypotenusa, & $cc-2ca$ plus aa ∞ $bb+aa$, & demptis utrinque, aa erit $cc-2ca$ ∞ bb, vel $2ca$ ∞ $cc-bb$ & $\frac{cc-bb}{2c}$ ∞ a.

Aliter,

Sit à hypotenusa erit $c-a$ basis, & aa ∞ $cc-2ca+aa+bb$ & $\frac{cc+bb}{2c}$ ∞ a.

CANON.	CANON.
Quadratum summæ laterum minutum qudrato basis, & per duplum laterum summam divisum exhibebit cathetum. Auctum vero quadrato basis, & per duplum laterum summam divisum exhibebit hypotenusam.	*If from the square of the sum of the sides, you take away the square of the base, and divide the residue by double the sum of the sides, the quotient shall be the quantity of the perpendicular. But if the square of the sum of the sides, be increased by the square of the base, and that sum divided by double the sum of the sides, the qnotient shall be the hypotenuse.*

Example in numbers. Let the base be 3.

Let the sum of the sides be 9, *the Square* 81, *the Square of the base* 9, 81 + 9, *is* 90, *divided by* 18, *shall give* 5 *the*

hypote-

hypotenuse, or 81 − 9 *is* 72, *that divided by* 18, *shall give* 4, *the perpendicular, so the sides shall be* 3, 4, 5.

Fig. 2. Geometricè sic. Diametro AB ∞ *c* describatur semicirculus. Cui inscribatur B*f* continuata infinite, & sit B*f* ∞ *b* erit A*f* cui æquatur *fi*, √q. *cc* − *bb*. continuetur *fa* in *l*, ita ut *fl* sit æqualis 2 *c* inter quam ut prima, & *fi* ut secunda inveniatur, *fm* tertia, quæ æquabitur catheto quæsito, nam *fi*q. ∞ *fa*q. ∞ *cc* − *bb* dividitur per *lf* ∞ 2 *c*. & fit *fm* quotiens geometricus. Nam *fl*. *fi*. *fm* sunt continuè proportionales; per 8 El. sexti *Eucl.* ergo $\frac{fi\text{q.}}{fl}$ producit *fm*, per 4 sexti *Eucl.*

Geometrically thus. *Upon* AB *made equal to* c, *the sum of the hypotenuse, and cathetus, describe a semicircle, in which, inscribe* B f, *from the term* B *equal to* b *the base given, and continue it infinitely, so shall* A f, *to which make* f i *equal, be the root square of* c c − b b, *continue* f a *in* l, *so that* f l *be made equal to* 2 c, *between this as the first, and* f i *the second, find* f m *the third in continual proportion, it shall be equal to the perpendicular sought, for* f i *q.* ∞ f a *q.* ∞ c c − b b *is divided by* f l ∞ 2 c, *and* f m *is the geometrical quotient, for* f l. f i. f m *are continually proportional, by the* 8*th. of* 6 Eucl. *Therefore* $\frac{fi\text{q.}}{fl}$ *produces* f m; *by the* 4*th. of the* 6 Euclid.

PROBLEMA II.

In triangulo rectangulo datis p, *perpendiculo ab angulo recto in hypotenusam dimisso, &* b *differentia segmentorum hypotenusæ, invenire triangulum.*

PROBLEM II.

In a right angled triangle *p*, the perpendicular, let fall from the right angle upon the hypotenuse, and *b* the difference of the segments of the hypotenuse are given, to find the triangle.

Sit â minus segmentum, *b* + *a* erit majus, & *b* + *a* in *a*, hoc est, *ba* + *aa* ∞ *pp*. Ergo

Canon. $\sqrt{\tfrac{1}{4}bb + pp} : \pm \tfrac{1}{2}b \infty a.$

Potest

Poteſt Problema ſic aliter proponi. Data media trium quantitatum continue proportionalium cum differentia extremarum invenire reliquas.

Geometrice ſic. Perficitur ſuper diametro E F infinita erigatur ad rectos *m* I æqualis *p* datæ & menſuretur *m* H æqualis $\frac{1}{2}b$ erit H I $\sqrt{\frac{1}{4}bb+pp}$ ſuper hâc ut ſemidiametro ſcribatur ſemicirculus, & obſervatur Canon *Algebricus*. Nam E *m* eſt $\sqrt{\frac{1}{4}bb+pp}:+\frac{1}{2}b$, & *m* F eſt $\sqrt{\frac{1}{4}bb+pp}:-\frac{1}{2}b$. Nam E *m*. *m* I. *m* F ſunt $\div$ propter ſimilitudinem triangulorum, E I *m*. *m* I F.

THEOREMA.

SI quadratum perpendiculi augeatur quadrante quadrati differentiæ datæ. Aggregati radix quadrata aucta dimidio differentiæ datæ erit E *m* majus ſegmentum. Minuta vero dimidio differentiæ erit *m* F minus ſegmentum, & inventum eſt triangulum.

PRO-

The probleme may be thus otherwiſe propounded. In three quantities in continual proportion, the middle term is given, and the difference of the extremes. To find the reſt.

Geometrically thus. Upon the diameter E F, *produced infinitely erect* m *I at right angles, equal to* p *the perpendicular given, and meaſure off* m H, *equal to* $\frac{1}{2}$ b, *draw* H *I which ſhall be* $\sqrt{\frac{1}{4}bb+pp}$ *by the* 47 *of the* 1 *of* Eucl. *upon that as ſemidiameter deſcribe a ſemicircle, and the analytical Canon is obſerved. For* E m *is* $\sqrt{\frac{1}{4}bb+pp}:+\frac{1}{2}b$, *and* m *F is* $\sqrt{\frac{1}{4}bb+pp}:-\frac{1}{2}b$. *For* E m, m *I*. m *F* $\div$ *by reaſon of the ſimilitude of the triangles* E I m. m *I F*. Fig. 3.

THEOREME.

IF *to the ſquare of the perpendicular, you adde a quarter of the ſquare of the difference given, and from the ſum extract the ſquare root, that root increaſed by half the difference, ſhall be equal to* E m *the greater ſegment, but diminiſhed by half the difference, ſhall be equal to* m F *the leſſer ſegment. And all the parts of the triangle are known.*

PRO-

PROBLEMA III.

Fig. 4. *Data media* c *trium quantitatum* ∺ a, c, d, *unâ cum* b, *differentia qua major terminus excedit duplum minoris invenire terminos. Est octavam* Oughtredi, *in Clave aliter propositum, & resolutum.*

Data $b.\ c.\ d.$

Quæritur minor terminus, a.

PROBLEM III.

In three terms a, c, d, ∺ c the middle term is given, with b, the difference between the greater term, and double the lesse. The terms are required.

Given b. c. d.

Sought the lesser term a.

Puta factum quod requiritur. Sitque a, minor extrema, erit $b + 2a$ major, & $ba + 2aa \propto cc$. vel $2aa \propto cc - ba$. Ergo $\sqrt{\frac{1}{4}bb + 2cc:} - \frac{1}{2}b \propto 2a$, scilicet duplo minoris termini. Vel quod idem est, $aa \propto \frac{cc - ba}{2}$, & $\sqrt{\frac{1}{16}bb + \frac{cc}{2}:} - \frac{1}{4}b. \propto a$.

THEOREMA.

DUplo quadrato mediæ datæ addatur quarta pars quadrati differentiæ datæ. Hujus summæ radix quadrata minuta dimidio differentiæ erit dupla minoris termini.

Fig. 5. Geometrice. Ad punctum c rectæ bc mensuretur $co \propto c$ datæ, cui æqualis statuatur om ad rectos: erit $cm \propto \sqrt{2cc}$. cui æquatur cf ducta à termino c ad rectos fac $ci \propto \frac{1}{2}b$ erit if æqualis $\sqrt{\frac{1}{4}bb + cc}$, cui æquetur ib subduc ih æqualis $\frac{1}{2}b$, erit $bh \propto 2a$, & $hc \propto$ differentiæ, super bc igitur diametro describatur semicirculus blc, in cujus circumferentia accomodetur $bl \propto c$, & dividatur bh bifariam in n, erit bl

THEOREME.

IF *to double the middle term squared, you adde a quarter of the square of the difference: the square root of this sum being diminished by half the difference, shall leave the lesser extreme sought.*

Geometrically. At the point c *of a right line* c b, *measure* c o *equal to* c, *the middle term given, to which, make* o m *at right angles equal. Then shall* c m *be the* $\sqrt{2cc}$, *to which* c f *is by construction equal & perpendicular to* b c, *make* c i *equal to half* b, *then shall* i f *be* $\sqrt{\frac{1}{4}bb + cc}$, *to which, make* i b *equal: from* i b *subduct* i h *equal to* $\frac{1}{2}$ *the difference*, b h *shall be equal to double the lesser term sought, and* h c *shall be*

b l media, *b n* minor, & *b c* major trium quantitatum $\div\!\!\!\cdot\cdot$, nam triangula *b l n*, *b l c*, sunt similia, ergo *c b. b l. b n* sunt $\div\!\!\!\cdot\cdot$ & observatur præscriptum Theorematis.

be equal to the difference given. Upon b c, *as a diameter, describe a semicircle* b l c, *into which fit* b l ꝏ c, *and divide* b h *into two equal parts*, b l *shall be the middle*, c b *and* b n *the two extremes, in continual proportion. For the triangles* b l n, b l c *are alike.*

Problema IV.

In triangulis duobus rectangulis dantur summa basium, & utriusque cathetis ea conditione, ut angulus ad F *sit rectus. Quæruntur bases.*

Problem IV.

In two right angled triangles, the sum of the two bases, each perpendicular, and a right angle at F are given. The bases are sought. *Fig. 6.*

Dantur *b*, *c*, *d*, & angulus ad F rectas.

$aa \,.\, ꝏ\, aa$

$ee \,.\, ꝏ\, cc + aa - 2ca$

$gg \,.\, ꝏ\, cc + \overline{dd + bb - 2bd}$

$hh \,.\, ꝏ\, bb + aa$

$kk \,.\, ꝏ\, dd + ec$ (id est) $+ cc + aa - 2ca$

$gg \,.\, ꝏ\, (hh + kk)$ vel $bb + aa + dd + cc + aa - 2ca.$

$gg \,.\, ꝏ\, bb + aa + dd + cc + aa - 2ca.$

$gg \,.\, ꝏ\, cc + dd + bb - 2bd$, ergo hæ duæ species æquantur inter se, *viz.*

$cc + bb + dd - 2bd \;ꝏ\; bb + dd + cc + 2aa - 2ca.$

Et sublatis utrinque æqualibus.

$2aa - 2ca + 2bd \;ꝏ\; 00$. Ergo mutatis signis $2aa \;ꝏ\; 2ca - 2bd$; & $aa \;ꝏ\; ca - bd$, & resoluta æquatione

$\frac{1}{2}c \pm \sqrt{\frac{1}{4}cc - bd} : ꝏ\, {}^{a}_{e}$

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$\frac{1}{2}c \pm \sqrt{\frac{1}{4}cc - bd} : ꝏ\, {}^{e}_{a}$

In verbis,

EX quadrato dimidii summæ basium tolle planum ex uno cathetorum ducto in alterum. Residui

In words,

OUt of the square of the sum of both the bases; take the plain made by one of the perpendi-

Fig. 6. Residui radix quadrata aucta dimidio summæ basium, erit basis trianguli majoris. Sed dimidium summæ basium minutum radice quadrata dicti residui erit basis trianguli minoris.

Geometrica effectio patet in figura. Est enim F A summa cathetorum, & quadratum BC æquatior plano F B A (hoc est *b d.*) E B est semis. (*c*) B E D est semic. B D ∞ BC, ergo D E est $\sqrt{q.\ \frac{1}{4}cc. - bd}$: D E. ∞ E G, ergo B G est $\frac{1}{2}$ c + $\sqrt{\frac{1}{4}cc - bd}$ H G est $\frac{1}{2}$ c − $\sqrt{\frac{1}{4}cc. - bd}$ quod requirit Theorema.

pendiculars, multiplyed by the other. The square root of the residue, being increased by half the sum of the sides, shall be the base of the greater triangle: but half the sum of the sides diminished by the root of the said residue, shall be the base of the lesser triangle.

The effection is evident in the figure. For F A is the sum of the perpendiculars. And the square of B C is equal to the plain F B A. ∞ b d, *B E is half* (c) *B E D is a semicircle B D is equal to B C, therefore D E is* $\sqrt{\frac{1}{4}cc - bd}$ *D E* ∞ *E G, therefore B G is* $\frac{1}{2}$ c + $\sqrt{\frac{1}{4}cc - bd}$: *and H G is* $\frac{1}{2}$ c − $\sqrt{\frac{1}{4}cc - bd}$ *as the Theoreme requires.*

PROBLEMA V.

ANno 1644. *Johannes Pellius* Coritano Regnus *Anglus,* Matheseos in illustri *Amstelodamensium* Gymnasio Professor, chartulam quandam excudi curavit, & in varia loca dimisit continentem Theorema, quoddam cujus medio *Cristiani Severini, Longomontani, Cimbri, &c.* Librum de absoluta circuli mensura solide, & nervose refutavit, uti fusius in prædicti D. *Pellii* libello postea contra *Longomontanum* divulgato apparet. Hujus chartulæ prius impressæ exemplar unicum ad me misit D. *Guilielmus Beecher* Eques Auratus

PROBLEM V.

IN *the yeer* 1644. *Mr.* John Pell *Professor of the Mathematicks in* Amsterdam *caused a certain paper to be printed, and dispersed abroad conteining a Theoreme, by help of which he hath both solidly, and substantially confuted* Longomontanus *his Book of the absolute measure of a circle, as may appear more largely in a Book since published by Mr.* Pell *against* Longomontanus. *One of those first papers, Sr.* William Beecher *then living at* Roven, *sent me to* Paris, *to whom I returned my answer after some dayes, whither it miscarried*

Auratus meamque postulavit sententiam, & Theorematis demonstrationem. Nonnullis ab accepta chartula diebus solutionem, & demonstrationem analyticam à *Parisiis* ad illum tunc Rothomagi degentem misi. Utrum vero ei in manus venerit ignoro. Erat autem Theorema ut sequitur.

ried or no, I know not. The Theoreme was as followeth.

Fig. 6.

Tangens cujuslibet arcus minoris quam 45 g. 00 m. ducatur in duplum quadratum radii; à quadrato radii auferatur tangentis quadratum illud productum dividatur per hoc residuum: Quotus erit tangens arcus dupli.

Let the tangent of any arke lesse then 45 deg. 00 m. be multiplyed by double the square of the radius, from the square of the radius, take the square of the tangent. Let the first product be divided by this residue, the quotient shall be the tangent of the double ark.

Ego ad formam Problematis reduxi.

I reduced it into the form of the following Probleme.

Datis trianguli rectanguli basi (*r*) perpendiculi segmento angulo recto contermino (*t*), & angulo ad A bifariam secto invenire perpendiculum, & totum triangulum.

In a rightangled triangle, there is given the base (r,) tht segment of the perpendicular conterminous to the right angle (t,) with the angle at A bisected, to find the perpendicular and the whole triangle.

A E, est $\sqrt{q.}\ rr + aa$. per 47. 1 *Euclid.* Fig. 7.

Data. *r.* & *t.* Quæritur *a.* Quia per tertium sexti *Euclidis.*

$r.t :: \sqrt{q.}\ rr + aa.\ a - t$ erit

$ar - tr.\ \infty\ \sqrt{q.}\ rrtt + ttaa$, ergo eorum quadrata erunt æqualia.

$rraa + rrtt - 2rrta\ \infty\ rrtt + ttaa$, vel subductis æqualibus.

$rraa - 2rrta\ \infty\ ttaa$, & dividendo,

$rra - 2rrt\ \infty\ tta$, vel transponendo terminos.

$2rrt\ \infty\ rra - tta$. Ergo

$rr - tt.\ 2rr :: t.\ a$, & propterea ex $\frac{2rrt}{rr-t}$ orietur *a.*

Quod est ipsissimum Theorema D. *Pellii.* Posita enim basi
Fig. 7. trianguli pro radio erit *t*, tangens arcus simpli, & à tangens arcus dupli. Ergo si tangens cujuslibet arcus minoris quam 45 gr. 00 min. &c.

DETERMINATIO.

Hinc patet quod segmentum perpendiculi (hoc est tangens arcus simpli) non debet radium excedere (hoc est tangentem arcus 45 gr. 00 min.) aliàs enim subductio nequit fieri quod requirit Theorema.

Fig. 8. Geometricè sic. Super E G circuli radio ut diametro describatur semicirculus: mensuretur E C ∞ E T tangenti datæ, erit G Cq. ∞ *r r* — *t t*, cui æqualis statuatur A B, B *m* q. verò sit æqualis, lateri seu radici 2 E G q. hoc est 2 *r r*. Inter A B, & B *m*, hoc est, inter √q. *r r* — *t t*, & √q. 2 *r r* quæratur, tertia proportionalis quæ invenietur ∞ A E, & per 18 octavi *Eucl.* *r r* — *t t*. 2 *r r* :: A B. A E. Ergo, ut A B. A E :: *t*. *a*. Nam ut *r r* — *t t*. 2 *r r* :: *t*. *a*. Erigatur igitur à puncto B, perpendicularis B D ∞ E T, hoc est *t*. cui parallelæ ascendat infinita E F, & à puncto A per terminum D, ducatur A F erit E F ∞ (*a*) quæsitæ qua cognita compleatur triangulum Theoremati congruum.

PRO-

DETERMINATION.

From hence it appears, that the segment of the perpendicular, (to wit the tangent of the simple ark) must not exceed the radius (that is the tangent of 45 gr. 00 m.) for otherwise the subduction cannot be made as the Theoreme requires.

Geometrically thus. Upon E G *the Radius of your circle, as a diameter describe a circle. Set off* E C ∞ E T *the tangent given.* G C*q. shall be equal to* r r — t t *to which make* A B *equal. And let* B m *q. be equal to* 2 G E *q. that is,* 2 r r. *Then between* A B, *and* B m, *that is, between* √q. r r — t t, *and* √q. 2 r r *find the third proportional, which let be* A E. *by the* 18 *of the* 8*th* Eucli. A B*q. shall be to* B m *q.* :: A B. A E, *that is,* r r — t t. 2 r r :: A B. A E. *for as the first is to the fourth, so shall the square of the first, be to the square of the second, in terms continually proportional, since it is therefore* r r — tt. 2 r r :: A B. A E, *and* r r — tt 2 r r :: t. a. *it shall be* A B. A E :: t. a. *therefore from the term* B, *erect a perpendicular,* B D ∞ E T, *that is, to* t, *to which draw* E F, *an infinite line parallel,*

rallel, and from the point A, by D, draw A F. E F shall be equal to (a,) which being found, finish the triangle agreeable to the Theoreme. Fig. 8.

PROBLEMA VI.

Data tangente arcus dupli quæratur tangens arcus simpli, hoc est data a *quæratur* t, *quia antea inventa est hæc æquatio* $tta + 2rrt \text{ ꝏ } rra$. *erit* $tt. \text{ ꝏ } rr - \frac{2rrt.}{a}$ *Ergo* $\frac{\sqrt{rrrr + rr:}}{aa} - \frac{rr}{a} \text{ ꝏ } t$.

Praxis geometrica facilis est loco $\frac{rrrr}{aa}$ scribe ss. Hoc modo, ut $aa.\ rr :: rr.\ ss.$ Ergo $\frac{aass}{aa} \text{ ꝏ } \frac{rrrr.}{aa}$ & $\sqrt{ss + rr:} - \frac{rr.}{a} \text{ ꝏ } t$.

PROBLEM VI.

The tangent of a double ark being given, if it be required, to find the tangent of the single ark, the equation will be $\frac{\sqrt{rrrr + rr:}}{aa} - \frac{rr}{a} \text{ ꝏ } t$.

The geometrical effection is easie, in the place $\frac{rrrr}{aa}$, *write* ss. *Thus* $aa.\ rr :: rr.\ ss.$ *then* $\frac{aass}{aa} \text{ ꝏ } \frac{rrrr}{aa}$, *and* $\sqrt{ss + rr:} - \frac{rr}{a} \text{ ꝏ } t$.

PROBLEMA VII.

Dato triangulo quadratum inscribere.

PROBLEM VII.

To inscribe a square into a given triangle.

Sit basis trianguli *b* Fig. 9.
Perpendiculum *p*
Sit latus quadrati inscribendi *a*.
Ergo segmentum perpendiculi superius, erit $p - a$.
Et erit $p - a.\ a :: p.\ \frac{pa}{p-a} \text{ ꝏ } b$.
Ergo $pa \text{ ꝏ } bp - ba$, & $pa + ba \text{ ꝏ } bp$. Ergo $p + b.\ b :: p.\ a$.

Praxis Geometrica est facillima, sit A C, ꝏ $p + b$. & C D ꝏ b, & sit B A ꝏ p. erit B E latus quæsitum ꝏ a.

The effection is very easie, & requires no more then in the three terms given, to find the fourth. Therefore, make A C ꝏ b + p, *and C D.* ꝏ b, *and B A* ꝏ p.

Eodem

∞ p. *B E shall be the side sought.*

Eodem modo circulus qui inscribi potest maximus inveniatur, cujus diameter erit quadrati, latus diagonium.

So may the greatest possible circle be inscribed, whose diagonium shall be equal to the diameter of the circle.

Fig. 10. Hoc idem Problema sic aliter absolvitur.

This Probleme is thus otherwise performed.

Sit (a) segmentum perpendiculi inter trianguli verticem, & latus quadrati inscibendi, erit $p-a$ latus quadrati. Et erit,

$$\left.\begin{array}{l} p \,.\, c :: a \,.\, \frac{ca}{p} \\ p \,.\, d :: a \,.\, \frac{da}{p} \end{array}\right\} \begin{array}{l} \text{Secundo erit} \\ \text{Ergo } \frac{ca+da}{p} \infty p-a, \, \& \; ca+da \infty pp-pa. \end{array}$$

Et $ca+da+pa \infty pp$. Ergo $\frac{pp}{c+d+p} \infty a$. quâ sublatâ à perpendiculo residuum, erit latus quadrati inscribendi.

Canon. $\frac{pp}{c+d+p} \infty a$.

Fig. 11. Geometricè sic. Ducatur ab æqualis $c+d+p$, & super hâc ut diametro, describatur semicirculus acb, mensuretur $bc \infty$ perpendiculo, cui æquatur $b\,e$ per structuram, & à puncto (c) descendat perpendicularis (cd,) erit (bd) quotiens Geometricus, & æqualis (a,) nam $a\,b.\ c\,b.\ b\,d \div$

Geometrically thus. Make a b *equal to* c + d + p, *and upon it as a dinmeter, describe a semicircle* a c b, *measure* b c *equal to the perpendicular, to which* b e *is equal by structure, from the point* (c) *let fall the perpendicular* (c d,) (b d) *is the Geometrical quotient equal to* (a,) *for* a b. c b. b d $\div$

Ducatur a termino (e) (ei) æqualis basi trianguli, & ad (ba) normali, agantur denique (dg) (bf) parallelæ, & compleatur triangulum.

Lastly, from the point (e) *draw* (e i) *equal to the base of your triangle, and square to* (a b) *draw* (d g) *and* (b f) *parallels, and complete the triangle.*

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PROBLEMA VIII.

Dato triangulo rectangulum inscribere, cujus area sit ad aream trianguli in ratione possibili data.

r *ad* s. *Et sit area trianguli* m m.

PROBLEM VIII. *Fig.* 12.

In a triangle given, to inscribe a rectangle, whose area shall be to the area of the triangle in any possible proportion, as

r to *s*, and the area of the triangle let be *m m*.

Puta factum sitque latus quæsitum *a*.

Primo $p.c::p-a.\ \frac{pc-ca}{p}$ ∞ lateri rectanguli majori.

Secundo $p.d::p-a.\ \frac{dp-da}{p}$ Ducatur in *a*.

Erit $\frac{pca-caa+dpa-daa.}{p}\ mm::s.r.$

Ergo smm ∞ $\frac{rpca-rcaa+rdap-rdaa.}{p}$ vel $\frac{psmm}{r}$

∞ $c+d$, hoc est,

$bpa-caa-daa$, vel $\frac{psmm}{r}+baa$ ∞ bpa, & tandem, baa ∞ $bpa-\frac{psmm}{r}$ vel aa ∞ $pa-\frac{psmm,}{br}$

Et $\sqrt{\frac{1}{4}pp-\frac{psmm}{rb}}:+\frac{1}{2}p$ ∞ a.

The Æquation.

$\sqrt{\frac{1}{4}pp-\frac{psmm}{rb}}:+\frac{1}{2}p.$ ∞ a.

Determinatio. Absolutum datum non debet excedere quadratum semissis perpendiculi. Nam si superaverit rectangulum inventum erit areæ majoris quam inscribi potest.

Constructio Problematis. Primo reducatur *r b* planum ad quadratum, sit illud *n n*. Similiter

Determination. *The absolutum datum must not exceed the square of half the perpendicular, for otherwise the area found will be greater then can be inscribed.*

For the Geometrical construction. *First reduce* r b *to a square, let that be* n n. *In like manner*

Fig. 12. Similiter reducatur (ps) ad quadratum sit illud xx, & loco $\frac{psmm}{rb}$ scribatur $\frac{xxmm,}{nn}$ deinde fiat $nn.xx::mm.tt,$ ergo $\frac{xxmm}{nn}$ œ $\frac{nntt,}{nn}$ & æquatio constructionis facillimæ, sic stabit

$$\sqrt{\tfrac{1}{4}pp - tt} : + \tfrac{1}{2}p \,\infty\, a.$$

manner (ps,) *let that be* xx, *then in the place of* $\frac{psmm}{rb}$ *you will have* $\frac{xxmm}{nn}$ *then find the third proportion between* nn *and* xx. *As* $nn.xx::mmtt$, *and then your Equation fit for construction will stand thus*

$$\sqrt{\tfrac{1}{4}pp - tt} : + \tfrac{1}{2}p \,\infty\, a.$$

Problema IX.

Proposuit mihi vir ingenuus, & Philomathematicus, hanc quæstionem solvendam.

Dantur duæ lineæ sive numeri A & B, quarum summa (z) æquatur differentiæ quadratorum. Summa vero quadratorum subducta ex quadrato summæ relinquet b planum.

A

B

Postquam paululum mecum ruminavi venit mihi in mentem Lemma sequens.

Lemma. Summa duorum quorumlibet numerorum unitate differentium, erit æqualis differentiæ quadratorum. Sin differant binario differentia quadratorum, erit dupla summæ, trinario tripla, &c.

Fig. 13. *Demonstratio.* A est major numerus & A + E. major, & E est

Problem IX.

An ingenuous person, and lover of the Mathematicks, propounded unto me this question.

Two lines or numbers A and B are given, whose sum (z) *is equal to the difference of their squares. But the sum of their squares being taken out of the square of the sum, the residue shall be equal to* b *planum.*

After I had a while thought upon, it there came into my mind this Lemma.

Lemma. *The sum of any two numbers differing by an unite, shall be equal to the difference of their squares. If their difference be two. Then shall the difference of the squares be double to the sum, &c.*

Demonstration. *Let A be the lesser number, and A + E the*

est 1. Differentia quadratorum erit $2 AE + EE. \infty z$. Nam A in E. vel A in 1. hoc est A est A, est summa minoris numeri vel unius portionis. $AE + EE$, hoc est, A in $1 + 1$. est summa majoris numeri unitate tantum excedentis minorem, ergo $2 A + 1$ est summa utriusque numeri quod erat demonstrandum.

In numeris. Sit $A + E. \infty 5$. $A - E \infty 4. 2 AE + EE$, hoc est, $8 + 1 \infty 9$. summæ numeri utriusque.

the greater. And let E be an unite. The difference between the squares of A, and A + E. is 2 AE + EE. But 2 AE + EE is equal to z. For AE that is A, because E is an unite, is the lesser number, and AE + EE, that A + 1 is the greater number, therefore 2 AE + EE is the sum of both numbers equal to z. Which was to be demonstrated. Fig. 13

Sit jam major linea a. Minor erit $a + 1$.

Summa $2a + 1 \infty$ differentiæ quadratorum per Lemma præcedens.

$4aa - 4a + 1 \infty$ Zq. Quadratum summæ.

$2aa - 2a + 1 \infty$ Z summa quadratorum utriusque numeri.

Differentia. $2aa - 2a \infty b$ plano. Ergo $2aa. \infty b$ pl. $+ 2a$. vice b pl. scribe $(bb.)$ Ergo $2aa. \infty bb + 2a$.

Et $\sqrt{2bb + 1} : \begin{matrix} + \\ - \end{matrix}\ 1. \infty$ Duplo majoris lineæ. / Duplo minoris lineæ.

Theorema. Si duplo residuo dato addatur 1. Hujus aggregati radix quadrata aucta unitate erit dupla majoris lineæ, minuta vero unitate erit dupla minoris.

PROBLEMA X.

In triangulo rectangulo dantur (b) differentia hypotenusæ à crure minore, & (p) perpendiculo. Quæritur triangulum.

PROBLEM X.

In a right angled triangle there are given b, the difference between the hypotenuse, & the lesser leg or cathetus, together with (p) the perpendicular. The triangle is sought. Fig. 14.

Puta factum, & sit (a) pars quæsita, reliqua consonantibus notata dantur. Imo $b + a. p : : p. \frac{pp}{b+a} \infty a$.

Ergo $pp \infty aa + ba$, & $\sqrt{\frac{1}{4}bb + pp} : - \frac{1}{2}b. \infty a$.

Canon

Fig. 14. *Canon.* Quadrato perpendiculi, adde quartam partem differentiæ quadratæ aggregati radix quadrata, minuta dimidio differentiæ erit segmentum quæsitum.

Geometrice. Fiat $st \infty \frac{1}{2}b$, & $tq. \infty p$. & sit angulus ad (t) rectus sq erit $\sqrt{\frac{1}{4}bb+pp}$: dematur $qm \infty \frac{1}{2}b$, erit $sm. \infty a$.

Canon. *To the square of the perpendicular, adde a fourth part of the square of the difference, the square root of this aggregate shall exceed the segment sought, by half the difference given.*

Geometrically. *Make* (st) $\infty \frac{1}{2}$b, *and* tq ∞ (p), *and let the angle at* (a) *be right,* (sq) *shall be* $\sqrt{\frac{1}{4}bb+pp}$: *take away* qm ∞ *to half* b, sm *shall be equal to* a.

PROBLEMA XI.

Fig. 15. *Inscribere in circulo rectam (f) diametro minorem: ita ut si producatur infinite occurrat diametro productæ in puncto* (m) *dato.*

Data

Punctum *m*. Recta *f*.

QUæritur portio lineæ *f* productæ à peripheriâ ad punctum *m*.

PROBLEM XI.

To inscribe in a circle the right line f, *which must be less then the diameter, so that, if it be infinitely continued, it shall occurre with the diameter in the given point* m.

Given

The point m. *The right line* f.

SOught. *The portion of the line* f, *continued from the periphery to the point* m.

Puta factum. Sit portio quæsita *a*. Per demonstrata à *Pitisco* ad Axioma quartum Triangulorum Planorum erit,

$$f+a.\ b+c :: b-c.\ a. \text{ Ergo}$$

$$\frac{bb-cc}{f+a} \infty a. \text{ ergo } bb-cc \infty fa+aa. \ \& \ aa \infty bb-cc-fa.$$

Et $\sqrt{\frac{1}{4}ff+bb-cc}: -\frac{1}{2}f. \infty a.$ quia vero $cc. \infty dd+\frac{1}{4}ff$.

Erit $\sqrt{\frac{1}{4}ff+bb-dd-\frac{1}{4}ff}: -\frac{1}{2}f. \infty a.$ vel $\sqrt{bb-dd}: \infty a+\frac{1}{2}f$.

Canon. $\sqrt{bb-dd}: \infty a+\frac{1}{2}f$.

Constructio patet in ipsâ figurâ. Est enim $\sqrt{bb-dd}: \infty a + \frac{1}{2}f$. per 47. 1 *Eucli.*

The construction is apparent in the figure, for by 47. 1 Eucl. $\sqrt{bb-dd}. \infty a+\frac{1}{2}f$.

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PROBLEMA XII.

Ex dato rectangulo (y z) à puncto (t) dato triangulum exterius abscindere æquale trapezio superiori (h h) dato.

PROBLEM XII.

From the given rectangle $(y\,z,)$ and from the known point (t) to cut off the exterior triangle equal to the upper trapezium.

Fig. 16.

Puta factum, & sit (a) basis trianguli majoris. Erit

$a+c.b::a.\frac{b\,a}{a+c}$ ∞ istius trianguli catheto, & $\frac{b\,a\,a}{a+c}$ ∞ $2\,d\,b$,

vel $b\,a\,a$, ∞ $2\,d\,b\,a+2\,d\,b\,c$, vel $a\,a$ ∞ $2\,d\,a+2\,d\,c$, &

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$$\sqrt{d\,d+2\,d\,d\,c}:+d.\ \infty\ a.$$

Geometrice. Fiat A c ∞ $2\,d$, & $c\,e$ ∞ C, ergo $c\,g$. ∞ $\sqrt{q}.2\,d\,c$, & $z\,g$ vel $z\,d$ ∞ $\sqrt{q}.d\,d+2\,d\,c$. fiat $d\,f$ ∞ d, erit $z\,f$ quantitas quæsita.

Geometrically. Make A c ∞ 2 d, *and* c e. ∞ *to* C, *therefore* c g *is* $\sqrt{q}.2\,d\,c$, *and* z g, *or* z d ∞ $\sqrt{d\,d+2\,d\,c}$ *make* d f ∞ *to* d, z f *shall be the quantity sought.*

PROBLEMA XIII.

Ex *dato rectangulo* (y z) *à puncto* (t) *dato triangulum abscindere æquale spatio* (h) *dato.*

PROBLEM XIII.

From the given rectangle $(y\,z,)$ from the given point $(t,)$ to cut off a triangle equal to a trapezium known.

Fig. 17.

Puta factum, & sit basis trianguli abscindendi (a)

Erit $a+d+n$. hoc est $a+c.\,b::a.\ \frac{b\,a}{a+c}$ æquale lateri alteri triangulo ignoti (*viz.*) Catheto. Sed $\frac{b\,a\,\bar{a}}{a+c}$ æquatur duplo areæ trianguli, id est $(2\,h.)$ Ergo $\frac{b\,a\,a}{a+c}$ ∞ $2\,h$, vel $b\,a\,\bar{a}$ ∞ $2\,h\,a+2\,h\,c$, & $a\,a$ ∞ $\frac{2\,h\,a+2\,h\,c.}{b}$ Ergo

Canon $\sqrt{\frac{h\,h}{b\,b}+\frac{2\,h\,c}{b}}:+\frac{h.}{b}$ ∞ a.

E Ad

Fig. 18. Ad constructionem hujus Problematis, reducatur (h) superficies ad quadratum quod perinde vocetur ($h\,h$,) & Æquatio sic stabit: $\sqrt{\frac{bbhh}{bb} + \frac{2bhc:}{b}} + \frac{h\,b}{b}$ ∞ a. inter $b\,b$ & $h\,h$. inveniatur tertia proportionalis ff, erit $\frac{bbff}{bb} \infty \frac{bbhh}{bb}$ Fiat secundo $b.\ h :: h.\ f.$ erit $\frac{2bfc}{b}$ ∞ $\frac{2hhc}{c}$ similiter $\frac{bf}{b}$ erit ∞ $\frac{h\,b}{b}$ & sic æquatio integra constructioni apta sic stabit,

$$\sqrt{\frac{bbff}{bb} + \frac{2bfc:}{b}} + \frac{bf}{b} \infty a.$$

vel $\sqrt{ff + 2fc:} + f. \infty a.$

Fig. 18. In hoc Schemate, sit h latus trapezii ad quadratum reducti, & sit ff tertium proportionale inventum. Fiat $2f + 2c$, diameter circuli dg, erit $\sqrt{q.\ ff + 2cf}$, cui si addatur $dm \infty f$. erit gm trianguli quæsiti basis qua cognita linea recta, à puncto t. ad terminum istum ducta abscindet trapezio dato triangulum æquale.

Fig. 19. Problema præcedens potest generalius proponi hoc modo.

Posito D angulo recto, à puncto t, dato supra basim DE

For the construction of this Probleme, you must first reduce the trapezium (h) *to a square, which may be called* (h h,) *& the Equation will stand thus,*

$$\sqrt{\frac{hhhh}{bb} + \frac{2hhc:}{b}} + \frac{hh}{b} \infty a.$$

then between b b, *and* h h, *find* ff *the third proportional. Then* $\frac{bbff}{bb} \infty \frac{hhhh.}{bb}$ *Secondly, make* b. h : : h. f. *then* $\frac{2bfc}{b}$ ∞ $\frac{2hhc}{c}$ *in like manner, because* bf ∞ hh. $\frac{bf}{b}$ *shall be equal to* $\frac{hh,}{b}$ *& the whole Equation will stand thus,*

$$\sqrt{\frac{bbff}{bb} + \frac{2bfc:}{b}} + \frac{bf}{b} \infty a.$$

or $\sqrt{ff + 2fc:} + f. \infty a.$

In the Scheme, let (h) *be the side of a square equal to the trapezium, and* ff *the third proportional, between* (b b) *&* (h h:) *make* 2 f + 2 c *the diameter of a circle,* d g *shall be* $\sqrt{q}$ ff + 2 c f, *to which if you adde* d m *equal to* f, g m *shall be the base, of your triangle, and a streight line drawn from* t, *to that base shall cut off a triangle equal to the trapezium.*

The preceding Probleme may be more generally propounded in this manner.

From a given point t, *the angle at* D *being right, by position,*

Fig. 1

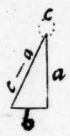


Fig. 2

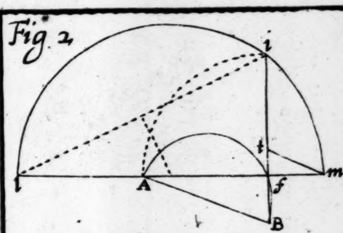


Fig. 3

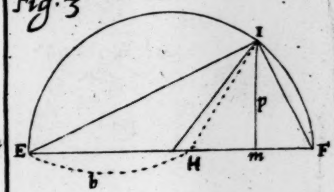


Fig. 4

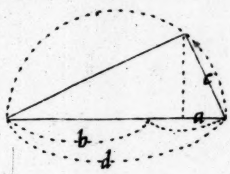


Fig. 5

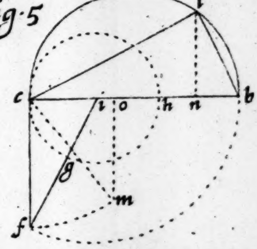


Fig. 6

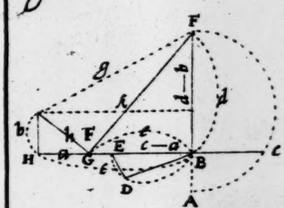


Fig. 7

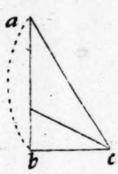


Fig. 8

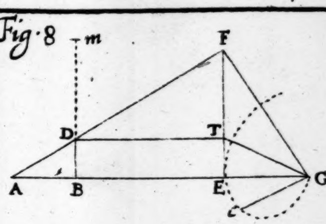


Fig. 9

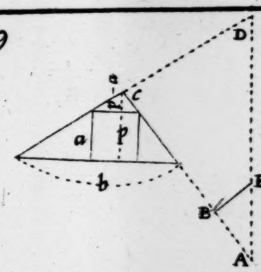


Fig. 01

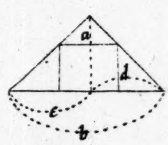


Fig. 11

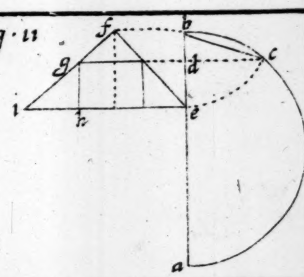


Fig. 12

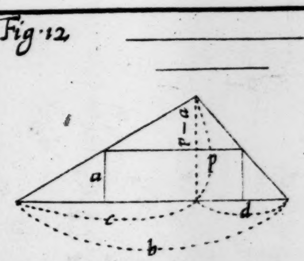


Fig. 13

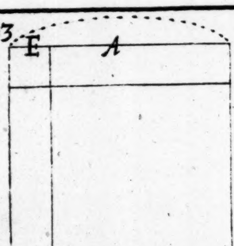


Fig. 14

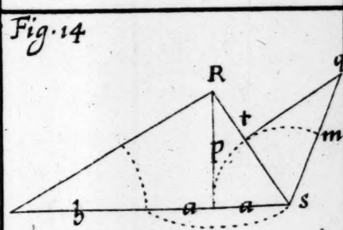


Fig. 15

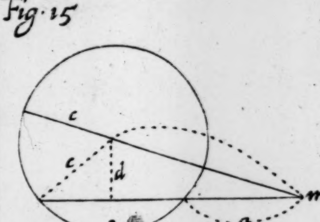


Fig. 16

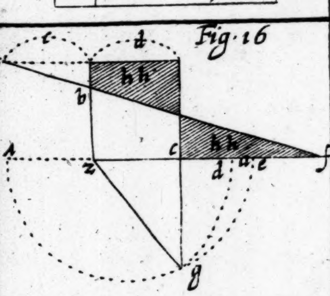
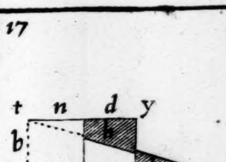


Fig 17



Place this at Pag: 18
of
Problematum quorundam
Mathematicorum

DE triangulum abscindere æquale spatio dato, C.

tion, upon a base DE, to cut off a triangle equal to any space given, C.

PROBLEMA XIV.

Datam *b* lineam ita secare ut quadratum partis unius sit equale plano ex altera parte cum externa data contentum.

Vel,

Data (ex tribus proportionalibus) una extremarum cum summa reliquarum invenire reliquas.

PROBLEM XIV.

T Is required to divide the given line b, *so that the square of one of the parts, may be equal to the plain contained between the other part, and an external line given. Or,*

In three terms continually proportional, one of the extremes being given, and the summe of the other two, to find the terms.

Sit *b* linea data secanda. *Fig.* 20.

Sit *d* externa data.

Sit pars lineæ *b* secanda *a*.

Erit reliqua pars *b* — *a*.

Et $aa. \infty\ bd - da.$ vel $\sqrt{\tfrac{1}{4}dd + bd}: - \tfrac{1}{2}d. \infty\ a.$

In verbis.

Si quadrato dimidii externæ datæ, addatur planum ex externa data in summam itidem datam. Aggregari radix quadrata minuta dimidio externæ datæ, erit media trium quantitatum $\div$

Geometricè. Super A B. ∞ *b* + *d* describatur semicirculus, & sit A C ∞ *b*. à termino C erigatur perpendicularis C D. ducatur D E bisecans C B: erit D E vel E F $\sqrt{\tfrac{1}{4}dd + db}$: & F C segmentum quæsitum, & erunt A F. F C. C B $\div$

PRO-

In words.

If to the square of half the external line given, be added the plain made by the summe given, and the external line. The square root of this aggregate lessened by half the external line given, shall be equal to the middle term sought.

Geometrically. Upon A B. ∞ b + d *describe a semicircle, & let A C be equal to* b, *from C erect a perpendicular C D. draw D E bisecting C B. D E or E F shall be the* $\sqrt{\tfrac{1}{4}dd + db}$: *and F C the segment sought: So that A F. F C. C B* $\div$ *Fig.* 21.

PRO-

PROBLEMA XV.

PRoposuit mihi (*Rothomagi*) Amicus quidam hanc quæstionem solvendam, cujus voto satisfeci, & canonem addidi quo omnes hujus naturæ quæstiones solvantur Quæstio.

Datur rectangulum cujus area est $1345.\frac{5}{49}$ latitudo $\frac{2}{3} - 13$ longitudinis. Quæruntur latera.

Adhibeantur loco numerorum species.

PROBLEM XV.

A Friend of mine (at Roven) desired of me the solution of this question, whom I not only satisfyed, but gave him a Rule for the solution of all such like Questions.

A rectangle is given, whose area is $1345.\frac{5}{49}$, the latitude $\frac{2}{3} - 13$ of the longitude. The sides are sought. First the longitude.

In the place of the numbers put Letters.

$$\text{Sint}\left\{\begin{array}{l} b\ .\ \infty\ 2 \\ c\ .\ \infty\ 3 \\ d\ .\ \infty\ 13 \\ f\ .\ \infty\ 1345.\frac{5}{49} \end{array}\right. \quad \left.\begin{array}{l}2\\3\end{array}\right\}\frac{b}{c} \infty \frac{2}{3}$$

Puta factum, & sit (a) longitudo quæsita, erit latitudo $\frac{ba}{c} - d$, & $\frac{b}{c}aa - da \infty f$. vel $\frac{b}{c}\ aa \infty f + da$, & terminis omnibus in (c) ductis $baa \infty fc + dca$. Ergo $\frac{1}{2}dc + \sqrt{\frac{1}{4}ddcc + bfc} \infty ba$. Hinc Theorema, sive

Canon. In omni rectangulo, ubi latitudo est longitudinis pars aliquo deficiens. Dico,

Si quarta pars quadrati defectus dati ducatur in quadratum denominatoris fracti, & huic facto adjiciatur area ducta in utrosque fractionis terminos. Hujus aggregati radix quadrata aucta dimidio defectus in fracti denominatorem ducti, exhibebit longitudinem totuplam quotupla est fracti numerator.

Canon. *In every rectangular figure, whose latitude is any aliquot part of the longitude deficient. I say,*

If a quarter of the square of the defect given, be multiplyed into the square of the fractions denominator, and to this product be added the area drawn into both the terms of the fraction. The root square of this aggregate, increased by half the given defect, shall exhibit a longitude so much greater

numerator. Unde nec longitudo nec latitudo ignorabitur.

Experiamur in numeris juxta Canonem.

greater then the truth, as the numerator of the fractions consists of units. So that the true longitude and latitude cannot be unknown.

Let us examine by numbers according to the Canon.

$$\tfrac{1}{2}dc + \surd\overline{\tfrac{1}{4}ddcc + bfc:}\ ꝏ\ ba.$$

$\tfrac{1}{2}dc.$ ꝏ $\tfrac{39}{2}$
$\tfrac{1}{4}ddcc$ ꝏ $\tfrac{1521}{4}$
$bfc.$ ꝏ $\tfrac{395460}{49}$ $\Bigg\}$ $\dfrac{ddcc}{4} + bfc.$ ꝏ $\dfrac{1656369}{196}$ terminis (ſc.) ad unum idemque nomen prius reductis. Hujus radix quadrata eſt $\tfrac{1287}{14}$ huic addi debent $\tfrac{39}{2}$ facit $\tfrac{3120}{28}$, id eſt, $111.\tfrac{3}{7}$ vel $\tfrac{780}{7}$ ꝏ ba, cujus dimidium, quia (b) eſt (2) $\tfrac{780}{14}$ vel $55.\tfrac{5}{7}$ æqualis longitudini quæſitæ, & latitudo invenietur $24.\tfrac{1}{7}$. Nam $\tfrac{520}{14}$ eſt $\tfrac{2}{3}$ $\tfrac{780}{14}$. Sed $\tfrac{520}{14} - \tfrac{13}{1}$ hoc eſt minus, $\tfrac{338}{14}$ eſt $24.\tfrac{1}{7}$. Duc $55.\tfrac{5}{7}$ in $24.\tfrac{1}{7}$, hoc eſt $\tfrac{390}{7}$ in $\tfrac{169}{7}$ facit $\tfrac{65910}{49}$, vel $1345.\tfrac{5}{49}$ æqualis areæ datæ, ergo latera verè ſunt inventa.

PROBLEMA XVI.

Requiritur ſecare $\surd q.125 + 5$ extrema, & media ratione.

Quia planum 125 provenit ex ductu 25 in 5. ergo media proportionalis inter 25 & 5. erit $\surd q.$ 125.

Sit jam AB 25 talium partium qualium B C eſt 5. BD eſt media proportionalis inter AB, & B C, dico BD eſſe $\surd q.$ 125, cui ſi adjiciatur D E ꝏ B C. erit B E $\surd q.$ $125 + 5$. linea data ſecanda.

PROBLEM XVI.

Fig. 22.

Tis required to cut $\surd q.$ 125 $+ 5$ in extreme, and mean proportion.

Becauſe the plain 125 is produced by the multiplication of 25 into 5, therefore a mean proportional between 25 and 5 ſhall be the $\surd q.$ 125.

Let A B be 25 ſuch parts as B C is 5. B D is a mean proportional between A B & B C. I ſay therefore B D is $\surd q.$ 125 to which, if you adde D E equal to B C. B E ſhall be $\surd q.$ $125 + 5$. The line given to be cut.

Fig. 22. Sit $a.$ major portio
$b \propto$ B E linea integra.
Erit $b - a.$ minor portio, &
$b. a :: a. b - a$ ergo $aa \propto bb - ba.$ &
$\sqrt{\frac{1}{4}bb + bb :} - \frac{1}{2}b. \propto a.$ majori portioni.

Geometrice, sit B F $\propto \frac{1}{2}$ BE erit F E $\sqrt{q.} \frac{1}{4}bb + bb$ fiat FG $\propto$ FB erit EG $\sqrt{\frac{1}{4}bb + bb:}$ $- \frac{1}{2}b.$ fiat E H $\propto$ G E. Dico E B hoc est, $\sqrt{q.}$ 125. + 5 esse sectam in H extrema, & media ratione Geometrice cujus major portio est E H, minor B H.

Sed quia quæstio proponitur numerose. Numerose rem aggrediamur.

Sit $\sqrt{q.}$ 125 + 5. secanda extrema, & media ratione.

Sit majus segmentum 1 $\sqrt{}$. Erit Ut $\sqrt{q.}$ 125 + 5. $1\sqrt{} :: \frac{1\sqrt{}.\ 1\,q.}{\sqrt{q.}\,125 + 5.}$ Ergo $\frac{1\,q}{\sqrt{q.}\,125 + 5} + 1\sqrt{} \propto \sqrt{q.}$ 125 + 5. Et $\frac{1\,q}{\sqrt{q.}\,125 + 5} \propto \sqrt{q.}$ 125 + 5 $- 1\sqrt{}$, & 1q $\propto$ 150 + $\sqrt{q.}$ 12500 $- \sqrt{q.}$ 125 q. $- 5\sqrt{}$. Hæc æquatio est jam solvenda.

Dimidium Radicum
$\sqrt{q.}$ 31 $\frac{1}{4}$ q. + 2 $\frac{1}{2}$
$\sqrt{q.}$ 31 $\frac{1}{4}$ + 2 $\frac{1}{2}$
Ejus quadratum est

Geometrically. Make B F $\propto \frac{1}{2}$ *B E. F E shall be the root square* $\frac{1}{4}$ b b + b b, *make* F G *equal to F B. E G shall be* $\sqrt{q. \frac{1}{4}bb + bb} - \frac{1}{2}$b. *make* E H $\propto$ *to G E. I say, E B that is,* $\sqrt{q.}$ 125 + 5 *is Geometrically cut in extreme and mean proportion, whose greater portion is E H, the lesser H B.*

But because the question is propounded in numbers, let us attempt it in numbers.

$\sqrt{q.}$ 125 + 5 *is to be cut in extreme, and mean proportion.*

Let the greater segment be 1 $\sqrt{}$. *It shall be, As* $\sqrt{q.}$ 125 + 5. 1 $\sqrt{} ::$ 1 $\sqrt{}$ $\frac{1\,q}{\sqrt{q.}\,125 + 5.}$

Therefore $\frac{1\,q}{\sqrt{q.}\,125 + 5} + 1\sqrt{}$ *equal* $\sqrt{q.}$ 125 + 5. *And* $\frac{1\,q}{\sqrt{q.}\,125 + 5} \propto \sqrt{q.}$ 125 + 5 $- 1\sqrt{}$, *and* 1q. $\propto$ 150 + $\sqrt{q.}$ 12500 $- \sqrt{q.}$ 125 *q.* $- 5\sqrt{}$. *This equation is now to be solved.*

Half the number of Roots is
$\sqrt{q.}$ 31 $\frac{1}{4}$ q. + 2 $\frac{1}{2}$
$\sqrt{q.}$ 31 $\frac{1}{4}$ + 2 $\frac{1}{2}$.
The square of $\frac{1}{2}$ *the number of Roots is*

31 $\frac{1}{4}$ + $\sqrt{q.}$ 781 $\frac{1}{4}$ + 6 $\frac{1}{4}$, id est, 37 $\frac{1}{2}$ + $\sqrt{q.}$ 781 $\frac{1}{4}$.

Idem

Idem hoc quadratum adnexum numero absoluto facit $37\frac{1}{2} + \sqrt{q}.781\frac{1}{4} + 150 + \sqrt{q}\ 12500$. Hoc est $\sqrt{}\ 187\frac{1}{2} + \sqrt{q}.\ 781\frac{1}{4} + \sqrt{q}.\ 12500$. Et quia duo surdi numeri sunt commensurabiles, & proportio quadratorum est $\frac{16}{1}$ erit ergo proportio radicum, $\frac{4}{1}$ multiplicanda igitur est minor $\sqrt{q}$. per 5, hoc est ducenda est $\sqrt{q}.\ 781\frac{1}{4}$ in $\sqrt{q}.\ 25$, & producetur $\sqrt{q}.\ 19531\frac{1}{4}$ pro summa surdarum quantitatum. Jam igitur summa numeri absoluti & quadrati è dimidio radicum numero est $187\frac{1}{2} + \sqrt{q}.\ 19531\frac{1}{4}$ hujus autem binomii $\sqrt{q}$ est $\sqrt{q}.156\frac{1}{4}$ plus $\sqrt{q}.\ 31\frac{1}{4}$ vel $12\frac{1}{2} + \sqrt{q}.\ 31\frac{1}{4}$, atque hæc radix minuta dimidio radicum numero, id est, $\sqrt{q}.\ 31\frac{1}{4} + 2\frac{1}{2}$ est valor $1\sqrt{}$ primo positæ. Sic igitur stabunt termini.

This square added to the absolute number, makes $37\frac{1}{2} + \sqrt{q}\ 781\frac{1}{4} + 150 + \sqrt{q}.12500$, that is, $187\frac{1}{2} + \sqrt{q}.\ 781\frac{1}{4} + \sqrt{q}.\ 12500$. And because these two surd numbers are commensurable, and the proportion of their squares, is as $\frac{16}{1}$ the proportion of their roots shall be $\frac{4}{1}$. Therefore the lesser $\sqrt{q}$. is to be multiplyed by 5, that is, $\sqrt{q}.\ 781\frac{1}{4}$ in $\sqrt{q}.\ 25$, the product will be $\sqrt{q}.\ 19531\frac{1}{4}$ for the sum of the surd quantities. Now the sum of the absolute number, and the square of half the number of roots is $187\frac{1}{2} + \sqrt{q}.19531\frac{1}{4}$. The root square of this binome is $\sqrt{q}.\ 156\frac{1}{4} + \sqrt{q}.31\frac{1}{4}$, or $12\frac{1}{2} + \sqrt{q}.31\frac{1}{4}$, & this root diminished by half the number of roots, that is, $\sqrt{q}.\ 31\frac{1}{4} + 2\frac{1}{2}$ is the value of that which at first was supposed $1\sqrt{}$. The terms will stand thus: **Fig. 22.**

$12\frac{1}{2} + \sqrt{q}.\ 31\frac{1}{4} - \sqrt{q}.\ 31\frac{1}{4} - 2\frac{1}{2}$, id est $12\frac{1}{2} - 2\frac{1}{2}$, id est 10.

Tota igit. lin. secanda est	$\sqrt{q}\ 125 + 5$	*The whole line to be cut.*
Majus segmentum est	10	*The greatest segment.*
Minus segmentum est	$\sqrt{q}\ 125 - 5$	*The lesser segment.*

PROBLEMA XVII.

Datâ (mm) area trianguli æquilateri invenire latera.

PROBLEM XVII. **Fig. 23.**

The area (*mm*) of an equilateral triangle being given to find the sides.

Esto *p* perpendiculum bisecans basim, & sit *a* semissis basis, ergo 2 *a* erit basis integra, & $4aa \propto pp + aa$, ergo $3aa \propto pp$ & $\sqrt{q}.\ 3aa \propto p$, sed $pa \propto mm$, ergo $\sqrt{q}$.

3 *aa*

Fig. 23. $3aa$ in a, hoc eſt, $\surd$q. $3aaaa \infty mm$, vel etiam $\surd$q. $aaaa \infty \surd$q. $\frac{1}{3} mmmm$, vel $aaaa \infty \frac{1}{3} mmmm$. Ergo $\frac{1}{3} mm.\ aa :: aa.\ mm$, vel denique $\surd$q. $\frac{1}{3} mm.\ a :: a.\ m$. Nam ſi quadrata ſint proportionalia erunt, & radices quadratæ eorum proportionales. Ergo media proportionales inter m & $\surd$q. $\frac{1}{3} mm \infty a$.

Theorema. $$aaaa \infty \frac{mmmm.}{3}$$

Ex tertia parte areæ in ſe multiplicatæ educ radicem biquadratam quotiens exhibebit ſemiſſem lateris trianguli æquilateri.

Extract the biquadratick root of the third part of the area biquadrated, the quotient ſhall give half one of the ſides of the equilateral triangle.

Fig. 24. Geometrica praxis. Quia $\surd$q. $\frac{1}{3} mm.\ a :: a.\ m$. inveniatur media proportionalis inter m, & $\frac{1}{3} mm$.

Fiat $cd \frac{1}{3} m$, cui æquatur de, & ſit $bd \infty m$, erit $cl \frac{1}{3} mm$ & dh quadratum æquale cl oblongo quadrata $bd.(m)\ bd :: hd.\ de\ (\frac{1}{3} m,)$ ergo dh æqualis $\surd$q. $\frac{1}{3} mm$. Nam be eſt diametrus circuli deſcripti ſuper $bd \infty m$, & $de \infty \frac{1}{3} m$. Fiat $df \infty hd$, & diametro bf deſcribatur ſemicirculus erit $di\ q \infty m$ in $\surd$q. $\frac{1}{3} mm$, & perinde æqualis ſemiſſi lateris cujuſlibet incogniti. Fiat igitur $ng \infty 2di$, & compleatur triangulum.

Geometrically. Make c d $\frac{1}{3}$ m *to which, let* d e *be equal, and* b d *equal to* m, *the oblong* c l *ſhall be equal to* $\frac{1}{3}$ mm. *Therefore if upon* b e *you deſcribe a ſemicircle,* h d *ſhall be equal to* $\surd q\ \frac{1}{3}$ m m, *becauſe the ſquare of it is equal to the oblong* c l $\infty \frac{1}{3}$ m m. *Make* d f *equal to* d h, *and upon* b f *as a diameter deſcribe a ſemicircle*; d i *q. ſhall be equal to* m *in* $\surd q.\ \frac{1}{3}$ m m, *and therefore* d i *ſhall be equal to* (a) *half the ſide unknown, double* d i, *that is* n g, *ſhall be equal to the ſide of the equilateral triangle.*

In Fig. 24. No. 2 *linea* t t, *pars quarta erit* ∞ a, *vel dimidium erit* g n, *lateri trianguli integro.*

Aliter Geometrice. Quia $aaaa \infty \frac{mmmm}{3}$ fiat $3.\ mm :: mm.\ tt$: erit $\frac{3tt}{3} \infty \frac{mmmm}{3}$ vel $tt \infty \frac{mmmm}{3} \infty aaaa$, ergo

Otherwiſe Geometrically. Becauſe $aaaa \infty \frac{mmmm,}{3}$ *make* $3.\ mm :: mm.\ tt.$ *and* $\frac{3tt}{3} \infty \frac{mmmm}{3}$ *therefore* $tt \infty \frac{1}{3} mmmm \infty aaaa$, *and* t. ∞ a a,

ergo $t. \propto a\,a$. linea quadrato quadretur igitur linea inventa hoc est assumatur pars quarta erit æqualis a.

$\propto a\,a$, *a line to a square. If therefore you square the line found, that is, take a fourth part of it shall be equal to* a *sought. I say, in the figure* No. 2 $\frac{1}{4}$ t. $\propto$ a, *or* $\frac{1}{2}$ t. $\propto$ *to the side of the triangle.*

PROBLEMA XVIII.

In triangulo rectangulo a x b *datis* a, b, *& recto ad centrum circuli invenire* x.

PROBLEM XVIII.

In a right angled triangle *a x b*, *a* and *b* are given, and the right angle at the center of the circle, to find *x*.

Fig. 25.

Vide Doctissimi Franc. à Schooten, *commentaria in lib.* 3. *Rei Geometricæ* Renat. des Cartes *pag.* 273, 274 *quæ seriùs vidi post Probl. solutionem.*

Puta factum, & sit latus quæsitum x.

Erit, Ut $b.\ x+a :: x-a.\ \frac{xx-aa}{b}$ Ergo

Quadratum $\frac{xx-aa}{b} \propto (c)$ erit æquale $xx+aa$

Viz. $\frac{x^4+a^4-2xxaa}{bb} \propto xx+aa$.

Et $x^4+a^4-2xxaa \propto xxbb+aabb$

Et $x^4-2xxaa-xxbb \propto aabb-a^4$, vel per transpositionem terminorum.

$2xxaa+xxbb-x^4 \propto a^4-aabb$

$aa+\frac{1}{2}bb$. Ergo

Quadratum $aa+\frac{1}{2}bb$ est $a^4+bbaa+\frac{b^4}{4}$

$aa+\frac{1}{2}bb \pm \sqrt{\frac{1}{4}b^4+a^4+bbaa-a^4+bbaa} \propto xx$

$aa+\frac{1}{2}bb \pm \sqrt{\frac{1}{4}b^4+2aabb} \propto xx$, vel denique

$aa+\frac{1}{2}bb \pm b\sqrt{\frac{1}{4}bb+2aa} \propto xx$

THEOREMA.

$$aa+\frac{1}{2}bb \pm b\sqrt{\frac{1}{4}bb+2aa} \propto xx$$

$$\text{Vel } \sqrt{aa+\frac{1}{2}bb+b\sqrt{\frac{1}{4}bb+2aa}} \propto x.$$

Praxis

Praxis Geometrica est facillima, & patet in Schemate.

The Geometrical effection is very easie, and appeares in the Scheme.

Fig. 26. Fiat $AB \infty b$ } erit $BC \sqrt{q.\ \tfrac{1}{4}bb}$ } Sit $BFq. \infty 2aaq.$
$BD \infty \tfrac{1}{4}b$ } & $BE \infty b. \sqrt{\tfrac{1}{4}bb}$ } Erit $EF \infty b\sqrt{\tfrac{1}{4}bb + 2aa}$,
cui in directum adjiciatur
$FG \infty \sqrt{.\ aa + \tfrac{1}{2}bb}$. Erit $EG \infty \sqrt{aa + \tfrac{1}{2}bb + b.\sqrt{\tfrac{1}{4}bb + 2aa}}$.
œ x, qua cognita compleatur triangulum Schemati congruum.

Problema XIX.

In triangulo plano rectangulo. Dato perpendiculo una cum aggregato basis, & duplæ hypotenusæ invenire ipsas, tum hypotenusam tum basim.

Problem XIX.

In a right angled triangle there are given the perpendicular, the sum of the base & double the subtense. The subtense & base are sought.

Puta factum & sit basis quæsita.

Fig. 27. Sit basis a. Erit

$$aa + bb \infty \frac{dd + aa - 2da}{4} \text{ vel}$$

$4aa + 4bb \infty dd + aa - 2da$, & demptis utrinque aa.

$3aa + 4bb \infty dd - 2da$, &

$3aa + 2da \infty dd - 4bb$ vel

$aa + \frac{2da}{3} \infty \frac{dd - 4bb}{3}$ & $\sqrt{\frac{4dd}{9} + \frac{dd - 4bb}{3}} : - \frac{d}{3} \infty a$, vel

$\sqrt{\frac{4dd - 12bb}{9}}$: minus $\frac{d}{3} \infty a$.

Praxis in numeris.

$d \infty 14$
$b \infty 3$
$dd \infty 196$
$bb \infty 9$
$4dd \infty 784$
$12bb \infty 108$
Differentia $\infty \frac{676}{9}$

Hujus

Hujus $\surd$q. $\frac{26}{3}$
Hinc tolle $\frac{d}{3}$ $\frac{14}{3}$

Restat $\frac{12}{3}$ ∞ 4

Ergo basis est	4	Tria latera
Dupla hypot.	10	3
Hypotenusa	5	4
Et perpend.	3	5

THEOREMA.

$$\surd \overline{\frac{4dd - 12bb}{9}} : \text{minus } \frac{d}{3} \infty a.$$

Praxis Geometrica.

Fig. 28.

Fiant
$ce \infty d$
$ch \infty \frac{4}{9} d$
$ep \infty \frac{1}{3} d$
$ci \infty b$
$ek \infty \frac{4}{3} b$

Inde $ce\,(d)\; ek\,(\frac{4}{3}b) :: ci\,(b)\; il\,(f)$

Fiat $hm \infty f$.
& $en \infty em$ ergo
▭ $cen \infty \frac{4}{9} dd - \frac{4}{3} bb$ &
eo $\surd$q. ejusdem
Fiat $oq \infty \frac{1}{3} d \infty ep$
erit $eq \infty$ basi (a) &
$\frac{1}{2} cq \infty$ hypotenusæ.

Idem fieri poterit pro aggregato basis, & triplo quadruplo, quintuplo, &c, hypotenusæ. Pro triplo hypotenusæ æquatio sic stabit,

The same thing may be done where the sum of the base, and treble quadruple, quindruple, &c. of the hypotenuse are given. Where treble the hypotenuse is given the equation will be,

$$\surd \overline{\frac{9dd - 72bb}{64}} : - \frac{d}{8} \infty a.$$

PRO-

PROBLEMA XX.

In triangulo plano rectangulo. Data hypotenusa una cum aggregato perpendiculi & duplo basis invenire perpendiculum & basim.

PROBLEM XX.

In a plain triangle. The hypotenuse and aggregate of the cathetus, and double the base being given, to find the rest.

Fig. 29. Sit factum. Erit $\frac{z-a}{2} \infty$ basi, ergo $aa+\frac{zz+aa-2za}{4}$,

vel $\frac{4aa+zz+aa-2za}{4} \infty hb$, vel

$5aa+zz-2za \infty 4bh$, vel

$5aa-2za \infty 4bb-zz$, vel

$2za-5aa \infty zz-4bb$ vel

$\frac{2z}{5}a-aa \infty \frac{zz-4bb}{5}$ Ergo

Theorema $\frac{z}{5} \pm \sqrt{\frac{20bb-4zz}{25}} : \infty a$.

Idem fieri poterit pro aggregato perpendiculi & triplo (quadruplo quintuplo, &c.) pro triplo basis æquatio sic stabit

$\frac{z}{10} \sqrt{\frac{90bb-9zz}{100}} \infty a$.

PROBLEMA XXI.

In quovis triangulo plano. Datis basi, area, & differentia laterum invenire triangulum.

PROBLEM XXI.

In any plain triangle whatsoever. Having the base the area and difference of the sides, to find the triangle.

Fig. 30. Sit trianguli area æqualis gg, ergo $\frac{2gg}{b} \infty$ perpendiculo.

Dantur
$b. \infty$ Basi
$gg. \infty$ Area
$d. \infty$ differ. Crurum
Quæritur latus minimum a.

Ut

Ut $b.\ d + 2a :: d.\ \frac{dd + 2da}{b} \propto o.$ Hanc tolle ex b erit

$\frac{bb - dd - 2da.}{b} \propto 2e.$ Et $\frac{bb - dd - 2da:}{2b} \propto e.$

hujus autem quadratum est

$$\frac{bbbb - 2bbdd - 4bbda + dddd + 4ddddda + 4ddaa,}{4bb}$$

cui addatur quadratum perpendiculi

$\frac{2gg}{b}$ hoc est, $\frac{4gggg}{bb}$ sed prius reducatur sic $\frac{16gggg}{4bb}$

Eritque $\frac{16g^4 + b^4 + 2bbdd - 4bbda + d^4 + 4d^5a + 4ddaa}{4bb} \propto aa.$

Id est $16g^4 + b^4 + 2bbdd - 4bbda + d^4 + 4d^5a +$ $4ddaa \propto 4bbaa,$ vel

$16g^4 + b^4 - 2bbdd + d^4 \propto 4bbaa - 4ddaa - 4d^5a$ $+ 4bbda.$ Et hujus æquationis parte ultimâ diuisâ per $4bb - 4dd.$ Quotus erit $aa + da.$ Ergo etiam erit

$aa + da \propto \frac{16g^4 + b^4 + d^4 - 2bbdd}{4bb - 4dd}$ Nam ut prior ita, & posterior pars æquationis dividenda est per $4bb - 4dd.$ Ergo pro solutione Problematis.

$\sqrt{\frac{1}{4}dd + \frac{16g^4 + b^4 + d^4 - 2bbdd:}{4bb - 4dd}} - \frac{1}{2}d. \propto a,$ vel reducto $\frac{1}{4}dd$ ad idem nomen,

$$\sqrt{\frac{bbdd - d^4 + 16g^4 + b^4 + d^4 - 2bbdd:}{4bb - 4dd}} - \frac{1}{2}d \propto a.$$

Vel deletis æquivalentibus erit

sic $\sqrt{\frac{16g^4 + b^4 - bbdd}{4bb - 4dd}:} - \frac{1}{2}d \propto a,$ vel denique

sic $\sqrt{\frac{4gggg + \frac{1}{4}bb:}{bb - dd}} - \frac{1}{2}d. \propto a.$

Theorema. $\sqrt{\frac{4gggg + \frac{1}{4}bb:}{bb - dd}} - \frac{1}{2}d. \propto a.$

Geometrice. Fiat C med. proport. inter $b + d$ ($k\ l$) & $-d$ ($k\ m$) ergo Cq. $\propto bb - dd$ ($\propto b + d$ in $b - d$) atque

Geometrically. Make C *a mean proportional between* b + d (k l) *and* b − d (k m) *therefore* C *quad.* $\propto$ bb − dd ($\propto$ b + d *in*

Fig. 31.

Fig. 31. que C (kp) est radix quadrata ejusdem. Itidem $2gg$ est $\sqrt{q}.\ 4gggg$, applicentur igitur $2gg$ (vel Hq,) ad $(kp$ vel$)$ C, hoc est, fiat C. H :: H. F. Ergo C in F $\infty\ 2gg$ (∞ Hq.)& F est quasi quotus ex hâc applicatione. Quadratum igitur ex F(nimirum Fq.) $\infty\ \frac{4gggg}{bb-dd}$ huic adde $\frac{1}{4}bb$, id est, fiat rs(ad angulos rectos) $\infty\ \frac{1}{2}b$, & agatur (ks) igitur (ks) est $\sqrt{q}.\ \frac{4gggg}{bb-dd}$ plus $\frac{1}{4}bb$; ex quâ aufer $\frac{1}{2}d$ ($\infty\ st$) restabit $kt\ \infty\ a$; & si addas $(sx)\ \infty\ \frac{1}{2}d$ ad kt erit $kx\ \infty\ d+a$, ex tribus igitur jam datis lateribus $b.\ a.\ a+d$, vel etiam kn, kt, kx fabricetur triangulum nAk Schemati congruum

d *in* b − d, *but C is the root square of it. So also,* 2 g g *is* $\sqrt{q}.\ 4gggg$. *Divide therefore* 2 g g (*or Hq.*)(*by* k p *or*) C, *that is, make* C. H :: H. F. *therefore* C *in* F, *is* ∞ *to* 2 g g (∞ Hq) *and* F *is the geometrical Quotient that riseth by this division. Therefore the square of* F, (*to wit*) Fq. *is* ∞ *to* $\frac{4gggg}{bb-dd}$ *to this adde* $\frac{1}{4}bb$ *that is, make* r s (*rightangled at* r) $\infty\ \frac{1}{2}b$, *and draw* k s(k s) *shall be the* $\sqrt{q}.\ \frac{4gggg}{bb-dd}$ $+\frac{1}{4}bb$, *from this take out* $\frac{1}{2}d$ (∞ s t) *the remainder* k t *is* ∞ *a, and if you adde*(s x) $\infty\ \frac{1}{2}$ d *to* k t. k x ∞ d + a. *Therefore from the three sides given* b. a. a + d *or* k n, k t, k x *for the triangle* n A k *agreable to the Scheme.*

Problema XXII.

Fig. 32. *Datis trianguli rectanguli summa hypotenusæ, & perpendiculi* (b) *& area* p p *invenire Basim.*

Sit basis x

$\frac{2pp}{x}$ erit perpendiculum,

$b - \frac{2pp}{x}$ erit hypotenusa.

Ergo quadratum hypotenusæ bb

Problem XXII.

In a rectangled triangle (b) the sum of the hypotenuse, and perpendicular are given, and pp the area, the bases is required.

Let the Base be x, *the perpend. shall be* $\frac{2pp}{x}$ *&*

$b - \frac{2pp}{x}$ *shall be the hypotenuse. And the square of the hypote-*

Fig. 32.

$bb - \frac{4bpp}{x} + \frac{4pppp}{xx} \infty$ $xx + \frac{4pppp}{xx}$ & sublatis æquiponderantibus, & reducta æquatione. Erit $xxx \infty x\,bb - 4bpp$. Sed utrum hæc æquatio cubica dummodo speciebus remanet obvoluta possit reduci, ad quadraticam difficulter judicatur. Datis speciebus applicabimus numeros ut in apposita figura. Sit $b \infty 8$. & $pp \infty 6$. Erit

$xxx \infty 64\,x - 192$, vel $xx - 64x + 192. \infty\, oo$.

Vel 1c. $\infty\, 64\surd - 192$, variis modis solubilis. Nam si 1c. $\infty\, 64\surd - 192$. Erit etiam 1q. $\infty\, 64\surd - \frac{192}{1\surd}$ Ergo pars aliquota $\frac{192}{1\surd}$ subducta ex 64 relinquet numerum æqualem 1q. Partes aliquotæ 192 sunt eligatur istiusmodi quæ subducta ex 64 relinquet quadratum numeri collateralis.

Ex partibus aliquotis.

1 .	192
2 .	96
3 .	34
4 .	48
6 .	32
8 .	24
10 .	19.$\frac{1}{5}$

Experiamur. Subducatur 34 ex 64 relinquit 30, sed 30 non est quadratum 3. Subducamus

hypotenuse $bb - \frac{4bpp}{x} + \frac{4pppp}{xx} \infty\, xx + \frac{4pp}{xx}$ *let the terms of equal value be taken away, and then the equation reduced will be* $xxx \infty\, xbb - 4bpp$. *Now whether this cubick equation whilst it thus remains hid under species can be reduced to a quadratick is hardly judged. Let us therefore apply numbers to the species, and let* b *be equal to* 8, *and* pp *equal to* 6.

$xxx \infty\, 64\,x - 192$. *or* $xxx - 64\,xx + 192. \infty\, oo$.

Which equation may be resolved several ways. For if 1c. $\infty\, 64\surd - 192$ *it followes that* 1q. $\infty\, 64 - \frac{192}{1\surd}$ *therefore, some aliquot part of* 192 *taken out of* 64 *shall leave a number equal to a square. The aliquot parts of* 192 *are as in the Table.*

1 .	192
2 .	96
3 .	64
4 .	48
6 .	32
8 .	24
10 .	19.$\frac{1}{5}$

Let us try, and first subduct 34 *out of* 64, *there remains* 30, *but* 30 *is not the square of* 3 *the*

camus secundo, 48 ex 64 relinquit 16, quadratum 4, numeri collateralis, ergo 1q. ꝏ 16 & valor x 4.

the correspondent number, therefore let us try the second time, and subduct 48, there remaias 16, the square of 4 the collateral number, therefore 1q. ꝏ 16 and 1√ ꝏ 4.

Secundo. Quia antea inventa æquatio xxx − *bb*x + 4*bpp*. ꝏ *o*. vel xxx − 64x + 192. ꝏ *oo*. Quæratur binomium per quod æquatio dividatur absque fracto quod invenietur x − 4 supponamus igitur x − 4. ꝏ *o o*, & partiatur æquatio hoc modo,

Secondly, Because xxx − bbx + 4bpp ꝏ *oo. or* xxx − 64x + 192 ꝏ *oo. Seek a binome which will divide this equation without a fraction, which will be found* x − 4, *and the quotient will be as appeares.*

$$
\begin{array}{rl}
x-4.\ \infty\ oo)\ & xxx - 64x + 192\ (xx + 4x - 48 \\
 & xxx - 4xx \\
\hline
 & \quad +4xx - 64x + 192 \\
 & \quad +4xx - 16x \\
\hline
 & \qquad -48x + 192 \\
 & \qquad -48x + 192 \\
\hline
 & \qquad oo + oo
\end{array}
$$

Ergo valor unius radicis est 4, sed quia æquatio tres habet dimensiones restant duæ adhuc aliæ deducendæ ex æquatione quadratica in quoto inventa suntque reliquæ duæ + √q. 52 − 2, & − √q. 52 − 2, altera affirmativa altera negativa, & sic exprimantur, 2 ± 2. √ 13.

Therefore the value of one root is four. But because the equation hath 3 roots by reason of its 3 dimensions, there remains yet two to be deduced out of the quadraticks equation, and they are + √*q.* 52 − 2, *and* − √*q.* 52 − 2, *one affirmative the other negative, and may be thus expressed* − 2 ± 2. √ 13.

Fig. 33.

$$
\begin{array}{lll}
AB \infty 4 & Ab \infty -2 + 2\cdot\sqrt{13} & A\beta \infty -2 - 2\sqrt{13} \\
Ac \infty 3 & Ac \infty + \frac{1+\sqrt{13}}{2} & A\gamma \infty + \frac{1-\sqrt{13}}{2} \\
BC \infty 5 & bc \infty + \frac{15-\sqrt{13}}{2} & \beta\gamma \infty + \frac{15-\sqrt{13}}{2}
\end{array}
$$

Bas

Basi igitur existente 4 triangulum erit A B C.

Basi existente $-2+2\sqrt{}13$ triangulum erit A *bc*.

Basis fuerit $-2-2\sqrt{}13$ triangulum erit A $\beta\gamma$.

In quibus omnibus area erit 6, summa hypotenusæ, & perpendiculi 8, sumptis quantitatibus antrorsum ab A ad B *b*, & ad C *c* pro affirmativis, retrorsum vero ad $\beta\gamma$ pro negativis.

Alii istiusmodi æquationes solvunt methodo (ut sic dicam) empyrico, seu tentativo. Hoc modo, sit 1c. ∞ $2\sqrt{} + 4$. Assumatur pro valore radicis: radix quilibet cubica exempli causa 2, ergo 1 c erit 8, & 8 debet esse equalis $2\sqrt{} + 4$, uti revera est, ergo 1 c ∞ $2\sqrt{} + 4$, hoc est 8, ∞ 4 + 4. Sit denuo 1c. ∞ $12\sqrt{} + 16$.

Assumatur 2 vel 3 pro valore radicis unius invenientur minores justo nam cubus 3 est 27, ergo 27 debuit esse æqualis 36 + 16, *viz.* 52. Assumatur 4 pro radice, ergo 64 debet esse equalis 48 + 16 uti est: sin fuerit 1 c. ∞ $12\sqrt{} + 20$. 4 invenietur minor (5) justo major. Ergo valor erit inter 4 & 5, extrahatur radix cubica ex 48 + 20 (*viz.*) 68 adjectis cyphris, & habebis valorem radicis ut volueris precise,

PRO-

The base therefore being 4 *the triangle shall be A B C.* Fig. 33.

The base being $-2+2.\sqrt{}13$ *the triangle shall be A* b c.

The base being $-2-2.\sqrt{}13$ *the triangle shall be A* $\beta\gamma$.

In all which the area is 6, the sum of the hypotenuse and perpendicular 8, *the quantities being taken forward from A to* B b *and* C c *affirmative, but backward to* $\beta\gamma$ *negative.*

Others resolve these kinde of equations by an empyrical, and tentative way, as I may call it, not much unlike the first solution of this question. Suppose 1 *c* ∞ $2\sqrt{} + 4$. *Assume for the value of* $1\sqrt{}$ *the root of any cubical number whatsoever, as for example* 2. *then* 1 c ∞ $2\sqrt{} + 4$. *shall be* 8 ∞ 4 + 4, *as in truth it is, therefore* 2 *is the value of one root.*

Again, Suppose 1 c ∞ $12\sqrt{} + 16$. *Take* 2 *or* 3 *for the value of* $1\sqrt{}$, *they will be found too little, for* 27 *the cube of* 3 *should be equal to* 36 + 16, viz. 52, *which it is not. Take* 4, *then* 64 *should be equal to* 48 + 16 *as indeed it is, therefore* 4 *is the value of* $1\sqrt{}$, *but if* 1 c *had been equal to* $12\sqrt{} + 20$, 4 *will be found too little, and* 5 *too big, therefore the value of* $1\sqrt{}$ *is between these numbers. Therefore extract the cubick root of* 48 + 20, *viz.* 68 *adding*

I *cyphers*

PROBLEMA XXIII

Fig. 34. *Datâ summa areæ paralellogrammi rectanguli, & diagonii, & data etiam differentia, vel summâ laterum, invenire singula.*

PRoblema est numerose solvendum alias enim dari non potest summa areæ & diagonii.

Data.

s ꝏ 73. Summa areæ diagonii.

b ꝏ 7. Differentia laterum.

Quæro latus minus.

Sit *x* ergo $x+b$ est latus majus, & $xx+xb$. est area rectanguli sed quadrata duorum laterum simul addita sunt æqualia quadrato hypotenusæ, per penul. 1 *Eucl.* Ergo $2xx+2xb+bb$ ꝏ quadrato diagonii. Sed $xx+bx$ est area rectanguli, ergo quadratum diagonii æquatur duplo areæ rectanguli plus laterum differentia quadrata. Hoc est $2xx+2xb$. ꝏ □ diagonii $-bb$. Eo igitur deventum ut ad solutionem hujus problematis nihil aliud requiratur quam ut dividamus (73) summam diagonii, & areæ in duas istiusmodi partes, ut quadratum unius minus 49 (*bb*) sit æquale duplo partis alterius.

Sit

cyphers, and you may have the root as precisely as you desire.

PROBLEM XXIII.

The sum of the area of a rectangle parallelogram, and the diagonium being given, as also the difference, or sum of the sides being given, to find the rest.

THis Probleme must be resolved in numbers, otherwise the sum of the diagonal and area cannot be given.

Given. s ꝏ 73. *sum of the area and diagonal.*

b ꝏ 7. *The differ. of the sides.*

I seek the lesser side.

Let it be x. *therefore* $x+b$ *is the greater, and* $xx+xb$ *is the area of the parallelogram. But the aggregate of the squares of both the sides are equal to the square of the diagonal, by the* 47 1 Eucl. *Therefore* $2xx+2xb+bb$. ꝏ *to the* □ *of the diagonum, but* $xx+bx$ *is the area of the parallelogram. Therefore the square of the diagonal is equal to double the area of the parallelogram & the square of the difference of the sides. That is* $2xx+2xb$. ꝏ □ *diag.* $-bb$. *Therfore for the solution of this question there is no more required then to divide* (73) *the sum of the area and diagonal into 2 such parts, that the square of one of them, lessened by* 49 ꝏ bb *shall be equal to double the other part.*

Let

Sit jam x. pars una (*sc.*) diagonium erit $73-x$ pars altera, *viz.* area, & $xx - 49. \infty 146 - 2x$, vel $xx \infty 195 - 2x$, & $\sqrt{q.196}-1 \infty x \sqrt{q.196}$ est 14, tolle 1 erit 13 diagonium: & $73-13$. *viz.* 60 erit area. Hinc oritur novum problema.

Quære duos numeros differentes per 7, qui invicem multiplicati producant 60. Sit primus & minor numerus y, major erit $y+7$ & $yy+7y. \infty 60$. Ergo $\sqrt{q.\frac{49}{4}+60}$ hoc est $\frac{289}{4}$ *viz.* $\frac{17}{2}-\frac{7}{2}$, hoc est $\frac{10}{2} \infty 5$, est minor numer. Ergo major erit.

Majus latus parallelogrammi erit	12
Minus latus	5
Diagonium	13
Area	60

Let x be one part, to wit, the diagonal $73-x$ shall be the area, and $xx - 49$ shall be equal to $146-2x$. or $xx \infty 195 - 2x$. Therefore $\sqrt{q.196}-1$, to wit 13, shall be the diagonal sought, and $73-13$ to wit 60, shall be the area. From hence arises a new Probleme.

Prob. *What two numbers are they whose difference is 7, and the product of them 60, which are easily found to be 5, the lesser & 12 the greater, so that*

The greater side of the parallelogram is	12
The lesser	5
The area	60
The diagonal	13

Fig. 34.

Sin aliter rem tentaveris in magis operosam divenies æquationem. Nam sit 73, summa areæ, & diagonii, & laterum differentia sit 7, erit $xx+7x$ area, ergo $73 - xx - 7x$ erit diagonium, cujus quadratum erit $x^4 + 14x^3 - 97x^2 - 1022x + 5329 \infty 2x^2 + 14x + 49$, vel post debitam terminorum transpositionem, & reductionem, erit

$x^4 \infty - 14x^3 + 99xx + 1036x - 5280$, per communem *Algebræ* regulam, radix x^4 invenire non potest, invenietur tamen methodo in problemate precedenti indicata.

Supponamus $x \infty 3$. x^4 erit $\infty 81$. Invenietur minor justo.

Supponamus secundo $x \infty 5$. x^4 erit 625.

99 xx. ∞	2475
1036 x ∞	5180
summa	7655
$14x^3$ ∞	1750
5280 ∞	5280
summa	7030
Tolle ex	7655
Restant	625 ∞ 625.

Ergo recte divinavimus.

If you go about to solve this Probleme otherwise, you will at last come to this Equation.

$x^4 \infty - 14x^3 + 99x^2 + 1036x - 5280$, *whose root will be found by the method propounded in the preceding Probleme.*

Sit

Fig. 43. *Sit jam data summa laterum (s ꝏ 17) summa areæ, & hypotenusæ 73. Quærantur reliqua.*

Sit latus minus x, latus majus erit $s - x$, area erit $sx - xx$, $ss - 2sx + 2xx$ quadratum diagonii, vel ss ꝏ $2sx - 2xx$. hoc est duplo areæ parallelogrammi. Ergo quadratum summæ (73q.) minus duplo areæ parallelogrammi est quadratum diagonii.

Hinc oritur novum Problema. Divide 73 in duas istiusmodi partes ut duplum unius sit æquale quadrato alterius (289 ꝏ ss) & omnia invenientur ut in problemate præcedenti.

Let (s ꝏ 17) the sum of the sides be given, as also 73 the sum of the area and diagonum. The rest are sought.

Let the lesser side be x, the greater shall be s − x. The area shall be sx — xx. ss — 2sx + 2xx shall be the square of the diagonium, or ss ꝏ 2sx + 2xx, that is double the area of the parallelogram. Therefore the square of the sum (73q.) lessened by double the area shall be the square of the diagonium.

Hence ariseth a new Probleme Divide 73 into two such parts that the double of one may be equal to the square of the other (289 ꝏ ss) and every thing will again be found as in the precedent Probleme.

FINIS.

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Fig. 18

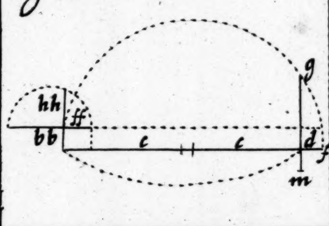


Fig. 19

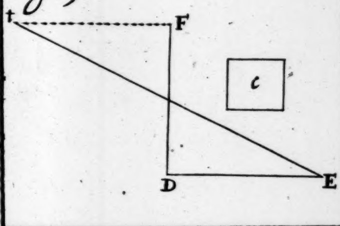


Fig. 20

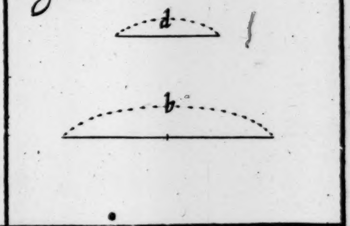


Fig. 21

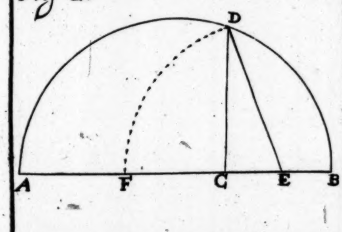


Fig. 22

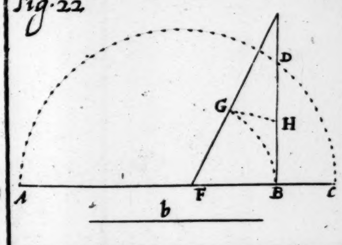


Fig. 23

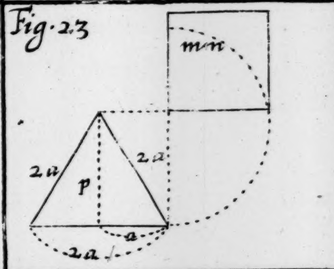


Fig. 24

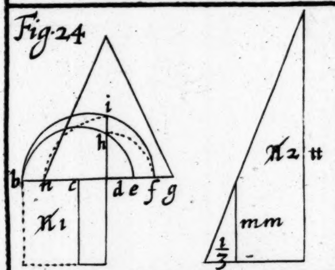


Fig. 25

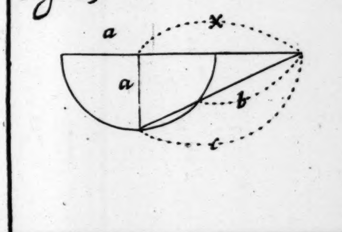


Fig. 26

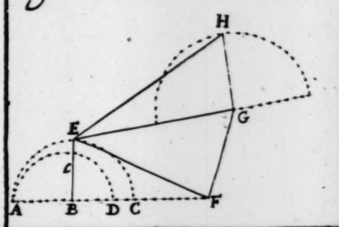


Fig. 27

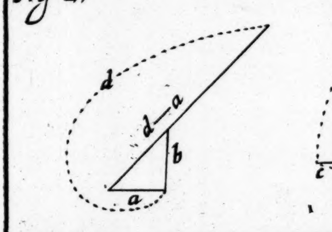


Fig. 28

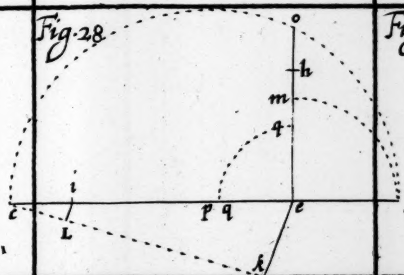


Fig. 29

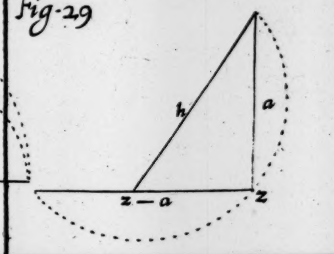


Fig. 30

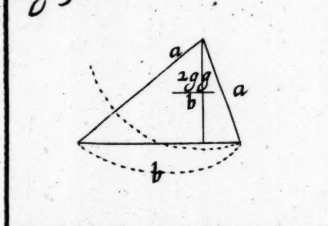


Fig. 31

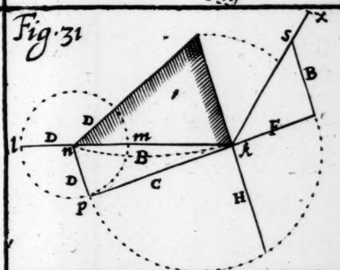


Fig. 32

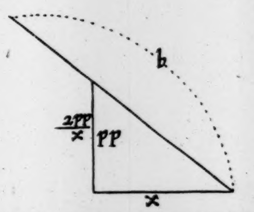


Fig. 33

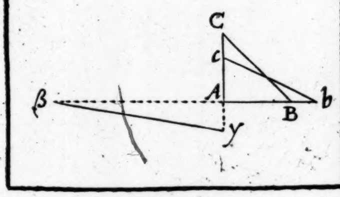
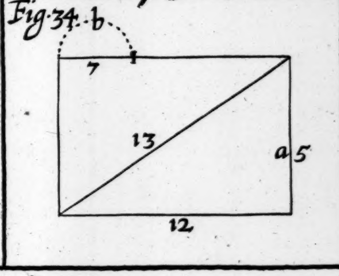


Fig. 34



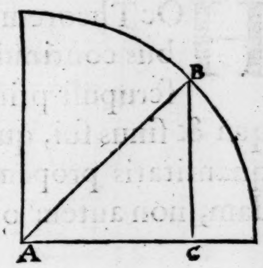
Place this at the end of Problematum quorundam Mathematicorum.

PROBLEMATA
Quædam succincta condendi Canones Sinuum, Tangentium, & Secantium.

PROBLEMA I.

Dato Sinu arcûs, Sinum complementi reperiri.

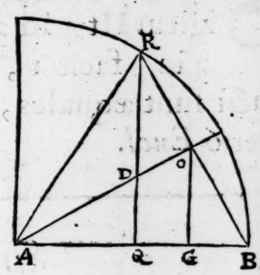
DAto B C invenire A C. Quoniam Triangulum ACB est rectangulum) per sinus definitionem) & latera A C, B C, æquè possunt hypotenusæ, id est, radio A B: si igitur quadratum Sinus B C subtrahatur de Quadrato radij A B, relinquitur quadratum A B, cujus latus est recta A C, sinus quæsitus.



PROBLEMA II.

Dato Sinu arcûs, unà cum sinu complementi, sinum arcûs dimidii reperire.

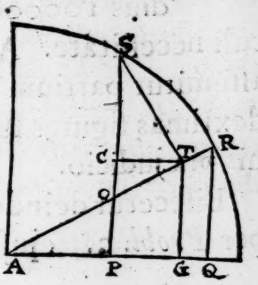
DAtis R Q, A Q invenire B O vel R O. Ut A *B* ad *B* O, ita *B* O ad *B* G. Erit ergo *B* O latus Quadratum plani ex A *B* radio & *B* G semisinu verso dato. Datur enim Q *B* sinus versus arcûs *B* R, quia A Q sinus complementi, & A *B* radius dantur ex hypothesi.



PROBLEMA III.

Datis sinibus duorum arcuum, & sinibus complementorum, sinum summæ reperire.

DAtis R Q, Q A, & S T, T A, quæratur S P. Ut A R ad R Q, ita A T ad T G, sive C P. Ut A R ad A Q, ita S T ad S C. S T & C P simul, faciunt S *P* sinum summæ duorum arcuum.



PROBLEMA IV.

Eisdem datis, sinum differentiæ reperire.

DAtis R Q, Q A, & *S P*, *P* A, quæratur *S* T. Ut A Q ad Q R, ita A *P* ad *P* O, unde innotescet O *S*. Ut A R ad A Q, ita O *S* ad *S* T.

His adnectantur Theoremata.

Theorema I. *Sinus minimi sunt in ratione suorum arcuum ferè.*

HOc Theorema verum esse infrà ostendetur in bisectionibus continuis. Arcus autem minimi sunt unius circiter scrupuli primi, vel infrà. Sunt ferè in eadem ratione qua & sinus sui, quia inter se ferè contigui ejusdemque adeò quantitatis propemodum, ad scrupulositatem satis profundam, non autem omnimodam.

Theorema II. *Si eadem linea secetur in partes numero inæquales, numerus partium primæ sectionis ad numerum secundæ, est (reciprocè) prout pars una sectionis secundæ, ad unam partem sectionis primæ.*

SEcetur eadem linea, primò in 4, deinde in 3 partes: Erit igitur Ut 4 ad 3, ita $\frac{1}{3}$ pars ad $\frac{1}{4}$ reciprocè. Ratio est quia 3 in $\frac{1}{3}$ facit 1, item 4 in $\frac{1}{4}$ facit 1. Quandoquidem verò facti sunt æquales, erunt factores reciprocè proportionales, per 6 *Eucl.*

Structura Canonis Sinuum.

TOtius quadrantis sinus, Radius dicitur; est enim semidiameter circuli. Statuatur autem in Canone Radius 100000 partium, vel etiam 100000.00, pro calculi necessitate. Ad structuram autem Canonis commodiùs assumitur partium 100000.00000, ita enim errores qui in dextimas figuras subrepunt deleri tutò possunt absque Canonis præjudicio.

Bisecetur deinde quadrans, & bisegmenti exquiratur sinus, per *Probl.* 2. ejusque cosinus per *Probl.* 1. Hoc rursus bisegmentum

segmentum bisecetur, & secundi bisegmenti investigetur sinus per *Probl.* 2. cosinus etiam per *Probl.* 1. Porrò & secundum hoc bisegmentum bisecetur, & investigentur ejusdem sinus & cosinus, per *Probl.* 2 & 1. Deinde verò & tertium bisegmentum bisecetur &c. continueturque bisectio tredecies, usque dum inventus sit sinus $\frac{1}{8192}$ partis totius quadrantis, prout hic in Tabella apponitur. Jam verò ad arcus minimos diventum est, ubi Theorematis primi veritas illustratur; Nam, Ut arcus quadrantis $\frac{1}{4096}$ est duplus ad arcum $\frac{1}{8192}$, ita & illius sinus ferè ad sinum hujus.

Quadrantis sinus	100000.00000
$\frac{1}{2}$ quadrant. sinus	70710.67811 +
$\frac{1}{4}$ quadrant. sinus	38268.34323 +
$\frac{1}{8}$ partis quadr. sin.	19509.03220 +
$\frac{1}{16}$ partis quadr. sin.	9801.71403 +
$\frac{1}{32}$	
$\frac{1}{64}$	
$\frac{1}{128}$	
$\frac{1}{256}$	
$\frac{1}{512}$	
$\frac{1}{1024}$	
$\frac{1}{2048}$	
$\frac{1}{4096}$	
$\frac{1}{8192}$	

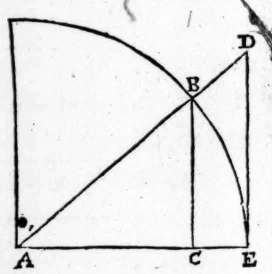
Post sinum hunc minimum sic inventum, inveniendus etiam est sinus unius scrupuli primi, id est, $\frac{1}{5400}$ partis de toto quadrante; vel unius centesimæ partis gradus, id est $\frac{1}{9000}$ partis totius quadrantis. Juxta igitur Theorema 2; Ut $\frac{5400}{9000}$ ad 8192, ita quantitas 1 partis hujus divisionis ad quantitatem 1 partis divisionis illius, & per Theorema 1, ita sinus $\frac{1}{8192}$ partis quam habes in Tabella ad sinum $\left\{\begin{matrix}\frac{1}{60}\\ \frac{1}{100}\end{matrix}\right.$ partis unius gradus.

Sinu

Sinu igitur 1 minuti, vel 1 centesimæ partis ita formato, per *Probl.* 1. erue sinum complementi, arcus scilicet 89 gr. $\left\{ \frac{59}{60} \atop \frac{99}{100} \right.$ Deinde, per *Probl.* 3. exquire sinum 2 min. ejusque cosinum per *Problem.* 1. Et ex his invenies sinum summæ 2 m. & 1 m. id est 3 min. per *Probl.* 3. ejusque rursus cosinum per *Probl.* 1. Ex sinu autem & cosinu 2 m. sive ex sinibus & cosinibus 3 m. & 1 m. investigabis sinum 4 m. per *Probl.* 3. & sinum complementi per *Probl.* 1. Item ex sinibus & cosinibus 2 m. & 3 m. vel 4 m. & 1 m. invenies sinum & cosinum 5 m. per *Probl.* 3 & 1 &c. usque ad $\left\{ 60' \atop 100 \right.$ vel 1 gradum. Ex sinu etiam gradus unius poteris eisdem mediis reperire omnes sinus 90 graduum integrorum: & ex prius inventis sinibus & cosinibus minutorum 60′ singulorum, facile erit per *Probl.* 3. adhibito etiam *Probl.* 4. quando è re fuerit, eruere singulorum omnium minutorum interspersorum sinus singulos.

Tangentium & Secantium deductio è Tabulis *Sinuum.*

Tangentes formantur sic.



Ut A C *cosinus, ad* C B *sinum*; *ita* AE *radius, ad* E D *Tangentem.*

Secantes autem sic.

Ut A C *cosinus, ad* AB *radium*; *ita* AE *radius, ad* AD *Secantem.*

Hoc modo integri Canones Tangentium & Secantium è sinuum Canone eliciuntur.

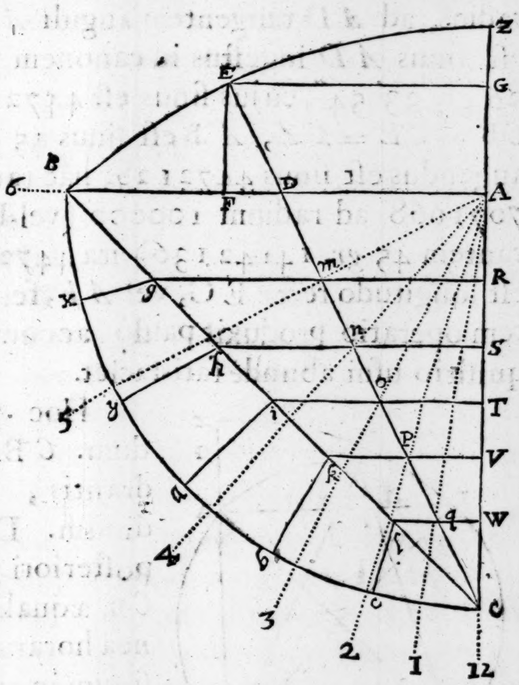
Compendia calculi prætermittimus omnia, Canones enim de novo condere non aggredimur; quandoquidem præstantissimorum Artificum pertinaci studio & labore hoc fasce liberamur. Nostro sufficit instituto si Syntaxeos ratio qualiscunque tantummodo intelligatur, & veritas numerorum in Canonem ingestorum: quod Propositiones suprapositæ abundè comprobant.

FINIS.

Demonstratio Quadrantis
HOROMETRICI.

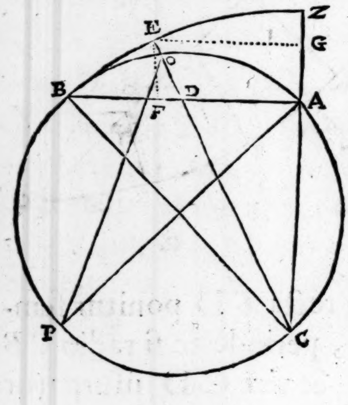
QVandoquidem *visum est Viro Erudito D.* Francisco à Schooten, Leydensi, *in Academia* Lugduno-Batava *Matheseos Professori, quadrantem Horometricum ab Authore nostro ante plures annos excogitatum; anno autem* 1638 Anglico *sermone impressum non solum laudare, sed praxeos veritatem ingeniosa demonstratione munire Sect.* Miscellan. *pag.* 510. *Placuit nobis Authoris ipsius demonstrationem qualem inter adversaria reperimus hic etiam subnectere.*

SIt radius A B vel A C: B C Horarum linea (in quadrante) artificialitèr divisa per filum *c* A, *b* A, *a* A, *y* A, *x* A, in punctis *l k i h g*: & ducantur *l* W, *k* V, *i* T, *h* S, *g* R, parallelæ ad B A. Sit porrò A D sinus 30 *gr.* respectu radii A *B*, & agatur recta *C* D E, quæ quidem dividet unamquamque rectarum parallelarum *g* R, *h* S, &c. in partes similes *B* D, D A; adeoque, Ut A *B* radius ad A D sinum 30 *gr.* ita R *g*, ad R *m*, & ita S *h*, ad S *n*, &c. Erunt ergo, Ut W *l* radius, ad W *q* sinum 30 *grad.* ita W *l* tangens 15 *gr.* ad W *q* tangentem anguli 1 A 12. & sic in horis reliquis.



Porrò autem in nostro Quadrante recta C D ponitur semper sub eadem longitudine cum *C B*, perindè ac si radio *C B* describeretur semiquadrans *B* E Z, & vice C D usurpatur

C E, pro C A etiam ponitur C G. Utcunque tamen triangulo E G C, & D A C sunt similia, & quoniam E C divisa sit in partes easdem cum partibus *B* C, dividetur etiam in partes similes iis quas continet recta D C; atque adeò idem opus absolvet. Linea igitur nostra latitudinum tota est A B, partes verò non sunt sinus A D, &c. sed G E, &c. vel *A* F, &c. designatæ per rectam C *D* protensam in E circumferentiam, ut C E sit æqualis *C* B. Inquiruntur autem hoc modo. Summæ quadratorum radii *C A*, & sinus *A D*, radix erit *C D*; Ut verò *CD* ad *D A*, ita *C E* = *C* B ad rectam E G, quæ inscribenda est lineæ latitudinum *A* B ad F; & *A* F erit pro latitudine 30 *gr.* Exempli gratiâ. Quadratum *A D* est 250000000000000, quadratum *AC* est — 1000000000000000, summa quadratorum 1250000000000000, cujus radix est *C D* recta — 11180340. At verò, Ut *CD* 11180340, ad *D A* 5000000; ita *C* B vel *C E* 14142136, ad E G vel A F 6324555. Tanta igitur est recta A F respondens 30 *grad.* in linea latitudinum. Et sic de partibus reliquis. Vel, Ut *C A* radius 100000, ad *A D* sinum 30 *gr.* 50000: ita *C A* radius, ad *A D* tangentem anguli *A C D* 26 *gr.* 33′ 54″. Hoc est, sinus *A D* ingestus in canonem Tangentium, dat arcum 26 *gr.* 33′ 54″, cujus sinus est 4472128; Atque posito radio *C* B = *C E* = *C Z*, *A* B est sinus 45 *gr.* 7071068, ideò rursus augendus est sinus 4472128, hâc ratione; Ut sinus 45 *grad.* 7071068 ad radium 100000 (vel, Ut rad. 10000000 ad secantem 45 *gr.* 14142136) ita 4472128 ad 6324544, quæ est longitudo rectæ E G, vel *A F*, ferè ut suprà. *S*uperior autem operatio produxit paulò accurationem. Hæc autem inquisitio usui abundè satisfaciet.



Hoc prætereà non omittendum. *C* B est linea Horarum quadrantis, & *A*B est linea latitudinum. Duo igitur, si circulus in posteriori parte describatur super *C* B, æqualis nempè diametri cum linea horarum, chordæ quadrantis 90 sinum in circulo, erunt eædem cum partibus 90, lineæ latitudinum. Nam (exempli gratiâ) ad *A*B radium, sit

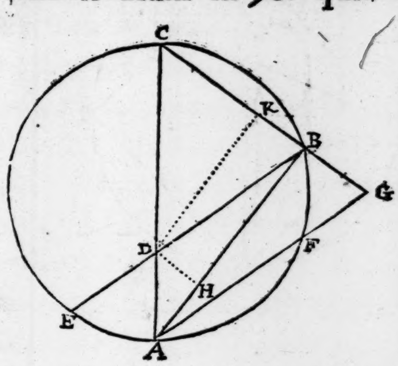
sit A D sinus 30 *grad.* C D O E secabit circulum in O; peripheriam Z B, in E; & efficiet tum E G (vel F A) 30 *gr.* in linea latit. (quod suprà probatur) tam A O 30 *gr.* in quadrante circuli A O B. At chorda A O, æquatur rectæ E G. Nam <, O P A, & O C A sunt equales, quod sunt in peripheria ad P & C, & insistunt eidem arcui O A. *Præterea*, P *A*, & *C* B vel *C* E sunt æquales; & *P* O *A*, *C* G E, sunt < recti. Ergo (cum *P* O *A*.*C* G E, sunt similia, vel æquiangula, latera homologa) E G, *A* O sunt æqualia. Quod probandum erat.

Demonstratio faciei posterioris Horometrici Quadrantis, adeoque Instrumenti totius Circularis.

Theorema I. *Si à diameter, diametrum circuli secet, erunt segmenta diametri proportionalia tangentibus arcuum oppositorum diametri segmentis conterminorum.*

SIt à diameter B E secans diametrum C A in D, dico primò, Ut segmentum C D ad D A, ita tang. arcus C B ad tang. E A. Notandum autem totum circulum hic dividi tantummodò in 180 *gr.* semicirculum in 90. quia de arcubus hic agitur prout angulos in peripheria obeunt, quorum sunt tantum subdupli.

Demonstrat. Fiat enim A G parallela ad adiametrum B E, & ducantur A B, C B G. Primum igitur quia C B A est rectus (in semicirculo quippe) erunt C B & B G tangentes angulorum C *A* B, B *A* G respectu radii *A* B, id est arcuum C *B*, & *B* F = E *A* quia uterque B F, E *A* includitur inter parallelas *B* E, G *A*. Deinde quia B E & G *A* sunt parallelæ, erit Ut C D segmentum, ad D *A* segmentum, ita recta *B* C tang. arcus B C, ad rectam B G tangentem arcus B F = E *A*. Dico secundò: Ut C D ad *D A*, ita tangens arcus C E, ad tangentem arcus oppositi *A B*, quod sic facile evincitur. Quia C B & B *A*, item C E, E *A*, sunt sibi invicem complementa, quorum tangentes sunt reciprocè proportionalia. Quare tangens C B

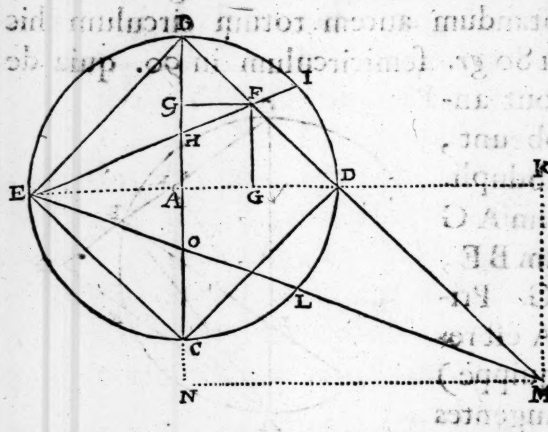


ad

ad tang. E *A* est in eadem ratione qua cotang. E *A* (id est tang. C *E*) ad cotang. C B (id est tang. B A,) ergo & tangentes C E, B A, sunt ut C D, D A.

Aliò modo sic evinco, Fiat D H perpendicularis ad B A, ac proinde parallela ad C B. Si DH sit radius, erunt HB, H A, tangentes angulorum *B* D H, H D A, id est arcuum C E, *B A*. C D. D A ∵ B H tangent. *B D* H = C B D = C *E*. H *A* tang. H D *A* = B C A = B A. Nam *B D* H est complementum D *B* H, vel arcus E *A*, cujus complementum est etiam arcus C E; item D A H vel C B arcus est complementum ADH vel B A arcus, ergo A D *H* & B A æquantur. Et quia C B & B H sunt parallelæ, ergo C *D*. *D* A ∷ B H tang. C E. H A tang. B A. Rursus per dimissionem perpendiculi D K. Ut C *D*. *D* A : : C K tang. C D K = C A B = C *B*. K B tang. K D *B* = D *B* H = E A.

Theorema II. *Si inter duas parallelas duæ rectæ ducantur se mutuò secantes, segmenta unius erunt proportionalia segmentis alterius, si similiter utrobique capiantur.*



Inter parallelas *BD*, E C, ducantur *B*C, E F, se mutuò secantes in H, dico segmenta C H, H *B*, esse proportionalia segmentis E H, H F. Ratio est. Quia triangula E H C, F H *B*; sunt æquiangula, propter æquales ad verticem H, & alternos æquales H C E, H *B* F; item H E C, H F *B*, alternos nempè inter parallelas B D, E C. Quapropter Ut C *H*, ad H *E*; ita H B, ad H F: & alternatim, Ut C H, ad H B; ita H E, ad *H* F.

Theorema II. *Si quadrati, diagonio intersecti, latus unum infinitè continuetur; & ab angulo utrique apposito, in continuatum recta ducatur, secans & continuatum & diagonium; erunt segmenta diagonii ut radius ad perpendiculum inter segmentum continuati, & diagonum: vel ut quadrati latus ad segmentum continuati diagonio conterminum.*

Demon-

Demonst. Sit quadrati E *B* D C diagonium *B* C (prolongatum si opus fuerit) & latus *B* D infinite continuatum, & ab E angulo utrique opposito ducatur recta $\left\{\begin{matrix} E\ F \\ E\ M \end{matrix}\right.$ secans continuatum in $\left\{\begin{matrix} F \\ M \end{matrix}\right.$ diagonium autem in $\left\{\begin{matrix} H \\ O \end{matrix}\right.$ dico segmenta diagonii $\left\{\begin{matrix} C\ H \\ C\ O \end{matrix}\right.$ & $\left\{\begin{matrix} H\ B \\ O\ B \end{matrix}\right.$ ita esse inter se, ut est E A radius quadrati, ad $\left\{\begin{matrix} S\ F = A\ G \\ N\ M = A\ K \end{matrix}\right.$ perpendiculum inter segmentum ad $\left\{\begin{matrix} F \\ M \end{matrix}\right.$ & diagonium *B C*. Nam (per 2 præcedens) $\left\{\begin{matrix} C\ H \\ C\ O \end{matrix}\right.$ est ad $\left\{\begin{matrix} H\ B \\ O\ B \end{matrix}\right.$ ut $\left\{\begin{matrix} E\ H \\ E\ O \end{matrix}\right.$ ad $\left\{\begin{matrix} H\ F \\ O\ M \end{matrix}\right.$ vel ut E A ad $\left\{\begin{matrix} A\ G = S\ F \\ A\ K = N\ M \end{matrix}\right.$ Quod erat probandum.

Pars posterior sic cogitur. Quia, Ut *E* A, ad $\left\{\begin{matrix} A\ G \\ A\ K, \end{matrix}\right.$ vel, Ut D A ad $\left\{\begin{matrix} A\ G \\ A\ K = N\ M \end{matrix}\right.$ ita D B latus quadrati, ad $\left\{\begin{matrix} B\ F \\ B\ M. \end{matrix}\right.$

Corollar. 1. Hinc sequitur. Si latus continuatum dividatur in partes quascunque (sive æquales, sive radices quadratas sive solidas, tangentes, sinusve rectos, vel versos) erit diagonium etiam in partes ejusdem nominis sectum atque tali modo, ut segmenta se semper habebunt ut latus quadrati ad partes continuato lateri inscriptas, sive ut radius quadrati ad longitudinem perpendiculi cujusque prædicti, si segmenta sumantur prout inter se respondeant. Causa manifesta est è superioribus.

¶ Notetur etiam (si cui bono) *H B* esse medium proportionale inter *H* F & *H* I. Nam ut *C H* ad *H E*, ita *H* I ad *H B*, per 3 *Eucl.* &, Ut *C H* ad *H E*, ita *H B* ad *H* F, ergo Ut *H* I ad *H B*, ita *H B* ad *H* F.

Corollar. 2. Hinc etiam. Si quadrato circumscribatur circulus, partes cujuscunque nominis projiciuntur à latere quadrati continuato in peripheriam; atque eo etiam modo, ita ut tangens quadrantis sive radius ad tangentes partium

inſcriptarum eandem ſemper ſervabit rationem quam tenet latus quadrati ad ſegmenta continuati lateris, ſive radius ad partes inſcriptas.

Demonſtr. Sit enim F pars in latere quadrati, inſcripta in circuli punctum *I.* Erit (per 1) Ut C *H* ad *H B*, hoc eſt (per 2) Ut radius E B ad partem inſcriptam B F, vel ſicut radius D A ad rectam A G; ita tangens quadrantis *E C*, hoc eſt radius rurſum, ad tangentem arcus *B I*, cujus tangens erit ideo æqualis A G rectæ.

Modus inſerendi. Ex his apparet modus inſerendi partes omnis generis, *viz.* Sinuum, Tangentium, partium æqualium, radium quadratum, cubicarum, ſinuum verſorum, &c.

In Peripheria ſic agendum eſt :

Omnes numeri cujuſcunque generis ut B F, B D, B M, &c. ingeſti in canonem tangentium dabunt arcus B I, B D, B L, &c. æquales pro tangentibus, inæquales pro numeris quibuſcunque reliquis.

In diametro, ſic :

Ut E B radius, & B F ſimul additi, ad B F partem radio additam; vel, Ut E G compoſita ex E A radio, & A G parte quacunque ad eandem rectam A G; ita diameter B C, ad ſegmentum B H.

Patet hinc, Partes non inſeri ultrà quadrantem in circulo, ultrà radium in diametro, ſi modò intrà radium ſive 100000 ſe contineant, quales ſunt numerorum ſeu partium æqualium, Sinuum rectorum, Semiſſinuum verſorum, Superficierum, Solidorum &c. At vero partes rectarum infinitarum quales ſunt tangentes & ſecantes per totum omninò ſemicirculum, totamque adeò diametrum diffundi: partes rurſum ad duplum radii extenſas, occupare $\frac{2}{3}$ diametri, peripheriæ autem ſemicircularis paulò plus duabus tertiis. Hinc rurſum, Quia eſt Ut tangens ad radium, ita radius ad cotangentem, perinde erit ſi dicas; Ut B H tangens ad H C radium, vel, Ut B H radius ad H C cotangentem; Nam ſegmenta diametri B H & H C repreſentant tangentes angulorum B E I, I E C, qui ſe mutuò complent.

Theor. 4. *Ubicunque punctum ſuſcipiatur in diametro, ſegmenta ſunt ut radius ad partes numero affixo denotatas; Vel contra, Ut partes ad radium, hoc eſt, Ut radius ad partes complementarias (ut ita dicam.)*

Ergo

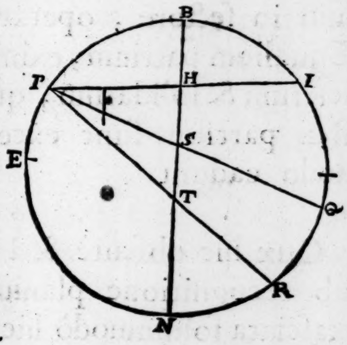
Ergo perinde est sive dicam, Ut radius ad partes, sive, Ut partes complementariæ ad radium. Analogicè dictum, juxta Theorema, Ut radius ad tangentem, sive Ut tangens ad radium. Hinc autem operandi methodus elucescet.

Modus operandi in istiusmodi lineis potest esse varius, est tamen unicus nobis & simplex. Fundatur autem in proportione Theor. 1. Quia segmenta diametri sunt ut ejus contermini arcus oppositi. *Modus operandi.*

PROPOSITIO.

Filo suprà planiciem circuli tenso, erunt radius, duo arcus, & segmentum diametri ab unâ quâcunque parte, quatuor proportionales.

Demonstr. HÆc Propositio est omnium operationum basis. Dico, Ut radius ad P *B*, ita B*I* ad B *H*. Vel, Ut radius ad P N, ita *NI* ad *NH*. Quia enim est, Ut tangens *PN* ad radium *EN*, ita tangens *IB*, ad segmentum *BH*, per Theor. 1. Et ut tangens *P* N ad radium E N, ita radius E N ad tangentem complementi P B; per Compend. Trigonometr. Erit ergo, Ut radius ad *PB*, ita *BI* ad *BH*. Eademque ratione, Ut radius ad *PB*, ita *B* Q ad B S, & ita B R ad B T. Quapropter etiam, filo ab ima parte ad punctum aliquod peripheriæ fixo, ab altera parte per peripheriam oppositam diametrumque moto.



Erunt arcus omnes cum segmentis diametri, proportionalis.

Demonstr. NAm sunt omnes, Ut radius ad *P B*, per præcedens. Ergo, Ut *B I* ad *BH*, ita B Q ad B S, & ita B R ad B T, &c. Vel contrà, Ut *P* B ad radium, ita *BH* ad *B I*, ita B S ad B Q, ita B T ad B R, &c.

Corollar.

Corollar. E tribus igitur terminis datis, filum per duos priores (quorum alter in peripheria alter autem in diametro numerandus est) debito peripheriæ loco figendum est; hinc autem à parte alterâ si moveatur in terminum tertium super eâdem circuli parte cum primo numeratum, exhibebit quartum in eadem circuli parte qua susceptus erat terminus secundus. Demonstratio hujus facilè resultabit ex superioribus.

Poterit etiam operatio institui juxta mentem Theorematis primi: Eam autem hic repetere non erit operæ-pretium.

Notandum etiam est: Quamvis propriè latet mysterium operationis in Tangentibus, diffunditur tamen in partes aliarum denominationum, & puta Sinuum, Superficierum, &c. Quod quidem fit applicatione harum partium ad Tangentes quæ longitudines earum emetiuntur: quemadmodum in sectore, operationes propriè pertinent ad lineas Æqualium partium, exindè verò derivantur in lineas superficierum & solidarum, quia harum scalarum partes ex æqualibus partibus sunt excerptæ, adeoque sub eodem operis modo cadunt.

Quæ hic obscurè & ἀμεθόδως tradita sunt, spero secundâ sub recognitione planiùs & limatiùs proditura. Nam quæ exasciata solummodò hic sunt, erunt olim meditationibus maturioribus dedolata.

FINIS.

EPITOME ARISTARCHI SAMII

De Magnitudinibus, & Distantiis trium Corporum,

SOLIS, LUNÆ, & TERRÆ.

POSITIONES I.

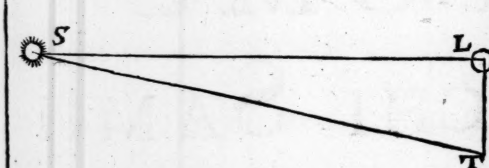
LUnam à Sole lumen accipere. *II.* Terram puncti ac Centri, habere rationem ad Sphæram Lunæ. *III.* Cum Lunæ apparet nobis dimidiata vergere in visum nostrum Circulum maximum qui Lunæ opacum, & splendidum determinat. *IV.* Eodem dichotomiæ momento Lunam à Sole distare minus quadrante, parte ejusdem trigesima, vel 3 gradibus distat, ergo 87 *gr.* circiter. *V.* Umbræ latitudinem esse duarum Lunarum (id est 4 *gr.* per positionem sequentem.) *VI.* Lunam subtendere $\frac{1}{15}$ signi, id est 2 *gr.* De hâc positione vide *Archimedem*, in libro de Numero Arenæ, ubi diameter Solis (ex *Aristarcho*) decernitur esse $\frac{1}{720}$ pars circuli, idest $\frac{1}{60}$ signi, & sic *Aristarchum* allegat. *Keplerus* Epitom. pag. 476.

Pappus

Pappus libro 6 Mathematicar. Collectionum pag. 136. ait, positiones 1, 3, 4 ferè, cum *Hipparchi*, & *Ptolomei* positionibus consentire reliquas autem 2, 5, & 6 discrepare.

PROPOSITIO. VII.

Distantia Solis à Terra est $19\frac{11}{100}$ *pla. distantiæ Lunæ à Terra.*



ANgulus enim S T L est 87 *gr.* per *Propos.* 4. & L rectus, ergo si T L sit radius, erit T S secans *gr.* 87 (sc.) $19\frac{11}{100}$ ferè.

PROPOSITIO. VIII.

APparentes diametri Solis & Lunæ sunt æquales, quia Sol totus in Eclipsi centrali deficit, at sine morâ etiam quod observationes confirmant.

PROPOSITIO. IX.

Solis igitur diameter vera est $19.\frac{11}{100}$ *pla. diametri Lunæ.*

DIametri enim sub eodem angulo apparentes erunt ut earundem distantiæ. Ut enim *u i* 1 ad *u a* $19\frac{11}{100}$: ita *i o* ad *a e*.

PROPOSITIO. X.

SOl ad Lunam est ferè, Ut 6979 ad 1. Sunt enim, Ut cubi $19\frac{11}{100}$ & *i*, id est, Ut 6979 ad 1.

PRO-

PROPOSITIO. XI.

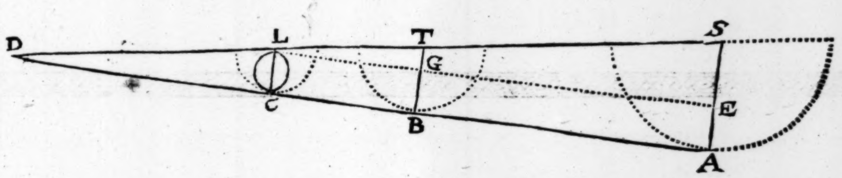
Diameter Lunæ est $\frac{7}{200}$ma distantiæ Lunæ à Terrâ circiter.

DIameter enim apparens Lunæ est 2 *gr.* per *Posit.* 6. at subtensa 2 *gr.* est ad radium, Ut 35 ad 100 ferè, hoc est, ut 7 ad 200.

PRROPOSITIO. XV.

Solis diameter ad diametrum Terræ est, ut 382 *ad* 57.

QUoniam enim diameter Lunæ, L C æquat. dimidium diametri umbræ (per *Posit.* 5.) auferatur E A diameter Lunæ, ex *S* A semidiametro Solis 9.$\frac{555}{1000}$, restabit *S E*, 8.$\frac{555}{1000}$, & quoniam *S* T est 19.$\frac{11}{100}$ quarum T L est 1 erit *S* L 20.$\frac{11}{100}$, & T L 1. Quapropter Ut *S* L 20.$\frac{11}{100}$ ad T L 1 : :



ita *S* E 8.$\frac{555}{1000}$ ad T G 0.$\frac{4254}{10000}$, adeoque diameter 2.$\frac{85}{100}$ qualium diameter *S*olis est 19.$\frac{11}{100}$. At vero 19.$\frac{11}{100}$ sunt ad 2.$\frac{85}{100}$ prout 382, ad 56.$\frac{97}{100}$, id est 57 ferè.

PROPOSITIO. XVI.

SOl ad Terram est, Ut 55742968 cubus diametri suæ, ad 185193 cubum diametri Terræ, id est, Ut 301 ad 1.

PROPOSITIO. XVII.

Diameter Terræ ad diametrum Lunæ est, Ut 57 ad 20. Nam qualium *S*olis diameter est 19.$\frac{11}{100}$ talium Lunæ est 1 per *Propos.* 9. & qualium idem *S*ol est 19.$\frac{11}{100}$, talium Terra est 2.$\frac{85}{100}$ per *Propos.* 15. Ergo in eisdem partibus Terræ & Lunæ semidiametri sunt, Ut 2.$\frac{85}{100}$ vel 2.$\frac{17}{20}$ ad 1, hoc est, ut 57 ad 20.

P RO-

PROPOSITIO. XVIII.

TErra ad Lunam eſt, Ut 185193, ad 8000, id eſt ferè $23.\frac{1}{10}$ pla. ſunt etenim, Ut diametrorum cubi, at diametrorum $\left\{ \begin{matrix} 57 \\ 20 \end{matrix} \right.$ cubi ſunt $\left\{ \begin{matrix} 185193 \\ 8000 \end{matrix} \right.$ quorum proportio eſt $23.\frac{3}{20}$ plani circiter. Ergo

Propoſitiones haſce noſtro modo demonſtravimus ex Theſibus *Ariſtarchi*, beneficio Canonum, *S*in. Tang. & *S*ecantium, qui quidem Canones Authoris tempore non erant in uſu. Unde etiam & terminos quantitatum præciſè (ex datis) figere non potuit, ſed inter binos plerunque ſtatuere coactus eſt. *I*ngenioſiſſime tamen demonſtrat & iſtas, & iſtis ſubſervientes quas (cum uſui nobis non ſunt) in hac Epitome omiſi. Vixit inter *Pithagoram*, & *Archimedem* 280 annis ante *Chriſtum*. Hunc librum *Schickardus* non vidit.

FINIS.

LEMMATA
ARCHIMEDIS,

APUD

GRÆCOS & LATINOS

jam pridem desiderata,

E VETUSTO CODICE *M. S.*
ARABICO.

à JOHANNE GRAVIO

TRADUCTA;

Et nunc primùm

CUM ARABUM SCHOLIIS PUBLICATA.

Revisa & pluribus mendis repurgata

à *S*AMUELE FOSTER.

LONDINI,
Ex Officina LEYBOURNIANA.
M. DC. LIX.

LEMMATA
ARCHIMEDIS,

Ex traductione *Thebit Ibn Coræ*: cum Commentariis Excellentis Viri, *Abi Alhonìn Ali*, filii *Almed Alnaſwæi*.

Propoſitiones Quindecim ſunt.

IN hoc Opuſculo *Archimedis*, demonſtrationes (inquit Vir Excellens [*Alhonìn*]) elegantes ſunt; ſi numerum ſpectes, paucæ, ſin uſum, multæ: eæque in *Elementis Geometriæ*, ſummam & utilitatem & voluptatem pariunt. Hæ, à *Neotericis* reponuntur, inter *Motawaſetât* (hoc eſt, media opera) quorum lectio, neceſſariò, inter *Euclidem* & μεγάλην σύνταξιν *Ptolemæi* requiritur. Quædam autem demonſtrationes, quæ hic collocantur, aliis opus habent, ut probentur. Et in harum nonnullis, *Archimedis* demonſtrationes citat, quas in reliquis ſuis operibus adhibuit. Inquit enim; quemadmodum demonſtravimus in Figuris Rectangulis, & quemadmodum demonſtravimus in Libro generali de Triangulis, & quemadmodum demonſtratum eſt in Figuris Quadrilateris. In quintâ etiam Propoſitione, peculiari modo demonſtrationem inſtituit. Poſt eum, *Abu Sohal Alkouhi* Librum edidit, eumque appellavit *Ornamentum libri Lemmatum Archimedis*, atque hanc (quintam) Propoſitionem cum Corollariis ipſius, de Compoſitione proportionis, generalius & melius probavit.

Quæ quidem, cum ita à me reperta eſſent, loca hujus Opuſculi obſcura, Commentariis, in modum Appendicis, illuſtravi, ſecundum viam communem *Alhawaſi*, eaque, ad quæ ab *Archimede* refertur, demonſtrationibus, ſi quid judico, propriis

propriis confirmavi. Duas insuper demonstrationes ex *Abi Sohal* excerpsi, quæ quintæ propositioni necessariæ sunt, reliquas vero omisi, ne nimis essem prolixus. In Deo est auxilium & fiducia mea.

PROPOSITIO I.

Fig. 1. *Si duo Circuli se mutuò, tangant (e. g. circuli* a h b, g h d, *in puncto* h) *sintque diametri, eorum parallelæ (v. d. diametri* a b, g d) *& conjungantur duo puncta*, b d, (*&*) d h, *erit linea*, b h, *recta.*

Sint duo centra H *r*, & describamus lineam inter H *r*, & producamus (eam) ad *h*, & ducamus *d t* parallelam H *r*. Quoniam ergo *t r* est æqualis *d* H, quæ est æqualis *h* H, erunt *r t*, *h* H, æquales, & remanebunt ex *r b*, *h r* æqualibus, H *r* [*t b*] hoc est *d t* & *t b*, æquales; & propterea duo anguli *t d b*, *t b d*, erunt æquales. Et quoniam duo anguli *h* H *d*, *h r b* (&) præterea duo anguli *b* H *d*, *d t b*, sunt æquales, reliqui duo anguli H *h d*, H *d h*, æquales, æquantur duobus angulis *t d b*, *t b d*, æqualibus; Ergo angulus *h d* H est æqualis angulo *d b r*, accipiamus angulum communem H *d b*, ergo duo anguli H *d b*, *r b d*, erunt æquales duobus rectis, qui sunt æquales duobus angulis H *d b*, H *d h*, ergo hi duo etiam erunt æquales duobus rectis, ergo *b d b* linea (erit) recta. Et hoc est quod volumus.

SCHOLIUM.

" Inquit Vir Excellens (*Albonîn*,) dici etiam potest, quod
" quoniam duo anguli *t d b*, *t b d*, sunt æquales, & angulus
" *d t b* rectus, angulus *b d t* erit dimidium recti, & propterea
" angulus *h d* H (dimidium recti) & angulus H *d t* rectus, ergo
" tres anguli sunt duo recti, linea ergo *h d b* erit recta. Simi-
" liter dico si fuerint duo circuli se mutuò extrà tangentes.

PROPOSITIO II.

Fig. 2. *Sit* a b g *dimidium circuli*, *&* d a, b d, *tangentes*, *&* b h *perpendicularis super* a g, *si conjungamus* g, d, *erit* b r *æqualis* r h.

Demonstratio. Jungamus *g*, *b*, eamque (lineam) ducamus in directum; & producamus *a d* donec occurrat alteri in H, & jungamus *a*, *b*. Quoniam angulus *a b g* est in Semicirculo,

Semicirculo, ergo est rectus, & reliquus *a b* H est rectus, & *d b h a*, est parallelogrammum rectangulum; ergo in triangulo *a b* H rectangulo, ducitur perpendicularis *b d*, à *b* recta super basim, & (lineæ) *b d*, *d a* sunt æquales quia ambæ tangunt circulum, ergo *a d* etiam æqualis est *d* H, quemadmodum demonstravimus in (Libro) de Figuris Rectangulis. Et quoniam in triangulo H *g a*, linea *b h* ducitur parallela basi, & linea *d g* ducitur à media basi, scilicet *d*. Ergo secat perpendicularem in *r*, eritque *b r* æqualis *r h*, & hoc est quod volumus. Fig. 2.

SCHOLIUM.

« Inquit Vir Excellens, quod (linea) *a d* sit æqualis *d* H, « probat è Libro suo de Figuris Rectangulis, quoniam duo « anguli *d a b*, *d b a*, sunt æquales, ergo (lineæ) *d b*, *d a*, sunt « æquales, & angulus *d b a*, cum angulo *d b* H (est æqualis) « recto, & propterea angulus *d a b*, cum angulo *a* H *b*, ergo « duo anguli *d* H *b*, *d b* H, necessariò erunt æquales, ergo duo « latera *d b*, *d* H, erunt æqualia.

« Dico, quòd sufficit dicere proportionem *a d* ad *d b* simi« lem esse proportioni *d b* ad *d* H; & (cum) *d a* æqualis sit « *d b*, ergo *d b* æqualis erit *d* H.

« Inquit, sit *b r* similis *r h*, & quoniam incidit *g d* in lineas « duas *b h*, H *a*, parallelas; in triangulo *g* H *a*, earum sectio « erit secundum eandem rationem. Hoc fit quia proportio « *g d* ad *g r* similis proportioni H *d* ad *b r*, & etiam similis est « proportioni *d a* ad *b r*; Ideoque proportio H *d* ad *b r* si« milis est proportioni *d a* ad *h r*, & alternè proportio H *d* ad « *d a* (quæ sunt æquales) similis est proportioni *b r* ad *h r*, « ergo hæ duæ etiam sunt sibi invicem æquales.

PROPOSITIO III.

Sit a b g *pars circuli, &* b *punctum quodlibet, &* b d *perpendicularis super* a g, *& ducatur* d h *æqualis* d g, *& arcus* b r *æqualis arcui* b g, *& ducatur* a r, *erit* (a r) *æqualis* a h. Fig. 3.

Demonstratio. Ducantur lineæ *g b*, *b r*, *r h*, *h b*, ergo quoniam arcus *b g* æqualis est arcui *b r*, erit *g b* æqualis *b r*, & quoniam *g d* æqualis est *b d*, & duo anguli ad *d* recti, & *b d* communis, ergo *g b* æqualis est *b h*, ergo *b r*,

Fig. 3. *b h*, æquales sunt, & duo anguli *b r h*, *b h r*, sunt æquales. Et quoniam quadrilaterum *a r b g* est in circulo, erit angulus *a r b*, cum angulo *a g b* ei opposito, hoc est cum angulo *b h g* æqualis duobus rectis; & (cum) sit angulus *a h b* cum angulo *b h g* æqualis duobus rectis, ergo duo anguli *a r b*, *a h b*, sunt æquales, et reliqui duo anguli *a r h*, *a h r*, sunt æquales, ergo *a h* est æqualis *a r*, Et hoc est quod volumus.

PROPOSITIO IV.

Fig. 4. (*Sit*) a b g *semicirculus*, & *statuantur super* a g *diametro*, *duo semicirculi*, *unus eorum* a d, *alter* d g, & b d *perpendicularis*, *figura inde orta ab* Archimede *appellatur* ARBELUS, & *hæc superficies continetur ab arcu semicirculi majoris*, & *à duobus arcubus semicirculorum minorum*, & *est æqualis circulo cujus diameter est* d b.

Demonstratio. Quoniam linea *d b* proportionalis est duabus lineis *d a*, *d g*, & media inter illas, ergo erit planum *a d* in *d g* æquale quadrato *d b*; et addatur *a d* in *d g*, cum duobus quadratis *a d*, *d g*, hoc est quadratum *a g* æquatur duplici quadrato *d b*, et duobus quadratis *a d*, *d g*, et proportio circulorum est ut proportio quadratorum (è diametris) ergo circulus cujus diameter *a g*, æquatur duplici circulo, cujus diameter *d b*, et duobus circulis quorum duæ diametri *a d*, *d g*, et semicirculus *a g* est æqualis circulo, cujus diameter est *d b*; et duobus semicirculis *a d*, *d g*, auferantur semicirculi *a d*, *d g*, communes, remanet figura quam comprehendunt semicirculi *a b g*, *a d*, *d g*, estque figura, quæ ab *Archimede* appellatur ARBELUS, æqualis circulo cujus diameter est *d b*. Et hoc est quod volumus.

PROPOSITIO V.

Fig. 5. *Si sit in semicirculo* (*linea*) a b, & *sumatur in diametro ipsius punctum quodlibet*, & *constituantur super diametrum duo semicirculi super* (*lineas*) a g, g b, & *ducatur à* g *perpendicularis* g d, *super* a b, & *ex utroque latere ipsius describantur duo circuli tangentes eam*, & *tangentes semicirculos*, *erunt duo circuli sibi invicem æquales.*

Demonstratio. Sit unus è duobus circulis tangens *g d* in *r*, & semicirculum *a b* in H, & semicirculum *a g* in *k*, et ducamus diametrum *r h*, ergo *r h* est parallela diametro

diametro *a b*, eò quòd duo anguli *h r g*, *a g d*, sunt recti, et jungamus H *b*, *h a*, ergo linea, *a h* est recta, per 1 Prop. (hujus Libri) et concurrant *a* H, *g r*, in *d*, quia ambæ ducuntur ab *a*, *g*, (angulis) minoribus duobus rectis. Jungamus etiam H *r*, *r b*, (linea) H *b* per 1 Prop. est recta, et est perpendicularis super *a d*, quia angulus *a* H *b*, est rectus, quia cadit in semicirculum *a b*. Et jungamus *b k*, *k g*, (linea) *b g* etiam est recta. Et jungamus *r k*, *k a*, (linea) *r a* etiam est recta, et producamus eam ad *l*, et jungamus *b l*, et hæc etiam est perpendicularis super *a l*, et jungamus *d l*, et quoniam *a d*, *a b*, sunt (lineæ) rectæ, et ducitur à *d a* ad lineam *a b* perpendicularis *d g*, et à *b* ad *d a* perpendicularis *b* H, ergo se intersecant in *r*. Ducatur *a r* ad *l*, et sit perpendicularis super *b l*, erunt *b l*, *l d*, duæ rectæ, quemadmodum demonstravimus in propositionibus quas adhibuimus in explicatione Libri de Triangulis rectangulis; et quoniam duo anguli *a k g*, *a l b*, sunt duo recti, ergo *b d*, *g k*, sunt duæ parallelæ, et proportio *a d* ad *d b* (quæ est similis proportioni *a g* ad *h r*) similis est proportioni *a b* ad *b g*, ergo planum *a g* in *g b* est equale plano *a b* in *h r*. Eodem modo demonstratur in circulo *t m n*, quod planum *a g* in *g b* est æquale plano *a b* in diametrum ipsius, & indè demonstratur, quod duæ diametri *r* H *k*, *t m n*, sunt æquales, ergo duo circuli sunt æquales. Et hoc est quod volumus. Fig. 5.

SCHOLIUM.

„Inquit Vir Excellens (*Alhonin.*) Id ad quod (*Archimedes*) refert in explicatione triangulorum rectorum demonstratur ex præcedenti, eaque est proportio in Elementis utilissima, & præcipuè in triangulis acutis. Opus autem habet sextâ Propositione hujus Libri. Ea autem hæc est. In triangulo *a b g*, ducantur duæ perpendiculares, *b h*, *g d*, se mutuò secantes in *r*, & jungatur *a r*, & ducatur ad H, erit perpendicularis super *b g*, jungamus *d h*, & erunt duo anguli *d a r*, *d h r*, sibi invicem æquales. Quoniam circulus qui comprehendit triangulum *a d r*, transit per punctum *h*, quia angulus *a h r* est rectus, & sunt in eodem arcu, et etiam angulus *d h b* æqualis est angulo *d g b*, quoniam circulus qui continet triangulum *b d g* transit etiam per punctum *h*, ergo in triangulis *a b* H, *g b d*, duo anguli *b a* H, *b g d*, sunt æquales, et angulus *b* est communis, ergo angulus *a* H *b* Fig. 6.

æqualis

Fig. 6. " æqualis est angulo *g d b* recto, ergo *a* H, est perpendicularis
" super *b g*.

Fig. 7. " His præmissis, describamus in figurâ ab *Archimede* allata,
" duas lineas *d a*, *a b*, & perpendiculares *d g*, *b* H, *a r l*, *b l*, &
" lineam *d l*. Dico, si *b l d* non sit linea recta, jungatur *b s d*
" recta, erit angulus *b s a* rectus, per præcedentem Proposi-
" tionem prædictam, & angulus *b l a* erit rectus, ergo interior
" (angulus) in triangulo *b l s*, æqualis est exteriori, eique op-
" posito, quod est impossibile, ergo linea *b l d* erit recta.

" Deinde adducit duas demonstrationes *Abi Sohal Alkonhi*.

" *Prior earum hæc est.*

Fig. 8. " Si duo semicirculi non fuerint se invicem tangentes, sed
" mutuò secantes, & perpendicularis fuerit à loco intersecti-
" onis, erit demonstratio ut præcedens.

" Sint semicirculi *a b g*, *a d h*, *r d g*, & duo semicirculi se
" intersecent in *d*, & sit *b* H perpendicularis super *a g* erectâ
" in H, & circulus *t k l* tangat circulum *a k g* in *k*, & circu-
" lum *a l g* in *l*, & perpendicularem in *t*, dico quòd sit æqua-
" lis circulo qui fuerit in alterâ parte, quæ est affectio
" (quæsita).

" Ducamus *t s* parallelam *a b*, & jungamus *g k*, ergo
" transit per *s*, quemadmodum *Archimedes* demonstravit, &
" producamus eam donec occurrat perpendiculari H *d b* in *m*,
" & jungamus *t g*, ergo transit per *l*, & producamus eam ad
" *m*, et jungamus *a m*, *m n*, et hæc est linea recta, et junga-
" mus *s r*, ergo transit per *l*, et jungamus *a k*, ergo transit
" per *t*, et linea, *a m n* est parallela lineæ *r s*. Et proportio
" *g n* ad *n s*, hoc est proportio *g* H ad *t s*, similis est proporti-
" oni *g a* ad *a r*, ergo planum *g h* in *a r* est æquale duobus
" planis *g a* in *t s*. Et quoniam H *d* perpendicularis, est in
" duobus circulis *g l r*, *h d a*, super duas diametros *g r*, *h a*,
" erit planum *g* H in H *r* æquale quadrato H *d*, et planum *a* H
" in *b* H etiam æquale ei, ergo planum *g* H in H *r* æquale
" est plano *a* H in *b* H, & proportio *g* H ad H *a*, similis est
" proportioni *b* H ad H *r*. Præterea similis proportioni *g b*,
" ad *r a* reliquum, ergo planum *g* H in *r a* est æquale plano
" *g a* in *t s* (quod est) æquale plano H *a* in *g b*; Et si sit cir-
" culus in alterâ parte, eodem in etiam modo proprietatem

prædictam

" prædictam demonstramus, quòd planum *g a* in diametrum *Fig.* 8.
" illius circuli, est æquale plano H *a* in *g h*, ergo manifestum
" est quòd duæ diametri sint circulorum æqualium.

Secunda (Demonstratio) hæc est.

" Inquit si duo semicirculi, neque se mutuò tangant, neque
" secent, sed à se invicem distent, & perpendicularis ducatur
" è concursu duarum linearum, æqualium, et tangentium duos
" semicirculos, indè etiam demonstratio constabit.

" Sint semicirculi *a b g*, *a d h*, *r* H *g*, quemadmodum descri- *Fig.* 9.
" psimus, et duæ lineæ *t d*, *t* H, tangentes duos semicirculos
" in *d*, H, et æquales et concurrentes in *t*, et linea *b t* perpen-
" dicularis transiens per punctum *t*, recta super *a g*, et circulus
" *m s* tangat eam in *m*, et circulus *m s* tangat circulum *a b g*
" in *k*, et circulum *r l g* in *l*, et ducamus diametrum *m s*,
" parallelam (lineæ) *a g*, et jungamus *g k*, ergo transit per *s*,
" et occurrit perpendiculari *t b* in *a*, et jungamus *a k*, ergo
" transit per *m*, et jungamus *s r*, ergo transit per *l*, et junga-
" mus *g m*, ergo transit per *l*, et ducamus eam ad *n*, et jun-
" gamus *a* A, ergo transibit per *n*, et erit parallela *r s*, et erit
" sicut proportio *g* A ad A *s*, hoc est, proportio *g t* ad *m s*, sic
" proportio *g a* ad *a r*, et planum *g t* in *a r*, æquale erit plano
" *g a* in *m s*. Simili modo demonstratur, quod planum *a t* in
" *h g* est æquale plano *g a* in diametrum circuli, qui fuerit ex
" alterâ parte. Et quoniam planum *a t* in *t h* æquale est qua-
" drato *t d*, et hoc equale est quadrato *t* H (quod est) æquale
" quadrato *g t* in *t r*, erit planum *a t* in *t h*, æquale plano *g t*
" in *t r*, et proportio *a t* ad *g t*, similis est proportioni *t r* ad *t h*,
" et similis proportioni totius *a r* ad totum *g h*, ergo planum
" *g t* in *a r*, æquale est plano *a t* in *g h*. Et jam demonstratum
" est quod *g t* in *a r* æquale est plano *g a* in *m s*, et quòd
" planum *a t* in *h g* æquale est plano *g a* in diametrum cir-
" culi alterius, ergo duæ diametri sunt æquales. Et hoc est
" quæsitum.

PROPOSITIO VI.

Fig. 10. *Si sit semicirculo (angulus)* a H b, *& sumatur in diametro ipsius punctum* g. *Sit* a g *ad* g b *in proportione sesquialterâ, & describantur super* a g, g b, *duo semicirculi, & describatur circulus ita inter tres semicirculos, ut tangat eos, & ducatur in eo diameter* d h *parallela diametro* a b, *reperienda est proportio diametri* a b, *ad diametrum* d h.

JUngamus duas lineas *a d*, *d* H, & duas lineas *b h*, H *h*, ergo erunt duæ lineæ *a h*, *b* H, rectæ per primam Prop. Describamus etiam duas lineas *h t a*, *d k b*: demonstratum est quòd hæ duæ etiam rectæ sunt, & prætereà duæ lineæ *g d*, *g h*: jungamus *g s*, *g m*, & *d r*, *h* A, & ducamus eas ad *l n*, ergo, quoniam in triangulo *a d g*, perpendicularis *a t*, & *g s*, etiam perpendicularis, se intersecant in *r*, ergo *d r l* etiam erit perpendicularis, quemadmodùm demonstravimus in explicatione tractatus de Triangulis universis. Et demonstratio ejus est eadem cum præcedente Propositione, & prepterea etiam *h n* erit perpendicularis super *b a*, & quoniam duo anguli qui sunt ad *m* & *h*, sunt recti; *g m*, erit parallela (lineæ) *a* H, & prepterea *g s* (parallela lineæ) *b h*, ergo erit proportio *a g*, ad *g b*, similis proportioni *a r* ad *r h*. Prætereà similis erit proportioni *a l* ad *l n*, & proportio *b g* ad *g a* similis erit proportioni *b* A ad A *d*. Prætereà similis proportioni *b n* ad *n l*. Est autem *a g* in sesquialterâ ratione *g b*, ergo *a l* est in sesquialterâ ratione *l n*, & *l n* est in sesquialterâ ratione *b n*. Ergo tres lineæ *a l*, *l n*, *n b*, sunt proportionales. Et in quâ quantitate *n b* erit quatuor, in eâ *n l* erit sex, *a l* novem, & *b a* novemdecim. Et quoniam *n l* similis est *d h*, erit proportio *a b* ad *d h* proportio XIX. ad VI. Ergo proportio dicta reperta est.

Si etiam fuerit proportio *a g* ad *g b* diversa ab eâ quam memoravimus, videlicet, proportio sesquitertia, aut sesquiquarta, aut alitèr, demonstratio & methodus (procedendi) similis est præcedenti. Et hoc est quod volumus.

PRO-

PROPOSITIO VII.

Si fuerit circulus circumscribens quadratum, & alter circulus (in quadrato) inscriptus, circulus exterior duplus est circuli interioris. Fig. 11.

SIt circulus *a h b* circumscribens quadratum *a b*, & sit circulus inclusus *g d*, & sit *a b* diameter quadrati (quæ etiam) est diameter circuli exterioris. Ducamus *g d* diametrum circuli inclusi parallelam (lineæ) *a h*. Et quoniam quadratum *a b* duplum est quadrati *a h*, hoc est *g d*, & proportio quadrati diametri circuli, ad quadratum diametri circuli, est sicut circulus ad circulum, ergo circulus *a b*, duplus est circuli *g d*. Et hoc est quod volumus.

SCHOLIUM.

" Inquit Vir Excellens (*Alhonîn*) libellum composui de
" descriptione Circuli, cujus proportio ad circulum datum
" similis foret proportioni datæ: Et eodem modo omnes fi-
" guræ rectiliniæ, & diversæ species operationum hujusmodi
" figurarum. Ex eo (libello) unam propositionem huc ad-
" duxi, quæ conducit explicationi hujus Opusculi, estque
" veluti universalis respectu harum Propositionum eas in-
" ferens. Et ea hæc est.

" Cupio constituere circulum *e.g.* in quintuplâ propor-
" tione circuli.

" Esto circulus cujus diameter *a b*, ei adde quintam ipsius Fig. 12.
" partem, sitque ea *b g*. Describatur super *a g* semicirculus
" *a d g*, & ducatur perpendicularis *b d*. Quoniam igitur pro-
" portio *a b* ad *b g*, similis est proportioni quadrati *a b* ad
" quadratum *b d*, erit totus circulus super *b d*, is qui à nobis
" quæritur ad solutionem Problematis. Et propterea si pro-
" portio circuli super *a b*, aut figura super eâ (lineâ) ad cir-
" culum, aut figuram, super *b d* constituatur, secundum con-
" structionem hujus figuræ, & ponatur secundum positionem
" ipsius, erit sicut *a b* ad *b g*. Et hoc est quod volumus.

PRO-

PROPOSITIO VIII.

Fig. 13. *Si ducatur in circulo linea qualibet* a b *recta*, *&* *ponatur* b g *æqualis dimidio diametri circuli*, *&* *ducatur linea inter* g *&* *centrum circuli, hoc est* d, *&* *ducatur ad* h, *arcus* a h *erit triplus arcus* b r.

DUcamus *b* H parallelam *a b*, & jungamus *d b*, *d* H, quoniam ergo duo anguli *d b* H, *d* H *b*, sunt æquales, erit angulus H *d g* duplus anguli *d b* H. Et quoniam *b d g* est æqualis angulo *b g d*, & angulus *g b* H æqualis angulo *a g b*, erit angulus H *d g* duplus anguli *g d b*, & omnes anguli *b d* H, tripli anguli *b d g*, & arcus *b* H, æqualis arcui *a b*, qui est triplus arcus *b r*. Et hoc est quod volumus.

SCHOLIUM.

Fig. 14. " Inquit Vir Excellens (*Alhonîn.*) Dicit (*Archimedes*) " arcum *b* H esse æqualem arcui *a b*. Hoc quidem fit propter " subtensas parallelas. Sint ergo in circulo *a b g*, subtensæ " *a g*, *b d*, parallelæ, dico quòd duo arcus *a b*, *g d*, sunt æquales. Ducamus *a d*, ergo duo anguli *g a d*, *a d b*, sunt æquales, & propterea erunt duo arcus æquales, & è converso per " similem demonstrationem.

PROPOSITIO IX.

Fig. 15. *Si duæ lineæ* a b, g d, *in circulo se mutuò intersecent in alio loco quàm in centro*, *&* *intersectio*, *fuerit ad rectos angulos*, *duo arcus* a d, g b, *sunt æquales duobus arcubus* a g, b d.

DUcamus diametrum *b r* parallelam *a b*, ergo intersecat *g d* in duas partes (æquales) in H, & erit *h g* æqualis *h d*; Et quoniam duo arcus *b d r* sunt semicirculus & arcus *h g* æqualis arcui *h a*, cum arcu *a d*, erit arcus *g r* cum arcu *h a* (&) *a d*, æqualis semicirculo, & arcus *h a* æqualis arcui *b r*, ergo arcus *g b* cum arcu *a d*, æqualis (erit) semicirculo, & relinquuntur duo arcus *h g*, *h a*, hoc est arcus *a g*, cum arcu *d b*, ei æquales. Et hoc est quod volumus.

PRO-

PROPOSITIO X.

Si fuerit circulus a b g, *&* d a *tangens circulum, &* d b *intersecans, &* d g *etiam tangens, & ducatur* H h *parallela* d b, *& jungatur* h a, *secans* d b *in* r, *& ducatur ab* r *perpendicularis* r h, *super* g h, *erit in medio ipsius* (g h) *in* H. Fig. 16.

IUngamus *r g*. Quoniam ergo *d a* est tangens,& *a g* secans circulum, erit angulus *d a g* æqualis angulo qui fit in sectione, quæ opponitur sectioni *a g*,hoc est, angulo *a h g*, hic autem est æqualis angulo *a r d*, quoniam *g h*, *d b*, sunt parallelæ, & anguli *d a g*, *a r t*, sunt æquales, & in duobus triangulis, *d a r*, *a t d*, duo anguli *a r d*, *t a d*, sunt æquales, & angulus *d* communis,propterea *r d* in *d t* erit æquale quadrato *d a* hoc est (erit æquale) quadrato *d g*. Et quoniam proportio *r d* ad *d g*, est similis proportioni *g d* ad *d t*, & angulus *d* communis, erunt duo triangula *d r g*, *d g t*, sibi invicem similia, & angulus *d r g* æquales *d g t*, qui æqualis est angulo *d a t*, qui æqualis est angulo *a r d*, ergo duo anguli *d r g*, *d r a*, sunt æquales, & *d r g* æqualis angulo *r g h*, & est *d r a* æqualis angulo *a h g*, ergo in triangulo *r h g*, duo anguli *g* & *h* sunt æquales, et duo anguli (ad) H duo recti, et latus H *r* commune, et propterea *g* H erit æqualis H *h*, ergo *g h* dividitur in duas æquales partes in H. Et hoc est quod volumus.

PROPOSITIO XI.

Si in circulo duæ lineæ a b, g d, *se intersecent ad angulos rectos in* h, *quòd non sit in centro, quadrata* a h, b h, h g, h d, *simul sumpta, sunt æqualia quadrato diametri.* Fig. 17.

DEscribamus diametrum *a r*, et jungamus lineas *a g*, *g r*. Quoniam igitur angulus *b h g* est rectus, erit æqualis angulo *a g r*, et angulus *a d g*, æqualis erit angulo *a r g*, quoniam habent eundem arcum *a g*,et relinquuntur ex duobus triangulis *a d h*, *a r g*, duo anguli *g a r*, *d a h*, æquales. Et propterea erunt duo arcus *g r*, *d b* (æquales) hoc est, subtensæ eorum æquales, et duo quadrata *d h*, *h b*, æqualia sunt quadrato *d b*, hoc est *g r*, et quadrata *a h*, *h g*,

Fig.17. æqualia ſunt quadrato *g a*. Et duo quadrata *g r*, *g a*, æqualia ſunt quadrato *r a*, hoc eſt diametro, ergo quadrata *a h*, *h b*, *g h*, *h d*, ſimul ſumpta, ſunt æqualis quadrato diametri. Et hoc eſt quod volumus.

SCHOLIVM.

" Inquit vir excellens (*Albonîn.*) Etiam hoc modo, qui " facilior eſt eo quem adducit *Archimedes*, (demonſtrari Fig.18. " poteſt.) Modus autem iſte eſt, Ut jungamus *a d*, *g b*, *b d*. " Quoniam ergo angulus *b h d* eſt rectus, erunt anguli *h b d*, " *h db*, quales recto, et duo arcus *a d*, *b g*, æquales ſemi- " circulo, et ſubtenſæ eorum æquales potentiâ diametro; " ſed quadrata *a h*, *h d*. æqualia ſunt quadrato *a d*, et qua- " drata *g h*, *h b*, æqualia ſunt quadrato *g b*, ergo quadrata " *a h*, *h b*, *g h*, *h d*, æqualia ſunt quadrato diametri. Et hoc " eſt quod volumus.

PROPOSITIO XII.

Fig.19. *Si fuerit ſemicirculus ſuper diametrum* a b, *& ducantur à* g, *duæ lineæ tangentes in punctis* d, h, *& jungantur* h a, d b, *ſe interſecantes in* r, *& jungantur* g r, *& producantur ad* H, *erit* g H, *perpendicularis ſuper* a b.

JUngamus *d a*, *h b*. Quoniam ergo angulus *b d a* eſt rectus, duo anguli *d a b*, *d b a*, reliqui de triangulo *d a b*, erunt æquales recto, & angulus *a h b* eſt rectus, Et quoniam *g d* tangit circulum, & *d b* interſecat ipſum, ergo angulus *g d b*, æqualis eſt angulo *d a b*. Et præterea angulus *g h r*, æqualis eſt angulo *h b a*, & duo anguli *g h r*, *g d r*, ſimul ſumpti ſunt æquales angulo *d r h*. Demonſtratum enim eſt in libro noſtro de figuris quadrilateris, quòd ſi ducantur inter duas lineas æquales, & concurrentes in puncto, nempè inter duas lineas *g d*, (&) *g h*, duæ lineæ ſe interſecantes, nempè lineæ *d r*, *h r*, & ſit angulus quem comprehendunt, ſcilicèt angulus *r*, æqualis duobus angulis, factis à concurrentibus cum duabus interſecantibus ſcilicet angulis *h*, *d*, ſimul, linea exiens à puncto concurſus ad punctum interſectionis, nimirùm linea *g r*, æqualis erit cuilibet ex duabus lineis concurrentibus, videlicet *g d* aut *g h*. Quapropter *g r* erit æqualis

g d,

g d, ergo angulus *g r d* est æqualis, hoc est, angulus *g r d* est æqualis *d a* H, sed angulus *g r d*, cum angulo *d r* H, æqualis est duobus rectis, ergo angulus *d a* H, cum angulo *d r* H, æqualis est duobus rectis, & relinquuntur de figurâ quadrilaterâ *a d r* H, duo anguli *a d r*, *a* H *r*, æquales duobus rectis, sed angulus *a d b* rectus est, ergo angulus *a* H *g* est rectus, & *g* H perpendicularis super *a b*. Et hoc est quod volumus. *Fig.19.*

SCHOLIUM.

" Inquit vir excellens (*Albonîn.*) In demonstratione *Fig.20.*
" (*Archimedes*) refert ad figuras quadrilateras: Sint ergo
" duæ lineæ æquales concurrentes *a b*, *a g*, & punctum con-
" cursus *a*, & (sint) duæ lineæ intersecantes *b d*, *d g*, & pun-
" ctum intersectionis *d*, & sit angulus *b d g* æqualis duobus
" angulis *a b d*, *a g d*, & jungamus *a d*, dico igitur quòd e-
" rit æqualis *a b*. Si non, erit aut minor *a b*, aut major eâ.
" Sit major, & abscindamus *a h* æqualem *a b*, & jungamus
" *b h*, *h g*, ergo duo anguli *a b h*, *a h b*, sunt æquales, sed
" angulus *a h b* major est angulo *a d b*, & prætereà angulus
" *a h g* æqualis est angulo *a g h*, qui major est angulo *a d g*, er-
" go omnes anguli *b h g*, hoc est anguli *a b h*, *a g h*, majores
" sunt duobus angulis, *a b d*, *a g d*, (hoc est) pars toto, quod
" absurdum est. Deindè sit *a d* minor *a b*, & faciamus
" *a r*, æqualem *a b*, & jungamus *b r*, *r g*. Demonstratur eo-
" dem modo quo anteà, quòd angulus *b r g*, hoc est, duo an-
" guli *a b r*, *a g r*, minores sunt duobus angulis *a b g*, *a g d*,
" totum parte; qnod absurdum est. Ergo demonstratio
" firma est.

PROPOSITIO XIII.

Si duæ lineæ, a b, g d, *in circulo se intersecent, & sit* a b *diameter ejus extrà* d g, *& ducantur à duobus punctis* a, b, *duæ perpendiculares super* g d, *eæque sint* a r, b h, *abscinduntur à* (g d) *lineæ* g h, d r, *æquales.* *Fig.21.*

JUngamus *r b*, & ab H, atque hoc est centrum, ducamus perpendicularem H *t* super *g d*, & producamus eam ad *k* in *r b*. Quoniam igitur H *t* est perpendicularis, à centro, super *g d*, ergo incidit in medium ipsius in *t*, et quoniam H *t*, *a r*, sunt duæ perpendiculares super eam, ergo sunt parallelæ.

Fig. 21. lelæ. Et quoniam *b* H, est æqualis H *a*, erit *b k* æqualis *k r*, quia illæ æquantur. Et quoniam *b h* est parallela *k t*, erit *h t* æqualis *t r*, et relinquuntur de *t g*, *t d*, æqualibus, *h g*, *r d*, æquales. Et hoc est quod volumus.

PROPOSITIO XIV.

Fig. 22. *Si* a b *sit semicirculus, & secentur ab* a b *diametro ejus, lineæ æquales* a g, b d, *& describantur super linas* a g, g d, d b, *semicirculi, & sit centrum circulorum* a b, g d, *punctum* h, *& sit* h r *perpendicularis super* a b, *et ducatur ad* H, *circulus, cujus diameter est* r H, *erit æqualis areæ quæ comprehenditur à semicirculo majori, et duobus semicirculis interioribus, et semicirculo medio, qui est exterior eo. Hæc figura ab* Archimede *appellatur*, SALINUS.

Quoniam *d g* bisecatur in *h*, & ei adjicitur *g a*, erunt quadrata *d a*, *g a*, dupla duorum quadratorum *d h*, *h a*, et sit *r* H æqualis *d a*, ergo duo quadrata *r* H, *a g*, sunt dupla duorum quadratorum *d h*, *h a*. Et quoniam (linea) *a b* dupla est *a h*, et *g d* dupla *h d*, erunt quadrata *a b*, *d g*, quadrupla quadratorum *d h*, *h a*, hoc est dupla quadratorum *r* H, *a g*, Et propterea erunt duo circuli, quorum diametri sunt *a b*, *d h*, dupli eorum, quorum diametri sunt *r* H, *a g*, et duo semicirculi quorum diametri sunt *a b*, *g d*, sunt æquales duobus circulis quorum diametri sunt *r* H, *a g*. Sed circulus cujus diameter est *a g*, est æqualis duobus semicirculis *a g*, *b d*, ergo abjiciamus ab eis duos semicirculos *a g*, *b d*, communes, relinquitur figura quam comprehendunt quatuor semicirculi, *a b*, *a g*, *g d*, *d b*, et est ea, quæ ab *Archimede* appellatur *SALINVS*, æqualis circulo, cujus diameter est *r* H. Et hoc est quod volumus.

PROPOSITIO XV.

Fig. 23. *Si* a b *sit semicirculus, et* a g *subtensa quintæ partis, et dividatur* a g *in* d, *et jungatur* g d, *et producatur, et cadat in* h, *et jungatur* d b, *et secet* g a *in* r, *et ducatur ab* r, *perpendicularis* r H *super* a b, *erit linea* h H *æqualis semidiametro circuli.*

Ducamus lineam *g b*, et sit centrum *t*, et jungamus *t d*, H *d*, *a d*, Et quoniam angulus *a b g* (cujus basis, (sive subtensa) est quinta pars circuli) est duæ quintæ partes

partes (anguli) recti, quilibet ex duobus angulis *g b d*, *d b a*, est quinta pars recti, & angulus *d t a* duplus est anguli *d b t*, ergo angulus *d t a* duæ quintæ recti. Et quoniam in duobus triangulis *g b r*, H *b r*, duo anguli (ad) *b* sunt æquales, & duo anguli H & *g* recti, & latus *r b* commune, erit *b g* æqualis *b* H. Et quoniam in duobus triangulis *g b d*, H *b d*, duo latera *g b*, *b* H, sunt æqualis, & prætereà duo anguli (ad) *b*, & latus *b d* commune, erunt duo anguli *b g d*, *b* H *d*, æquales, & quilibet ex eis, sex quintæ partes recti, & sunt æquales angulo *d a h* exteriori figuræ quadrilateræ *b a d g*, quæ in circulo est, relinquitur angulus *d a b* æqualis angulo *d* H *a*, & erit *d a* æqualis *d* H. Et quoniam angulus *d t* H est duæ quintæ recti, & angulus *d* H *t* sex quintæ recti, relinquitur angulus *t d* H, duæ quintæ partes recti, & erit *d* H æqualis H *t*. Et quoniam angulus *a d h* exterior figuræ quadrilateræ *a d g b*, quæ est in semicirculo, est æqualis angulo *g b* H, qui est duæ quintæ recti, & æqualis angulo H *d t*, & quoniam in duobus triangulis *h d a*, *t d* H, duo anguli *h d a*, *t d* H, sunt æquales, & prætereà duo anguli *d a h*, *d* H *t*, & duo latera *d a*, *d* H, erit *h a* æqualis *t* H, addamus *a* H communem, ergo *h* H erit æqualis *a t*. Et hoc est quod volumus.

Hinc perspicuum est, quod linea *h b* est equalis semidiametro circuli. Quoniam angulus *h* æqualis est angulo *d t* H, ergo linea *d t* erit æqualis lineæ *d h*. Dico etiam quòd *h g* divisa est secundum extremam & mediam rationem in *d*, & pars longior ejus est *h d*. Hoc autem fit, quoniam *h d* est subtensa sextæ partis, & *d g* subtensa decimæ, quod demonstratum est in libro Elementorum. Et hoc est quod volumus.

Finis Lemmatum ARCHIMEDIS.

Fig. 21. lelæ. Et quoniam *b* H, est æqualis H *a*, erit *b k* æqualis *k r*, quia illæ æquantur. Et quoniam *b h* est parallela *k t*, erit *b t* æqualis *t r*, et relinquuntur de *t g*, *t d*, æqualibus, *b g*, *r d*, æquales. Et hoc est quod volumus.

PROPOSITIO XIV.

Fig. 22. *Si* a b *sit semicirculus, & secentur ab* a b *diametro ejus, lineæ æquales* a g, b d, *& describantur super lineas* a g, g d, d b, *semicirculi, & sit centrum circulorum* a b, g d, *punctum* h, *& sit* h r *perpendicularis super* a b, *et ducatur ad* H, *circulus, cujus diameter est* r H, *erit æqualis areæ quæ comprehenditur à semicirculo majori, et duobus semicirculis interioribus, et semicirculo medio, qui est exterior eo. Hæc figura ab* Archimede *appellatur,* SALINUS.

QUoniam *d g* bisecatur in *h*, & ei adjicitur *g a*, erunt quadrata *d a*, *g a*, dupla duorum quadratorum *d h*, *h a*, et sit *r* H æqualis *d a*, ergo duo quadrata *r* H, *a g*, sunt dupla duorum quadratorum *d h*, *h a*. Et quoniam (linea) *a b* dupla est *a h*, et *g d* dupla *h d*, erunt quadrata *a b*, *d g*, quadrupla quadratorum *d h*, *h a*, hoc est dupla quadratorum *r* H, *a g*, Et propterea erunt duo circuli, quorum diametri sunt *a b*, *d h*, dupli eorum, quorum diametri sunt *r* H, *a g*, et duo semicirculi quorum diametri sunt *a b*, *g d*, sunt æquales duobus circulis quorum diametri sunt *r* H, *a g*. Sed circulus cujus diameter est *a g*, est æqualis duobus semicirculis *a g*, *b d*, ergo abjiciamus ab eis duos semicirculos *a g*, *b d*, communes, relinquitur figura quam comprehendunt quatuor semicirculi, *a b*, *a g*, *g d*, *d b*, et est ea, quæ ab *Archimede* appellatur *SALINUS*, æqualis circulo, cujus diameter est *r* H. Et hoc est quod volumus.

PROPOSITIO XV.

Fig. 23. *Si* a b *sit semicirculus, et* a g *subtensa quintæ partis, et dividatur* a g *in* d, *et jungatur* g d, *et producatur, et cadat in* h, *et jungatur* d b, *et secet* g a *in* r, *et ducatur ab* r, *perpendicularis* r H *super* a b, *erit linea* h H *æqualis semidiametro circuli.*

DUcamus lineam *g b*, et sit centrum *t*, et jungamus *t d*, H *d*, *a d*, Et quoniam angulus *a b g* (cujus basis (sive subtensa) est quinta pars circuli) est duæ quintæ

partes

partes (anguli) recti, quilibet ex duobus angulis *g b d*, *d b a*, est quinta pars recti, & angulus *d t a* duplus est anguli *d b t*, ergo angulus *d t a* duæ quintæ recti. Et quoniam in duobus triangulis *g b r*, H *b r*, duo anguli (ad) *b* sunt æquales, & duo anguli H & *g* recti, & latus *r b* commune, erit *b g* æqualis *b* H. Et quoniam in duobus triangulis *g b d*, H *b d*, duo latera *g b*, *b* H, sunt æqualis, & præterea duo anguli (ad) *b*, & latus *b d* commune, erunt duo anguli *b g d*, *b* H *d*, æquales, & quilibet ex eis, sex quintæ partes recti, & sunt æquales angulo *d a b* exteriori figuræ quadrilateræ *b a dg*, quæ in circulo est, relinquitur angulus *d a b* æqualis angulo *d* H *a*, & erit *d a* æqualis *d* H. Et quoniam angulus *d t* H est duæ quintæ recti, & angulus *d* H *t* sex quintæ recti, relinquitur angulus *t d* H, duæ quintæ partes recti, & erit *d* H æqualis H *t*. Et quoniam angulus *a d b* exterior figuræ quadrilateræ *a d g b*, quæ est in semicirculo, est æqualis angulo *g b* H, qui est duæ quintæ recti, & æqualis angulo H *d t*, & quoniam in duobus triangulis *b d a*, *t d* H, duo anguli *b d a*, *t d* H, sunt æquales, & præterea duo anguli *d a b*, *d* H *t*, & duo latera *d a*, *d* H, erit *b a* æqualis *t* H, addamus *a* H communem, ergo *b* H erit æqualis *a t*. Et hoc est quod volumus.

Hinc perspicuum est, quod linea *b b* est equalis semidiametro circuli. Quoniam angulus *b* æqualis est angulo *d t* H, ergo linea *d t* erit æqualis lineæ *d b*. Dico etiam quòd *b g* divisa est secundum extremam & mediam rationem in *d*, & pars longior ejus est *b d*. Hoc autem fit, quoniam *b d* est subtensa sextæ partis, & *d g* subtensa decimæ, quod demonstratum est in libro Elementorum. Et hoc est quod volumus.

Finis Lemmatum ARCHIMEDIS.

AD LEMMATA

ARCHIMEDIS

Animadversiones D. S. Foster.

I. DEMONSTRATIO.

INdiget explicatione multis in locis nisi accersantur quædam Propositiones ex *Euclide* aut aliquovis Geometra.

In Scholio ad primam Propositionem. Se mutuo extra tangentes. (*Et angulus* d t b *rectus*) *erit* b d t ½ *recti*.) At vero hoc non obtinet nisi solum quando diametri *d g* & *b a* sint perpendiculares communi, diametro *h r*. Propositio autem non adeo anguste enunciatur, sed æque vera est generaliter etiam ad diametros obliquas.

Figuræ 1^ma^. & 2^da^. Non sunt aptes satis formata.

Proposit. 2^da^. *d b h a Est parallelogrammum Rectangulum.* falsissimum.

ducitur perpendicularis d b falsissimum. Fateor si prius esset verum foret & posterius: at tunc demonstratio restringit Propositionem generalem ad particularem. Multa indiget explicatione.

In Scholio. *Et propterea*, non inquam propterea.

Inquit, *Sit* b r *similis*, non sub est sensus. Debet esse, sit *b r* æqualis.

Propos. 4. *hoc est quadratum A G*, at inquam hoc non ita est. Corrige sic & addatur bis A D in D G.

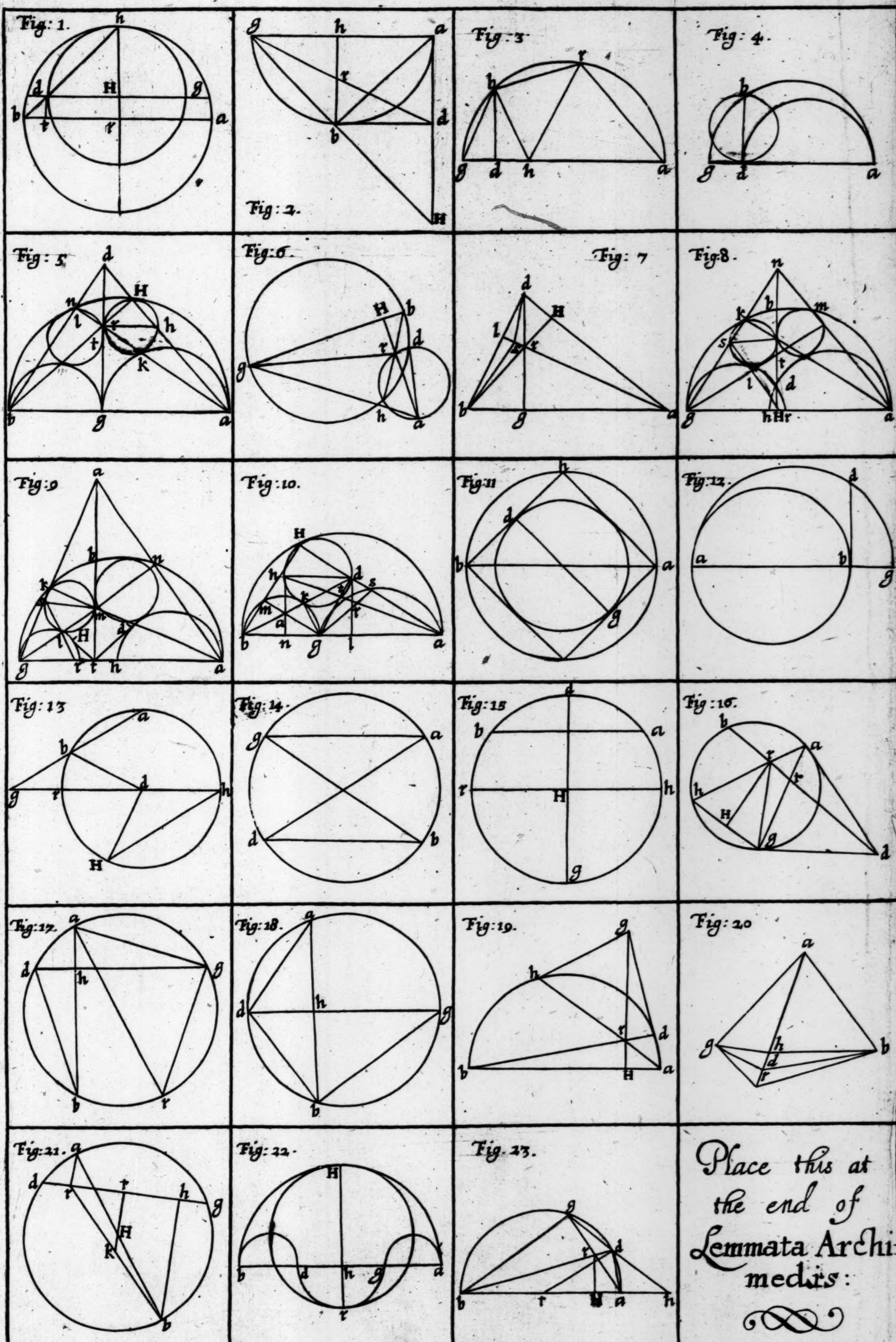
Propos. 10. Pro *h* dic H.

Propos. 14. Et sic *r h*, dic & est *r h* æqualis. Ergo duo quadrata, sed hoc nec *Alhonîn* nec ejus traductor demonstrarunt.

Propos. 15. Dicendum est & dividatur *a g* æqualiter in *d*.

Propos. 5. (linea) cur non diameter? nam est in semicirculo. Si quævis esset linea potius dicendum, si sit in circulo (linea) A B & sequitur etiam super diametrum, ergo traductor male addidit (linea.)

a h est recta, lege H *a* est recta, quod & sequentia innuunt ubi dicitur *a* H *g r*, &c.



Place this at the end of Lemmata Archimedis:

THE GEOMETRICAL SQUARE:

WITH THE USE THEREOF

IN

PLAIN and SPHERICAL

TRIGONOMETRIE.

Chiefly intended for the more easie finding of the HOUR and AZIMUTH.

By SAMUEL FOSTER, *Sometimes Professor of Astronomy, in* GRESHAM Colledge, LONDON.

LONDON,

Printed by R. & W. LEYBOURN.

M. DC. LIX.

A DESCRIPTION OF THE SQUARE.

THE whole superficies is divided into four lesser Squares, by the Diameters F G and H I.

Each of the 4 Semidiameters E F, E H, E G, E I, are divided as the lines of Sines upon the Sector, the Semidiameters being the whole Sine, And through the parts of each Semidiameter are drawn right lines perpendicular thereunto, quite over the face of the whole Square every 10th, 5th, &c. are to be distinguished from the rest, for the more easie and speedy account.

Upon the limb are inserted several Scales, for several uses. The edges of these Scales bordering close upon the sides of the inner Square, that it may be discerned which lines and parts of the Scales doe butt one upon the other.

On the sides A B, A C, are inscribed Scales of equal parts, the whole being divided into 10, and subdivided as quantity will give leave. The parts are numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10, and may stand, either for 1. 2. 3, &c. or for 10. 20. 30, &c. or for 100. 200. 300, &c. as occasion requireth.

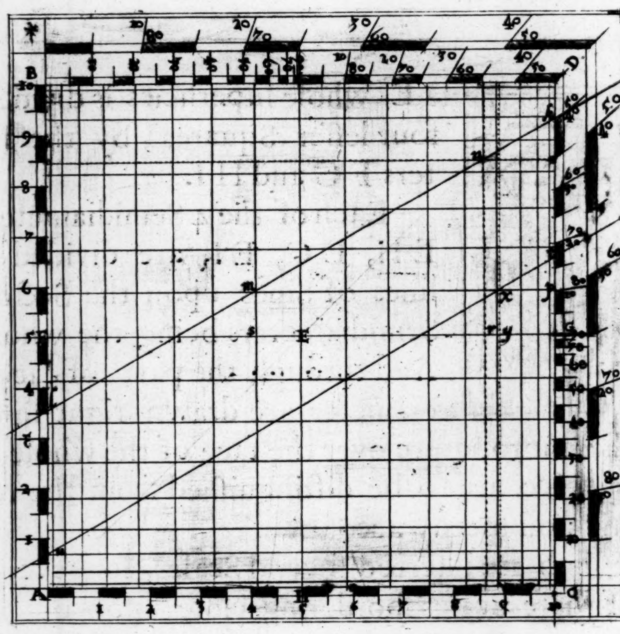
To the lines B I and C G, are annexed Scales of right sines whose beginning is at B and C, and the end at I and G, numbred by 10. 20, &c. to 90.

 The

The other parts I D, G D, have Scales of Tangents from o gr. to 45 gr. numbred either from G to D, and so to I. Or from I to D, and so to G, or rather both wayes, by 10, 20, 30, &c. till 90, and these divisions have respect to the Center E.

Lastly, a Scale of larger Tangents, lying behind these last named, Parallel to the sides BD, C D, beginning o gr. from B and C, and so proceeding to 45 gr. in D and ending in 90 gr. at C and B, accounted both wayes. These have respect to their Center A.

Τὸ μέγα βιβλίον ἴσον τῷ μεγάλῳ κακῷ.



There is further added a threed and plummet, which is to be used in every practice, and must be in length equal to the lines A F, and F G. And if the threed be found inconvenient in practice, because it will take up the use of both hands, there may instead of it, be used a little Bowe, the threed of it being at the least equal to A D, which will perform the office of the other threed by the help of one hand only, or a straight ruler may serve, if it be thought convenient for that purpose.

If the Square be applyed to the observation of Angles, it may

may be fitted thereto one of these two wayes, *Either by placing* two sights upon the side of the Square, one upon the Center A, the other upon the line A B, which issueth out of the Center A. *And a running sight* contrived upon the utter edge of the Instrument to move from B to C by D forward, and so from C to B by D, backward again; *Or else* if this be thought inconvenient, or not feasible because of the sights turning over at the Angle D, then this moveable sight may goe onely upon one of the sides B D or C D. And for that purpose the sight at A, is to stand precisely upon the Center, and both the sides A B, A G must have sights there fixed, as precisely, upon their lines that come from A.

Of the use of the Square in General for the Solution of Spherical Triangles.

In any Spherical Triangle whatsoever.

By having the Legs and Base, to find the Vertical Angle.

THe Angle given or sought is the Vertical Angle, The sides comprehending it are the legs. The side subtending it is the Base.

From the top of the Square, count the sum of the legs upon one side, the difference of them on the other side, To this sum and difference apply the threed, Then from the same top of the Square count the base also, And mark where it cuts the threed, for the line passing through the intersection, and standing Square to the top, (if it be numbred from that side of the Square whereon the difference of the legs was counted) gives the Vertical Angle required.

This is the general manner of work for this Proposition, which may be illustrated by these particulars.

FIRST,

Having the Latitude of the place, the Declination and Altitude of the Sun, to find the Hour of the day.

BY the declination of the Sun, may be had his distance from the elevated Pole, By subtracting it from 90 gr. when the Declination is of the same denomination with the said Pole,

Pole; Or by adding the Declination to 90 gr. when the Declination and elevated Pole are of ſeveral denominations.

In this caſe, we have the three ſides of a Spherical Triangle given, and an Angle ſought.

The two legs are *The complement of the Latitude*, and *The Suns diſtance from the Pole.* The baſe is, *The complement of the Suns Altitude :* The Angle is the *Hour* required, which muſt be accounted from the Coaſt of a contrary name, to the elevated Pole.

According then to the former general preſcript, and this particular declaration, *For the hour take the ſum and difference of the complement of the Latitude, and of the Suns diſtance from the Pole, and from the top of the Square, upon one ſide, count the differcnce, the ſum on the other, to theſe terms apply the threed; Then from the top of the Square alſo, count the complement of the Suns Altitude, and where it cuts the threed, the line that croſſeth it Square in the ſame point (being reckoned from that ſide whereon the difference of the legs was counted) gives the hour from the Meridian or noon.*

To make it plain by an Example.

In a North Latitude of 52 gr. 30 min. the Sun declining 20 gr. to the North, the Altitude of the Sun being by obſervation 43 gr. I would know the Hour of the day. The legs of this Triangle are the complements of the latitude and declination, that is 37 gr. 30 min. and 70 gr. 0 min. The ſum of them is 107 gr. 30 min. their difference is 32 gr. 30 min. Then from the top of the Square at D upon the ſide DG, I reckon this differencc 32 gr. 30 min. downward to *k*. And on the other ſide of the Square from the top at B, I alſo count the ſum of the legs 107 gr. 30 min. downward to *l*. To *k* and *l*. I apply the threed. Which done from the top of the Square, again, I count the baſe 47 gr. the complement of 43 gr. the altitude obſerved, downward alſo to o, and the line that there meets me, I follow till it cut the threed, which is at *n*, and the line that there ariſeth Square to it is *n r*. *I ſay now that n r*, if it be counted from the ſide DC whereon the difference of the legs was counted, ſhall give 44gr. 8 min. which turned into hours and minutes of an hour, (allowing 15 gr. to an hour; and 15 min. of a degree to one minute of an hour) will make two hours and 56 $\frac{1}{2}$ min. from the

the Meridian or South, And ſuch is the Hour for that *Latitude*, *Altitude*, and *Declination*.

So alſo, If in the ſame Latitude and diſtance of the Sun from the Pole, but in the altitude of 10 gr. I would know *The houre of the day*. Here becauſe the legs, that is, the complement of the latitude and the diſtance from the Pole, are the ſame, therefore the ſame poſition of the threed remaines ſtill, I therefore onely reckon the baſe (as before) which here is 80 gr. from D to *p*, then I follow the line *p*, till it cuts the threed at *m*, and the line there ariſing is *m s*, which counted from D C, whereon the difference of the legs was reckoned, ſhall give 99 gr. 50 min. that is 6 hours 39 $\frac{2}{3}$ min. of an hour from the Meridian or South.

Another Example.

In the ſame latitude of 52 gr. 30 min. let the declination of the Sun be 20 gr. to the South, where his diſtance from the elevated Pole is 110 gr. and let the altitude of the Sun be by obſervation 10 gr. I require the *Hour*. The legs are 37 gr. 30 min. the complement of the latitude, And 110 gr. the Suns diſtance from the Pole. The ſum of them is 147 gr. 30 min. The difference 72 gr. 30 min. which I count upon the ſides of the Square down to *u* and *t*; and the baſe which is 80 gr. the complement of 10 gr. I count alſo from D to *p*, then I follow the line *p*, till it cut the threed at *x*, and the line there ariſing is *x y*, which counted from D C, whereon the difference of the legs was reckoned, ſhall give 38 grad. 56 min. that is, two hours and almoſt 36 min. of an houre from the Meridian or South.

Note, That the threed in this ſituation, ſhewes on the diameter of the Square (which in this caſe repreſents the Horizon) the *Semidiurnal* and *Seminocturnal* Arks, for where the threed croſſeth the middle line, the line there ariſing, (counted from that ſide of the Square, whereon the difference was numbred) ſhewes the *Semidiurnal* ark, and counted from the other ſide, ſhewes the *Seminocturnal* ark.

Obſerve alſo, If you would known the *Crepuſculum* or *Twilight*, the threed is to be placed as before, according to the ſum and difference of the legs, and if you allow 18 gr. for the

Crepuſ-

Crepusculin line (as they usually doe) the base will alway be 108 gr. which in the two first Examples will not touch the threed at all, and therefore in that latitude and parallel of the Sun, the twilight continues all night. But in the last Example you shall find the *Crepusculin* line to cut the threed, 6 hours and 15 min. from the Meridian, which shewes that the twilight begins at $5\frac{3}{4}$ a clock in the morning, and ends at $6\frac{1}{4}$ in the evening, and the rest of the time is dark night which is $11\frac{1}{2}$ hours.

If the sum of the the legs be more then 180 gr. that is, if it would reach beyond the bottom of the Square, you must when you have reckoned to the bottom, count upward back again till you have ended the whole sum.

SECONDLY,

Having the Latitude of the place, the Declination and Altitude of the Sun, To find the Azimuth of the Sun.

HEre also the 3 sides are given, the same with the former, and an Angle sought. The two legs are the Complements of the latitude, and Suns altitude, The base is the Suns distance from the Pole which is elevated above the Horizon. The angle sought is the Suns *Azimuth*, from that part of the Meridian, which is of the same denomination with the elevated Pole.

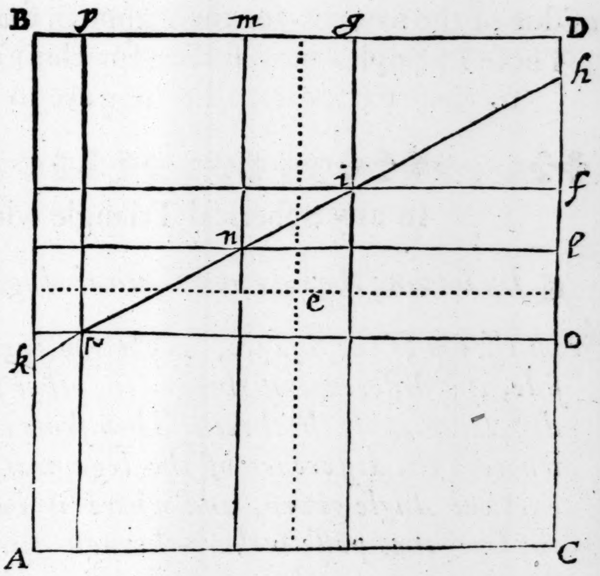
So then according to the former general prescript, and this particular declaration, for the *Azimuth*, doe thus.

Take the sum and difference of the Complements of the latitude and Suns altitude, and count from the top of the Square, the one upon one side, the other on the other side; and to these terms apply the threed; Then from the top of the Square also, count the Suns distance from the Pole, and where it doth crosse the threed, the line that there ariseth Square to the former, being reckoned from that side of the Square whereon the difference of the legs was counted, gives the Azimuth from that part of the Meridian which is of the same denomination with the elevated Pole, and counted from the other side, gives the Azimuth from the other coast.

To

To Illustrate it by an Example.

In a North latitude of 52 gr. 30 min. let the altitude of the Sun be 22 gr. and the declination 10 gr. Northerly, By these given I would know the Suns Azimuth, the two legs of the Triangle are the complements of the latitude and Suns altitude, that is 37 gr. 30 min. and 68 gr. the sum of them is 105 gr. 30 min. the difference is 30 gr. 30 min. The sum of them I count on the side B A, from the top at B down to *k*, The difference I count on the other side from D down to *h*, and to these points *k* and *h*, I apply the threed *k h*, And lastly, because the declination is 10 gr. North-ward in a North latitude, therfore his distance from the elevated Pole is 80 gr. which I count from the top D, down to *l*, and follow the line at *l*, till it meet with the threed at *n*, where I find the line *m n*, to crosse it also, which numbred from the side D C, whereon



the difference of the legs was numbred, gives 102 gr 38 m. the Azimuth from the North: And so also if it be accounted from the side B A, it gives the Azimuth from the South 77 gr. 22 min. the residue of the former, or the complement of it to 180 gr.

Another Example, In the same latitude and the same altitude, and therefore also the same situation, of the threed, let the declination be Northerly 23 ½ gr. therefore the distance from the Pole will be 66 ½ gr. which I count from D to *f*, and following the line *f* till it meet with the threed at *i*, I find the line *g i*, to crosse there also, which being counted from the side D C, whereon the difference of the legs was counted,

ſhewes 79 gr. 38 min. the Azimuth from the North, Or counted from the other ſide, gives the reſidue of the former, 100 gr. 22 min. The Azimuth from the South.

A third Example. In the ſame latitude and altitude, and therefore alſo in the ſame ſituation of the threed, let the declination of the Sun be 10 gr. to the South, then ſhall his diſtance from the elevated North Pole be 100 gr. and becauſe this 100 gr. is the baſe, I therefore count it from the top D, down to *a*, and following the line *a*, I find it to cut the threed at *r*, and the line *r p* there croſſing, ſhewes me from D C, (the ſide whereon the difference of the legs was counted) 146 gr. 32 min. for the Azimuth from the North, or if the ſame line be numbred from the ſide B A, it ſhewes 33 gr. 28 min. the reſidue of the former, for the Azimuth from the South.

These Examples may ſuffice for this kind, and according to theſe patternes, all others are to be framed.

In any Spherical Triangle whatſoever.

¶ *By having the legs and Vertical Angle, to find the Baſe.*

From the top of the Square, count the ſum of the legs upon one ſide, the difference of them on the other ſide. To this ſum and difference apply the threed: Then from that ſide of the Square whereon the difference of the legs was numbred, count the Vertical Angle given, and where it cuts the threed, mark the line that paſſeth there-through parallel to the top of the Square, for that line, counted from the top, gives the baſe required.

This is general for all works of this kind, which may be illuſtrated in particular: thus,

Having the latitude of the place, the declination of the Sun, and the hour of the day, to find the altitude of the Sun, for that latitude, declination, and hour.

Here have we the two legs of the Triangle, with the intercepted Vertical Angle, given, and the baſe ſought.

The legs are the complement of the latitude, and the Suns diſtance, from the Pole; The angle intercepted is the hour

hour whose altitude we seek. And the base is the complement of the altitude sought for.

Wherefore by the former general prescipt, and this particular explication, we may attain to the thing required thus, as in the former practice, so here; *Apply the threed to the sum and difference of the complement of the latitude, and of the Suns distance from the Pole, Then reckon the hour given from the Meridian, from the side whereon the difference of the legs was counted, and where it crosseth the threed observe the line that passeth there-through parallel to the top of the Square, for that line reckoned from the top, shewes the base, that is, The complement of the altitude, or reckoned from the middle line of the Square, it gives the altitude it self of the Sun, for that parallel and Hour; And so the threed (which now represents the Suns parallel:) lying still, you may count the altitudes for all the rest of the hours for that parallel.*

For Example.

In a North latitude of 52 gr. 30 min. let the Sun decline 20 gr. to the North, so that his distance from the elevated North-pole will be 70 gr. which is one of the legs given, and the complement of the latitude, 37 gr. 30 min. is the other, The sum of them is 107 gr. 30 min. The difference is 32 gr. 30 min. This difference is the complement of the Suns Meridian altitude; and I count it from D the top of the Square, to *k*: (in the first figure) thereto applying one end of the threed And on the other side from B, I count the sum, 107 gr. 30 min. down to *l*, thereto applying the other end of the threed. The threed thus laid, resembles the Suns parallel, for that declination, Now from the side D *k*, whereon the difference was numbred, I count the Vertical Angle, As first 15 gr. for the first hour from the Meridian, either 11 in the morning, or one in the afternoon, and where it cuts the threed, I observe the other line there crossing also, which counted from the top, gives for the base, 34 gr. 32 min. the complement of the altitude required.

Or rather. Count it from the middle line, which in this case represents the Horizon, and then you shall have 55 gr. 28 m. the altitude it self, for that hour and parallel, So the second

hour from the Meridian (10 or 2) gives for the altitude 50 gr. 4 min. The third hour (9 or 3) gives 42 gr. 31 m. The fourth hour from the Meridian (8 or 4) gives 33 gr. 53 min. The fifth (7 or 5) gives 24 gr. 48 min. The sixth (6 in the morning and evening) gives 15 gr. 45 m. The seventh (5 in the morning, or 7 in the evening) gives 7 gr. 6 m. And so farre the Sun is above the Horizon in that parallel, and then begins to go down.

And observe further, That the threed thus placed taken in that part below the Horizon, gives the altitudes for the hours in the declination which is equal to this, but to a contrary coast; so that the threed in this situation, gives the altitudes for the declination of 20 grad. towards the South, for that part of the threed that is under the Horizon or middle line, is the Semi-nocturnal ark for the parallel lying 20 gr. from the Equinoctial Northward, and is therefore equal to the Semi-diurnal ark that belongs to the parallel which lies 20 gr. from the Equinoctial Southward, and is of like situation below the Horizon that the other is above, wherefore the depressions belonging to the hours in this, are the same with the altitudes of the same hours in the other. To go on then where we left, The next hour counted from the Meridian of the Winter parallel is the fourth, that is, either 8 in the morning, or 4 in the afternoon, and his depression is 0 gr. 50 min. The next hour the third from the Meridian (either 9 or 3) is depressed 7 gr. 39 m. The second hour, (10 or 2) is 12 gr. 57 m. The first hour (11 or 1,) is depressed in this North parallel 16 gr. 20 min. that is, it is elevated so much in the like South parallel. Thus of each two opposite parallels of declination may the altitudes be had at one and the same situation of the threed. But if the other way seeme plainer, do as before. Let the Sun in the same latitude decline 20 gr. to the South, his distance from the North elevated Pole, is then 110 gr. the sum of the complement of the latitude 37 gr. 30 min. and this distance is 147 gr. 30 min. The difference is 72 gr. 30 m. This difference I count from D to *t*, (in the first figure,) The sum I also account as before, from B to *u*, And to *t u*, I placed the threed, Now from the side D *t*, I count the hours as I did before, and find the altitude of 11 and 1, 16 gr. 20 min. of 10 and 2, 12 gr. 57 min. of 9 and 3, 7 gr. 39 min. of 8 and 4, 0 gr. 50 min. All the same that the former depressions were;

And

And if now you take the depreſſions of the hours upon the threed in this ſituation, you ſhall find them all the ſame that the altitudes in the former parallel of 20 gr. North declination were; So that ever, one ſide of the threed will afford the altitudes for the hours in any two oppoſite parallels.

The Meridian altitude is the complement of the difference of the legs, And in the oppoſite parallel it is the exceſſe of the ſum of the legs above 90 gr. And as you have done for the Altitudes of the whole hours, ſo may you doe for their halves and quarters. Thus much for this alſo.

In any Spherical Triangle, whatſoever.

If the Proportions be in right Sines alone, they are reſolved in this manner.

Count the firſt ſine given (upon one of the ſides of the leſſer Square E I D G,) *from the Center* E, *and upon the line there ariſing count the ſecond ſine, whereto apply the threed, Then upon the ſame ſide with the firſt, count the third, and obſerve the line there ariſing, for from it doth the threed cut off the fourth ſine required.*

This general may be illuſtrated in particular thus.

Having the greateſt Declination of the Sun, and his diſtance from the next Equinoctial point, to find the Declination of the Sun for that diſtance.

THis particular belongs to the ſolution of a Rect-angled Spherical Triangle, yet the manner of the work in this is the ſame with the work belonging to the ſolution of the Obliquangled ones. The proportion ſtands thus; *As the radius, is to the ſine of the greateſt declination; So the ſine of the Suns diſtance from the next Equinoctial point, to the ſine of the declination of that point.*

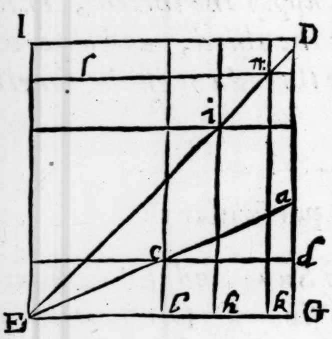
For an Example.

Let the diſtance from the Equinoctial be 30 gr. The greateſt declination 23 gr. 30 min. I would know the declination for that diſtance of 30 gr. The Proportion is, As the radius, is

is to the sine of 23 gr. 30 min. So the sine of 30 degrees, to what sine? Count E G for the radius, and upon G D the line there arising, reckon 23 deg. and 30 min. up to *a*, and thereto apply the threed, Then again upon E G, count *e b*, the sine of 30 gr. and follow the line there arising which is *b c*, till it cut the threed at *c*, and the line *c d*, there crossing also (being counted from E G) gives for the declination required, 11 gr. 30 min. So that the sine of 11 gr. 30 min. is the fourth Proportional Sine to the former three.

By the Hour of the Day given, with the Suns distance from the elevated Pole, and the complement of his altitude above the Horizon, to find his Azimuth.

The Azimuth thus gotten is counted in North latitudes from the North, in South latitudes from the South. Let the hour be 3 from the meridian. The Suns distance from the Pole 66 gr. 30 min. The complement of the altitude, 44 gr. 20 m. By these things known, I would find the Azimuth. The proportion whereby it is wrought is this.



As the Sine of the hour, is to the Sine of the complement of the altitude; So is the sine of the distance, to the sine of the Azimuth. Wherefore upon the side of the Square E G, I count the sine of 45 gr. to *h*, and upon the line there arising, I count the sine of 44 gr. 20 min. up to *i*, and thereto apply the threed. Then upon the same side with the first I reckon the sine of the Suns distance 66 gr. 30 min. from E to *k*, and following the line there arising till it meet with the threed at *n*, I find the line *n l*, to crosse there also, which counted from E G, gives the sine of 65 gr. for the fourth proportional, so that 65 gr. shewes the residue of the Angle required, that is to say, In our North latitude it shewes me the Azimuth from the South; because the Angle of the Azimuth from the North is an obtuse Angle, namely 115 gr. and the same sine serves both to it and to 65 gr. his residue.

¶ *And here also is to be noted, That when any Proportion in right sines alone is offered, and the radius is the first leader in*

in the Proportion, that then I say, it may be resolved by the former kind of work, by the sum and difference, counting the complement of the arks of the two Sines given, for the legs of the Triangle, and the ark of the radius or 90 gr. for the Vertical Angle, and the base found out to be the complement of the ark required. As in the first Example,

The two middle arks were 30 gr. and 23 gr. 30 min. their complements are 60 gr. and 66 g. 30 min. The sum of these is 126 gr. 30 min. there difference 6 gr. 30 min. to this sum and difference, I apply the threed, as in the former Examples, and then count the Vertical Angle 90 gr. which fals in the middle line and where the threed cuts it, there is the quantity of the Declination, 11 gr. 30 min. as before: And these degrees are counted from the Center of the Square at E.

And thus may all others of this nature, having the radius in the first place, be absolved. And not onely these of sines alone, but with sines intermingled with Tangents also, If it so fall out that these Tangents be lesse then the radius, And if instead of their proper arks be taken the complements of the arks of sines equal unto those Tangents.

And thus much for Exemplifying in this kind also. Those that follow are appropriate to rectangled Spherical Triangles only.

In any Rectangled Spherical Triangle whatsoever.

¶ *If the Proportion stand between right sines (whereof the Radius is alway one) and Tangents, they are to be resolved in this manner.*

Upon one of the sides of the lesser square EIGD. *Count the first term, and upon the line there arising count the second, whereto apply the threed. Then upon the same side whereon the first was reckoned, count the third, and follow it till it crosse the threed, for the quantity of it comprehended between the third term and the threed, gives the fourth proportional term required, alway remembring that every term be taken on his proper Scale.*

Here because the proportions are divers, we shall need more explication then in all the rest. Yet the variety herein, may be

be reduced to three wayes according as one of these three, either the Radius, Sine, or Tangent, doth lead in the Proportion, the three wayes are these :

1 *As the radius, is to a tangent , So is a sine to a Tangent.*

2 *As a sine, is to a tangent, So the Radius is to another tangent.*

3 *As a tangent, is to the radius; So another tangent, is to a sine.*

But this variety is not all, for each of these three wayes is subject to variation, and that upon this occasion. —— Upon the square we have no tangent greater then the radius, or tangent of 45 degrees. Wherefore the proportion must be so contrived, as that no tangent greater then of 45 gr. be ingredient into it. To that purpose serves this general direction, namely, —— If the tangent which is co-partner, in the proportion with the sine, be greater then of 45 gr. (alway provided that the two tangents doe never stand immediately together, which if they doe, may be brought into frame by transposition or alteration of the middle term.) Then, In the two first wayes the radius and sine must change places; and for the two tangents must be taken the tangents of their complements; In the third way, the co-tangents of the third and first terms must remove into the first and third places.

To shew this more particularly in the 3 former wayes.

In the first,

If the tangent required in the fourth place prove greater then of 45 gr. (which how to discover is shewed hereafter) then by the former direction this alteration must be made.

As the sine in the third place, is to the co-tangent in the second,

So is the radius in the first place, to the co-tangent of the fourth.

In the Second,

If the tangent in the second place, be greater then of 45 gr. then by the former direction this proportion must be thus changed.

As the radius in the third place, is to the co-tangent of the second.

So is the sine in the first place, to the co-tangent of the fourth.

In the third.

If the tangent in the third place, be greater then of 45 *gr. then according to the former prescript this proportion must thus be varied.*

As the co-tangent of the third place, is to the radius in the second;
So the co-tangent of the first place, is to the sine required in the fourth place.

Because in the first proportion it is unknown whether the tangent required in the fourth place be greater or lesser then of 45 *gr. and yet is necessary it should be known before it can be found out, you shall therefore in practice discover it thus.*

If the line whereon it is to be accounted doth not meet with the threed rightly situated upon the Square, then is it greater then of 45 *gr. and then the proportion must be altered as before, but if it doe meet with the threed, then is it lesse then of* 45 *gr. as it should be.*

Observe that the tangents are actually in the limb onely, yet may be understood to be all over the plain, for some line or other standing even against them in the plain will supply them as well as if they were actually there drawn.

And note that if such a Proportion as this do at any time happen, namely, *As a sine is to a tangent, So another sine, is to another tangent.* And that these tangents, be discovered to be one of them greater then of 45 gr. the other lesse, That then the radius is to be brought into the Proportion, by saying, *As the first sine, is to the first, So is the radius to another Tangent;* Then leaving out the first sine and tangent, and using for them the radius and this later tangent, say, As the radius is to the last found tangent; So is the sine in the third place to the tangent in the fourth; *Which Proportion suites* with those going before. But if both the tangents be either greater or lesser then of 45 gr. then may the solution be made without the help of the radius.

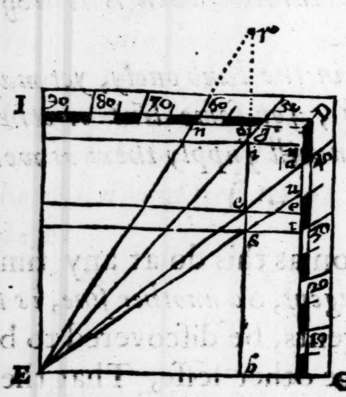
According to the former Rules generally delivered are these following Examples framed, and will fully illustrate every Case.

I. *For the first of the three several wayes* there are three cases, For either both the tangents are lesse then 45 gr. or both greater, or no lesse, the other greater.

1 As the radius is to the tangent of 40 gr. So the sine of 50 gr. to the tangent of what?

Upon the Square E I D G, I count E G for the radius, and upon the end of it in the Scales of tangents I reckon G *a*, 40 gr. thereto applying threed. Then upon E G, I count the sine of 50 gr. from E to *b*, and follow the line there arising till it cut the threed E *a*, at *c*, So that *b c* is the fourth term required, which because it is a tangent, must be numbred in the Scale of tangents, and therefore by help of the line *c e*, I transferre it thither, and find that the line lies even against 32 gr. 44 min. in the Scale of Tangents, and this is the ark of the fourth proportional term required.

2 *As the radius* is to the Tangent of 60 gr. So the sine of 50 gr. to the Tangents of what?



Upon the Square I count (as before) from G up to D and so forward to *n*, 60 gr. among the Tangents, and thereto apply the threed. Then I count the third term. E *b* the sine of 50 gr. and I follow the line there arising to the top of the Square, and yet it meets not with the threed, but beyond the Square I see that it would concurre with it at *r*, which shewes that the fourth Tangent required, is greater then the Tangent of 45 gr. and therefore standing as it doth, cannot be expressed upon the Scale of Tangents, wherefore I alter the Proportion, by transposing the first and third terms into one anothers places, and for the Tangents themselves I take their complement thus.

As the sine of 50, is to the Co-tangent of 60, So is the Radius, to a fourth Tangent, which will be the complement of that which should be produced by the former Proportion. By this alteration it comes to passe that the sine leads in the Proportion, and so this Example now fals under the Examples of the second General way, and therefore shall be resolved there.

3 As

3 *As the radius* is to the Tangent of 50 gr. So the ſine of 50 gr. to the Tangent of what?

Upon the Square I take E G for the radius, and at the end of it I reckon up to D, which is 45 gr. and ſo forward on the other ſide to *g*, that is to 50 gr. Then upon E G, I count *e b* the ſine of 50 gr. and follow the line there ariſing till it cut the threed at *i*, ſo that *b i* is the fourth term, and becauſe it is a Tangent; therefore by help of the line paſſing through *i*, that is, by the line *i m*, I transferre it to the Scale of Tangents, and find that lies even againſt 42 gr. 24 min. which is the fourth ark required.

For the ſecond of the three general wayes, there are two Caſes; For the Tangent that is Copartner with the ſine in the Proportion, may be either leſſer or greater then of 45 gr. for the leſſer, take the Example which before was preferred hither, namely, II.

1 As the ſine of 50 gr. is to the Tangent of 30 gr. So the radius is to the Tangent of what?

Firſt, upon the ſide E G, I count the ſine of 50 gr. and to the line there ariſing, I transferre G *t* the Tangent of 30 gr. by help of the line *t s*, and to *s*, I apply the threed, which threed cuts the limb in *u*, ſo that G *u* I find to be the Tangent of 37 gr. and this is the fourth term required in this Proportion; But in the ſecond Example going before, whereof this is alſo the ſolution, this 37 gr. is the complement of the fourth ark there required, ſo that the fourth ark there, ſhould be 53 gr. which becauſe it is greater then 45 gr. is therefore abſolved this way, and not the other.

2 As the ſine of 50 gr. is to the tangent of 50 gr. So the radius is to the Tangent of what?

Here becauſe the Tangent of 50 (being Co-partner in the Proportion with the ſine) is greater then the Tangent of 45 gr. and ſo cannot be expreſſed upon the ſquare, therefore the Proportion muſt be altered by changing the places of the firſt and third terms, and by taking the complement of the ſecond and fourth, after this manner.

As the radius is to the Tangent of 40 gr. So the ſine of 50 gr. to the Tangent of 32 gr. 44 min. the complement whereof 57 gr. 16 min anſwers to the queſtion in the former Proportion, and this laſt Proportion fals under the firſt general

 way

way where the radius leads, and was reſolved before in the firſt practice upon the Square, As E G, to G *a*, So E *b*, to *b c*, or G *e* the Tangent of 32 gr. 44 min.

III. *In the third of the three general wayes*, there are two caſes, according as the Tangent of the third place, which is Co-partner in the proportion with the ſine, is leſſer or greater then the Tangent of 45 degrees.

1 *As the tangent* of 40 gr. is to the Radius, ſo the Tangent of 32 gr. 44 min. to what ſine?

Upon the ſide G D, I count G *a*, the Tangent of 40 gr. and thereto apply the threed, then upon the ſame ſide G D, I reckon alſo the Tangent of 32 gr. 44 min. from G to *e*, and follow the line meeting at *e*, till it cut the threed at *c*, and the line there croſſing alſo is *c b*, which counted from *e*, the Center of the Square, gives the ſine of 50 gr. which is the fourth term required.

2 As the Tangent of 60 gr. is to the radius, So the Tangent of 53 gr. to what ſine?

Here becauſe the Tangent of 53 gr. being Co-partner in Proportion with the ſine, is greater then of 45 gr. therefore the firſt and third terms muſt change places, and their complements are alſo to be taken, thus, As the co-tangent of 53 gr. or Tangent of 37 gr. is to the Radius, So is the co-tangent of 60 gr. or Tangent of 30 gr. to the fourth ſine required.

Upon G D the ſide of the Square, I count G *u* the Tangent of 37 gr. thereto applying the threed. Then on the ſame ſide of the Square, I alſo count G *t*, the Tanegnt of 30 gr. and follow the line at *t*, till it cut the threed at *s*, and the line *s b*, there croſſing being counted from *e*, the Center of the Square, gives me the Sine of 50 gr. the fourth term required.

Theſe Examples are ſufficient to give light to the reſt, For no Proportion can fall out in theſe kinds, whereunto theſe Proportions and their Examples are not ſuitable.

And ſo much of Spherical Triangles.

Of

Of the uſe of the Square, in Right-lined Triangles.

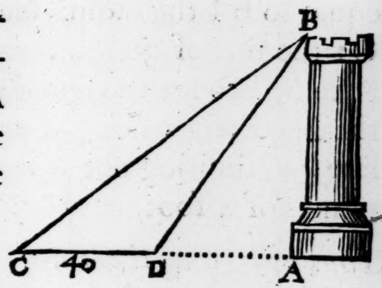
IF *the Proportions be between Tangents and equal parts, then are we to uſe the equal parts on the ſides* A B, A G, *as alſo the larger Tangents upon the two other ſides of the Square, and then the work will be the ſame, for form, that was before in Tangents and ſines, for the lines on the ſuperficies will carry the parts of either of theſe Scales to and fro, as they did before the parts of the Scales of the leſſer Tangents.*

If the Proportions be between ſines and equal parts, then are we to make uſe of the ſines inſcribed upon the Scales B I, C G, *together with the former equal parts, the lines upon the ſuperficies ſtill acting their former parts of carrying from the one to the other.*

Examples in theſe kinds, And firſt of ſines, and equal parts, or Numbers.

SUppoſe at the two ſtations D C, I had obſerved the angles B C A, 30 gr. B D A, 50 gr. and C D the difference of Stations 40 feet, and by theſe obſervations, I require to know the altitude A B.

First, I muſt find the length of the lines C B, or D B, in this Example of CB, after this manner, becauſe B C A is 30 gr. and B D A 50 grad. therefore their difference C B D is 20 gr. Now then, As the ſine of C B D 20 gr.



Is to C D 40 feet,

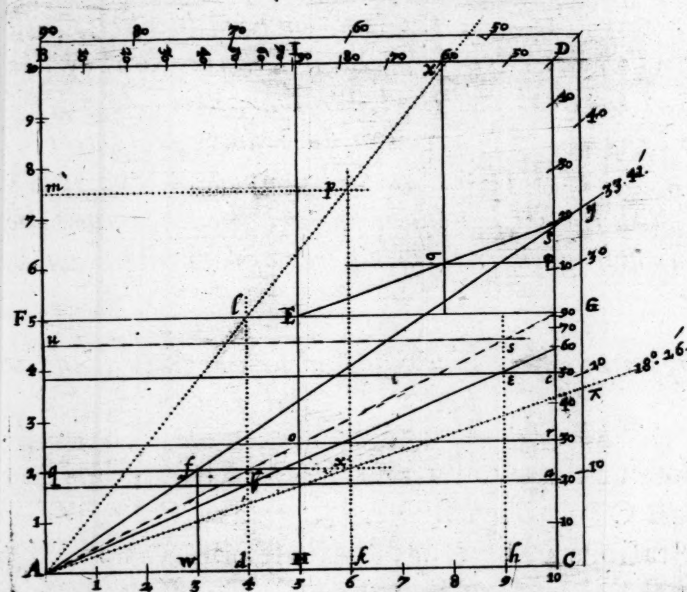
So is the ſine of BDC or BDA, 50 gr.

To the length of C B required.

To reſolve this upon the Square, from C, I count the line of 20 gr. to *a*, and obſerve the line there meeting me, then upon the ſide A C, I count A *d* 40 equal parts or feet, and thirdly, I reckon C *c* the third term, which is the ſine of 50 gr. and follow the line there meeting me, till it croſſe the threed (which was to be applied to *b*, the interſection of the lines *a b*, coming

coming from the first term, and *d b* rising from the second) at *e*, and there I find another line concurring, namely, *e h*, which I follow down to *h*, and there it shewes in the equal parts A *h* 89 feet, and 58 centesmes or hundredth parts of a foot, And this is the length of C B, now to get B A, by a second work, I say,

As C B the radius, to B A the sine of B C A 30 gr.
So B C 89 $\frac{58}{100}$ feet, to B A the altitude in feet.



To perform this Proportion, Upon the Square I take A H, equal to B I the radius, and upon H E, I count H *o*, equal to C *r* the sine of 30 gr. thereto applying the threed; Then from A to *h*, I count the length of C B, that is 89,58, and so follow the line there arising, up to the threed to *s*, where I find the line *s u*, limiting out A *u*, 44,79, that is 44 feet, and 79 centesmes of a foot, and such is the altitude of A B required.

Thus by having the three Angles of a plain Triangle, and one side you may find the two other sides; And by having two sides and an Angle opposite to one of them, you may find the other two Angles and third side, in any Right-lined Triangles whatsoever.

Examples of equal parts, and Tangents.

This kind of work may sufficiently be explained in the solution of this Probleme.

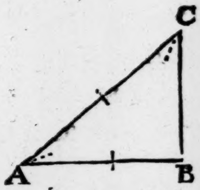
By

¶ *By having an Angle and the two sides comprehending it, to find the other Angles.*

First, if the Angle comprehended be a Right-angle the work is easie.

AND here we are to use the Scales of equal parts, with the larger Tangents onely. Suppose then in the Rect-angle Triangle A B C, By having the two sides including the right angle A B 30, B C 20 parts, I would find the angles at A and C, because this Proportion holds

As A B 30, to B C 20
So A B the radius to B C,
The Tangent of B A C.



Therefore upon the Square I count A *w* 30 equal parts, and follow *w f*, till it stand even with 20 equal parts counted on the side A B, and laying the threed at *f*, I find it to cut in the limb of the greater Tangent C *y*, which is 33 gr. 41 min. And such is the quantity of the angle C A B. And the complement of it 56 gr. 19 min. is the quantity of the angle *A C B*.

Further more it is to be noted, That if by having the right angle with the two including sides, you would find the sub tending side A C. In this case one of the acute angles must first be sought, and then by the Proportions of sines and equal parts, the side *A C* may be had.

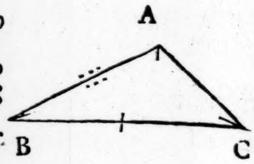
So also, If by having the distance *A B* 30 foot, and the angle *C A B* 33 gr. 41 min. I would know the *Height B C*, Upon the Square I lay the threed from C to *y* the Tangent of 33 gr. 41 min. then upon the equal parts I count *A w* 30, & follow the line rising at *w*, till it meet with the threed at *f*, and at *f*, I find the line *f q* crossing also, which followed to *q*, shewes in the limb 20 equal parts for the altitude *B C*.

By these mixt Proportions of equal parts with sines and Tangents, may all mensurations be performed, as also all conclusions upon the Common Sea-chart, with Mr. Gunters *corrections of it, to make it sufficient for* Sea-mens *use.*

Secondly,

Secondly, in any plain Triangle whatsoever.

The former Probleme may be resolved in general by this Proportion, As the sum of the two sides, is to their difference, So is the Tangent of half the sum of their opposite Angles, to the Tangent of the half difference of those Angles.



AS here, the two given parts, are *A B* 40 parts, & *BC* 20, the sum of them is 60, the difference is 20, the angle at *B* 120 gr. & therefore the sum of the two other 60 gr. the half sum of 30 gr. Now,

As 60 the sum of the sides, is to 20 their difference,
So is the Tangent of 30 gr. the half sum of the angles at A and C, to the Tangent of their half difference.

The best way for the solution of Proportions in this kind is first (as was before admonished in the joynt use of Sines and Tangents) to seek out a Tangent whereon the Radius is in Proportion as the sum of the legs is to the difference of them, which Tangent is ever lesse then the Tangent of 45 *gr. or radius, because the difference of the legs is alway lesse then the sum of them. And when the radius is brought in, the Proportion may be absolved upon the lesser Square, which is fitted for the Proportions of Sines and Tangents, in the same manner as was shewed in the like Examples before. And the Proportion will then stand thus.*

As the radius, is to this new found Tangent, So is the Tangent of half the sum of the angles, to the Tangent of half their difference.

To make it plain by the former instance, As 60 parts, are to 20; So is the radius, to what Tangent?

Upon the Scale of equal parts, I account A*k* 60 and on the line there arising I account *k x* 20, thereto applying the threed, and then I see it cut off in the greater Tangents C π 18 gr, 26 min. which is the Tangent sought. And now that the radius is brought in, the next Proportion will be thus,

As the radius, is to the Tangent of 18 gr. 26 min.
So is the Tangent of 30 gr. half the sum of the angles, to what Tangent?

Upon

Upon the lesser Square, I take E G for the radius, and count G ς the tangent of 18 gr. 26 min. thereto applying the threed, then upon I D, I count I ϰ the Tangent of 30 gr. and follow the line thence passing to the threed at σ, where the line σ φ shewes, in the limb the Tangent G φ, which is the Tangent of 10 gr. 54 min. the half difference of the angles required, which added to 30 gr. the half sum, makes the greater angle 40 gr. 54 min. And taken from the same 30 gr. leaveth 19 gr 6 min. for the lesser angle.

By having the three sides of a plain Triangle, to find the Angles.

THe first work here will be to let fall a perpendicular, and to know where it will fall, and so reducing the Triangle to two Rectangles, you may resolve them as Rectangles, either by sines and equal parts, or Tangents and equal parts.

The manner of dividing a Triangle, into two Rectangles, as also to find the place where the perpendicular falls, is shewed by Mr. *Gunter* in the first Book of his *Cross-staff*, and the Proportion for the solution of it, is a proportion of equal parts or numbers onely, the manner of which is hereafter shewed in the next use of the Square in numbers or equal parts alone.

Thus farre of the use in Right-lined Triangles.

Of the use of the Square in Proportions of equal parts or Numbers only.

THe equal parts with the lines on the superficies to carry them along, will perform them very sufficiently and expeditely, If any number be too great take $\frac{1}{10}$ or $\frac{1}{100}$ part of it, and count the rest as a fraction either decimal or centesimal. And as in the former works, so here, the first & third terms, must be counted upon one side of the Instrument, the second and fourth upon the lines arising out of the terms of the former, so the threed applyed to the second, will limit out the fourth.

The manner of the work is alwayes a like, and may sufficiently be declared in this one Example, I would know that number whereto 60 bears the same Proportion, that 40 doth to 50: The Proportion stand thus

As 40, is to 50: So 60, to what?

Upon A C the Scale of equal parts, I count A D 40, and upon the line arising out of *d*, (by help of the Scale of equal parts upon the other side A B) I count 50 up to *l*, and thereto apply the threed, Then upon A C, I count the third term, 60 to *k*, and follow the line there arising, till it meet with the threed at *p*, and there the line *p m*, meeting also, shewes in the Scale *A B* at *m*, 75 parts, which is the fourth proportional number required. And thus in all others.

Of the use of the Square in the observation of angles.

When the observation is made and the sight placed, then the threed from A, applyed to the running sight, will expresse the angle in the larger Tangent, And for observing any altitude or depth, the threed alone, without the help of the running sight, will expresse the Angle, if the observation be made as usually it is by other Instruments. —— The Square at the greatest cannot observe an Angle that is greater then 90. If therefore such an Angle come to be observed, you must observe the residue of it, which is his complement to 180 degrees.

Hitherto we have had a general view of the use of the Square in all Triangles and ordinary Proportions in Numbers. Now remains the bringing of it down to particulars in every kind, which would be an infinite labour, and un-necessary to those that are any thing experienced, in the use of Instruments, especially seeing we have here a tast of every of them, and the particular Proportions are every where extant. Hereafter I may adde something more on the other side, for the present I here make stay, and content my selfe with that which hath already been deli vered.

FINIS.

OF PROJECTION.

CHAP. I.

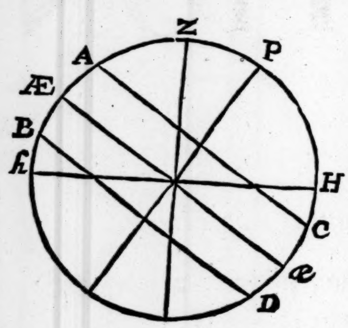
A Description of the Horizontal Projection.

PRojections of the Sphere, are best denominated from those great Circles upon which they are projected. This here, is called Horizontal, because the Circles on it are projected upon the plain of the Horizon: the eye being placed in the Nadir point thereof, upon the superficies of the Sphere; the whole delineation of it may be deduced Geometrically out of the Horizontal Circle. But because that way is in divers respects cumbersome and not so accurate, I rather choose to make such Tables as shall better suffice for the work, which what they are, and the manner how to make them, shall now be declared.

To describe the Æquinoctial and parallels of declination, belonging thereto, two Tables as requisite especially. The first, is to tell how farre from the Center each parallel in the Projection is to cut through the Meridian, which may be called a Table of Intersections. The second, is to find how farre the Centers of those Parallels do likewise stand from the Center of the Instrument, that so they may be described; and this Table may be called, A Table of Centers. And these two Tables are variable in every Latitude.

I. *To make the Table of Intersections.*

BEcause the Circle of the Instrument is the Horizon, and the Center, the Zenith, and the Diameter the Meridian, and it is required to know how far the parallels are to lie from the Center: It is meet therefore, first to enquire the same thing in the Sphere, (*viz.*) how many degrees each parallel of the Equator, from *Cancer*, to *Capricorn*, lies from the Zenith of the place, both on the South, and on the North part also, of the Meridian.



In this Scheam the work will be plain; wherein the Circle represents the Meridian of any place, as suppose of *London*, and in it, Z the Zenith, P the Pole, *h* H the Horizon, Æ *æ* the Equator, A C a North parallel, B D a South parallel.

Suppose, first, that A C, is the Tropick of *Cancer*, and B D, the Tropick of *Capricorn*, declining each 23 ½ grad. from the Equator Æ *æ*, the question is then, how far these two parallels are from the Zenith point Z, both wayes towards A B the South, and towards C D, the North? For answer, consider that from Z to Æ Southward, is the latitude of the place (as of *London* if you will 51 ½ gr.) and from Z to *æ*, Northwards is the supplement of that latitude, *viz.* 128 ½ gr.

And these are the South and North distances of the Equator or beginnings of *Aries*, and *Libra*. Then that Æ A, and Æ B, and so *æ* C, and *æ* D, are upon our supposition declining 23 ½ deg. from the Equator. So then if we take 23 ½ out of Æ Z, the latitude of the place 51 ½, there will remain A Z 28 gr. and so much is the parallel of *Cancer* distant from the Zenith of *London* on the South part of the Meridian. Again, because Z *æ*, is 128 ½ gr. and *æ* C 23 ½ gr. taking *æ* C, out of *æ* Z, there will remain 105 gr. and so much is the same parallel of *Cancer*, distance from the Zenith on the North part.

Now for the South parallel B D, belonging to *Capricorn*, adde Z Æ 51 ½ gr. to Æ B 23 ½ gr. the sum is 75 gr, shewing the

the distance of *Capricorn* from the Zenith on the South part of the Meridian, to be 75 gr. and Z æ, 128 $\frac{1}{2}$ gr. added to æ D, 23 $\frac{1}{2}$ gr. gives the distance of the same parallel of *Capricorn* from the Zenith to be 132 gr. So again, if we suppose A C to be a North parallel of the beginning of *Leo*, and B D of *Sagittarius*, each of which decline from the Equator 20 gr. 13 min. Take A Æ 20 gr. 13 min. out of Æ Z 11 $\frac{1}{2}$ gr. there remaines A Z, 31 gr. 7 min. the distance of the parallel of *Leo* from the Zenith of *London*, and the same æ c 20 gr. 13 min. taken out of æ Z 128 $\frac{1}{2}$ leaveth Z C, 108 gr. 17 m. for the distance of *Leo* from the Zenith, on the North part. Then for B D, the South parallel of *Sagittarius*, add B Æ, 20 gr. 13 min. to Æ Z, 51 $\frac{1}{2}$ gr. the sum is B Z, 71 gr. 43 m. for the South distance; and if æ D, 20 gr. 13 min. be added also to æ Z 128 $\frac{1}{2}$ gr. it will make Z D 148 gr. 43 min. the North distance of *Sagittarius*. In like manner, the beginnings of the signes *Virgo* and *Scorpio*, declining 11 g. 30 m. from the Equator, will give their distances South, for *Virgo* 40 gr. North, 117 gr. South, for *Scorpio* 63 gr. North, 140 gr.

The like computation must be made for the parallels of declination, belonging to each 5th degree of the fore-named signes, if you would have the Diameter of your Instrument to be about half a foot: but, if you would have it a foot (which is better) you may compute for each third degree of the Eclipticke.

And so having framed a Table of six Colomns, and written down the Characters of Signes in the first, and your numbers so produced into the second Column, as in this Example, made for the beginning onely of each Signe sheweth, you may proceed in this manner.

Write the halves of each of those arks forward into the the third Column of your Table, and in the natural Canon of Tangents, look out the particular Tangents belonging to each of these last arks, or halves, and set them down in the fourth Column, and the superiour of them also into the sixth Column.

The superiour Tangenrs in every cell of the fourth Column and sixth Column do make one of the Tables which is required, namely, the Table of *Intersections*.

I I. *To make the Table of Centers.*

FOr the effecting of this, add the two numbers standing in every cell of the 4th Column, and write the sums in the 5th Column, these are the Diameters of the several parallels. And if you take the half of each of those numbers, and write them in the 6th column, under the numbers there already placed, you shall then have the lengths of the Semidiameters of the same several parallels. And lastly, if from each of these Semidiamters, you take the Tangents of the 4th Column, which were before translated, and are already standing in the 6th Col. every couple as they stand, then shall you produce the distances of the Centers of every parallel, from the Center of your projection, which is the thing now required. And so the numbers of the second Table are made up also.

See here the form of the whole Calculation.

Tab. 1

	Arks	*Halfs*	*Tangent*	*Diamet.*		
♋	28.00 105.00	14.00 52.30	24932 130322	155254	24932 77627	♋
♌	31.17 108.17	15.38 54.08	27983 138313	166296	27983 83148	♊
♍	40.00 117.00	20.00 58.30	36397 163185	199582	36397 99791	♉
♎	51.30 128.30	25.45 64.15	48234 207321	255555	48234 127777	♈
♏	63.00 140.00	31.30 70.00	61280 274747	336027	61280 168013	♓
♐	71.43 148.43	35.51 74.21	72255 356956	429211	72255 214605	♒
♑	75.00 152.00	37.39 76.00	76732 401078	477810	76732 238905	♑
1 col.	2 Coll.	3 Col.	4 Col.	5 Col.	6 Col.	

Table 2.

Table of Centers		*Table of Intersect.*	
52695	♋	♋	24932
55165	♊	♌	27983
63394	♉	♍	36397
79543	♈	♎	48234
106733	♓	♏	61280
142350	♒	♐	72255
162173	♑	♑	76732

How to make the Table of Intersections, and of Centers more fully.

1 TAke half the complement of your latitude, and comparing it with 33 gr. 15 min. find out both the sum, and difference of them, and set this sum and difference in the uppermost cell of the Arches answering to *Cancer* 00 gr. and for better distinction, note them with A and B. These two numbers are for the first point of *Cancer*, and are as radical numbers, by help of which all the rest are made.

2 To

2 To these two numbers A and B, add such numbers of the Table following as doe stand at such degrees (of every signe) as you intend to put into your projection (as in a large one of 30 inches diameter you may insert every degree , or each two degrees for one of 15 inches, or each 3 gr. for 10, or each 5 gr. for *6* inches diameter) and place each couple of these last products in one cell, so shall you make up the column of Arches, such as in the following Table made for the latitude of 51 gr. 30 min. to the beginning of each signe, onely for an example.

3 For these arches of the first column, set the Tangents belonging thereto, as appeareth in the second column by the numbers C and D, &c.

4 Then for the third column of Diameters, and Semidiameters, it is thus perfected, add the two Tangents standing together in each cell, and put the sum of them in the third column, in the same line with the uppermost of the second collum; So shall these sums be the Diameters of such parallels in the projection as doe passe through the above-mentioned degrees, chosen for the projection. So C D added together, make E, G H make I, L M make the number N, &c.

5 If half these Diameters be taken, by a bi-section of the Diameters before found, the same will be the Semidiameters, and are to stand in the same third column, as the second line in each cell sheweth; So E being Bi-sected makes F, I makes K, N makes O, &c From these thus prepared the two fore-mentioned Tables will easily be excerpted in this manner.

6 The inferiour Tangents of each cell in the second column, are the very numbers which doe make up the Table of Intersections. If therefore, they be onely transcribed, you shall have the same Table perfected, as H M, &c. in the second column being transcribed, will make up P Q R, &c. the full Table of Intersections.

7 The differences of the inferiour numbers of the second and third columns being gathered into the particular cels of the Table of Centers, doe make up the numbers of that same Table; So D taken from F makes S, H taken from K gives T, and M from O, makes V, &c. and thus are the two Tables to be made up.

A Table

A Table for the Horizontal Projection,

made to the latitude of 51 gr. 30 min. shewing where every parallel that passeth through each degrees of the Ecliptick is to cut the Meridian line.

The Table of Intersections.

	♋	♌	♍	♎	♏	♐	
0	2493	2802	3640	4823	6128	7226	30
1	2493	2820	3676	4867	6172	7252	29
2	2493	2842	3709	4910	6212	7279	28
3	2496	2864	3746	4953	6253	7306	27
4	2500	2886	3782	4997	6297	7332	26
5	2503	2908	3819	5040	6338	7359	25
6	2506	2931	3855	5084	6379	7382	24
7	2512	2956	3892	5128	6420	7404	23
8	2515	2981	3929	5169	6457	7427	22
9	2521	3003	3966	5213	6498	7449	21
10	2527	3029	4006	5258	6540	7467	20
11	2537	3057	4047	5302	6577	7490	19
12	2543	3083	4084	5347	6615	7508	18
13	2552	3108	4122	5388	6652	7522	17
14	2561	3137	4163	5433	6690	7540	16
15	2571	3166	4200	5479	6728	7558	15
16	2583	3195	4241	5520	6766	7572	14
17	2596	3224	4283	5566	6805	7586	13
18	2608	3252	4321	5608	6839	7600	12
19	2620	3281	4362	5654	6873	7609	11
20	2633	3314	4404	5696	6911	7623	10
21	2645	3343	4445	5739	6946	7632	9
22	2661	3375	4487	5785	6976	7641	8
23	2676	3404	4526	5828	7011	7646	7
24	2692	3437	4568	5871	7046	7655	6
25	2708	3469	4610	5914	7076	7659	5
26	2726	3502	4653	5957	7107	7664	4
27	2745	3538	4695	6001	7137	7669	3
28	2764	3571	4738	6044	7168	7673	2
29	2783	3607	4781	6084	7199	7673	1
30	2802	3640	4823	6128	7226	7673	0
	♊	♉	♈	♓	♒	♑	

The

The Table of Centers, for the Latit. of 51 gr. 30 min.

This Table sheweth how farre the Centers of every of the former parallels, are from the center of the Horizontal Circle in the Horizontal projection.

	♋	♌	♍	♎	♏	♐	
0	5269	5519	6339	7954	10673	14235	30
1	5269	5535	6380	8026	10789	14342	29
2	5269	5554	6418	8099	10897	14451	28
3	5272	5573	6460	8173	11006	14562	27
4	5274	5593	6502	8249	11129	14674	26
5	5277	5612	6545	8326	11243	14787	25
6	5279	5631	6589	8404	11359	14883	24
7	5284	5654	6633	8484	11477	14980	23
8	5286	5677	6678	8558	11586	15078	22
9	5291	5697	6724	8641	11709	15177	21
10	5296	5720	6775	8726	11834	15258	20
11	5393	5746	6826	8812	11949	15359	19
12	5308	5770	6874	8899	12066	15441	18
13	5315	5794	6922	8981	12185	15503	17
14	5323	5820	6976	9072	12306	15587	16
15	5330	5849	7026	9165	12430	15670	15
16	5340	5876	7082	9252	12556	15735	14
17	5350	5904	7138	9348	12684	15799	13
18	5359	5933	7191	9438	12800	15864	12
19	5369	5961	7249	9538	12918	15907	11
20	5379	5994	7308	9682	13053	15973	10
21	5390	6023	7369	9727	13175	16017	9
22	5402	6056	7430	9833	13284	16061	8
23	5415	6086	7487	9932	13411	16083	7
24	5428	6120	7550	10034	13539	16128	6
25	5440	6154	7615	10137	13653	16150	5
26	5456	6189	7680	10242	13770	16172	4
27	5472	6228	7747	10349	13888	16195	3
28	5487	6263	7815	10458	14007	16217	2
29	5503	6303	7884	10560	14129	16217	1
30	5519	6339	7954	10673	14235	16217	0
	♊	♉	♈	♓	♒	♑	

Thus

Thus are these two Tables to be made up.

	Arkes	Tangents	Diameters & Semidiamet.	
♋	A 52.30 B 14.00	C 130322 D 24933	E 155255 F 77627	♋
♌	X 54.09 X 15.39	G 138399 H 28015	I 166414 K 83207	♊
♍	Y 58.30 Y 20.00	L 163185 M 36397	N 199582 O 99791	♉
♎	Z 64.15 Z 25.45	207321 48234	255555 127777	♈
♏	70.00 31.30	274748 61280	336028 168014	♓
♐	74.21 35.51	356957 72255	429212 214606	♒
♑	76.00 37.20	401078 76733	477810 238905	♑

	Intersections	Centers	
♋	P 24933	S 52694	♋
♌	Q 28015	T 55192	♊
♍	R 36397	V 63394	♉
♎	48234	79543	♈
♏	61280	106734	♓
♐	72255	142351	♒
♑	76733	162173	♑

Præcepta superiora, characteribus compendiosè expressa.

$$33\ gr.\ 15\ min. \pm \tfrac{1}{2}\ \text{Complement Latit.} = \frac{A}{B}$$

$$\frac{A}{B} \pm \text{b, c, d, \&c. in tab. sequ.} = X,\ Y,\ Z,\ \&c.$$

Quorum arcuum Tangentes sunt C, D, G, H, L, M, &c.

$$C+D=E.\quad G+H=I.\quad L+M=N,\ \&c.$$

$$\frac{E}{2}=F.\quad \frac{I}{2}=K.\quad \frac{N}{2}=O,\ \&c.$$

$D=P.\ H=Q.\ M=R,$ &c. Atque hæc est tabula Intersectionum

$F-D=S.\ K-H=T.\ O-M=V,$ &c. Atque hæc est tabula Centrorum

And here, it must not be forgotten; that the precepts of making up these Tables, are proper to those Latitudes that exceed 23 gr. 30 min. for in those latitudes, which are lesse then 23 gr. 30 min. some North parallels will not intersect upon the South part of the Meridian at all, but all together upon the North, and then, for such parallels, their North declinations must not be taken out of the latitude, but the latitude out of them, and so the superiour arkes of the second column will at first decrease in such latitudes, and after again increase, and the Diameters in the first column (for such parallels as are altogether North, of which onely we now speak) must

must be made by the differences (not sums) of the numbers of the fourth column; And the sums (not differences) of the numbers in the 6th column, give the distances of the Centers of such parallels we now mention, from the Center of the Instrument. Now to know how many degrees of declination will intersect both lines on the North side of the Meridian, in a North latitude is easie: namely, all those parallels whose declination from the Equinoctial is greater then the latitude, and none else. And for those onely, all this caution is made; The rest of the Table for other parallels must be finished as before prescribed, and what is true here of North latitude, and North parallels, is respectively true of South latitude, and South parallels.

III. *The delineation of the parallels upon the Instrument.*

AFter you have described the Circle upon the Horizon (which is to contain the whole work) and quartered the same, and set out partitions for the limbe , to divide it as the usual manner is into 360 gr. you are to make a decimal scale of the same length, with the innermost Semidiameter of your Instrument, for this scale by help of your Tables will pitch out the whole work.

For looking first into your Table of Intersections, see what the first number there is, namely, 24932, take this in your compasses upon your decimal scale from 2 forward as the letters of *a* and *b* doe declare; the same length of *a b* will reach upon the South part of the Meridian, from Z the Center of your Instrument unto A upon the Meridian line, which gives the point of Intersection between the parallels of ♋, and the Meridian. So the second number being taken in your compasses from the Decimal scale, will give the length Z*c*, shewing where the parallel of ♌ and ♊ is to passe through the Meridian. The third number so ordered, will give the point *i*, The foruth *o*, the fifth *u*, the sixth *m*, and the last will give *x*, where the rest of the parallels of ♍ and ♉, ♎ and ♈, ♏ and ♓, ♐ and ♒, and lastly ♑, must intersect with the Meridian.

After these points of intersections, you are next to prick

down your Centers answering to them. For the first number in the Table of Centers being taken from the decimal scale, and pricked down upon the North part of the Meridian from Z (towards N) and it will reach to A. And so the second number will be extended from Z to E, the third from Z to I, the fourth to O, the fifth to V, the sixth to M, and the seventh and last to X.

Having gone thus far, the rest will be easie ; For if you set your Compasses from A, the first Center to *a* the first intersection, you may describe the first parallel of *Cancer*, and so if from E the second Center, you extend to *e*, the second intersection you shall describe the second parallel passing through *e*, and so forward with the rest having due regard to every intersection, with his proper Center: and thus are the parallels to be described, amongst which that which passeth through *o*, is the Æquinoctial, and if it be true done, will passe through W and E, each tenth parallel must be distinguished with somewhat a bigger line then the rest, and where every fifth or third will not come in for want of due space, as about *Cancer* and *Capricorn*, where they grow close, there may you put in every fifth or tenth onely, which will serve in those narrow spaces as well as more.

IV. *The Delineation of the Hour Circles.*

First, you must prick down the North Pole, (which in our supposition is elevated 51 $\frac{1}{2}$ gr.) in this manner. Take the complement of the latitude, *viz.* 38 gr. 30 min. and half it, which will be 19 $\frac{1}{4}$, seek then the Tangent of 19 gr. 15 m. you shall find it to be 34921, take this upon your decimal scale, and prick it upon the North part of the Meridian from Z, towards N, you will find it to fall in P, that point P therefore is the North Pole in this projection, through which all the hours now to be drawn must passe.

The first hour Circle to be described is the hour of 6, upon which all the rest have their dependance ; now to effect this, you are to look for the Secant of your latitude (which is as before 51 $\frac{1}{2}$) which will be found to be 160638, this number taken out of the decimal scale must be extended upon the South end of the Meridian from P, and you shall find it will reach

reach unto B; upon B therefore as your Center with the distance B P describe the hour circle of 6, which, if all be right will passe through the points of E and W exactly, where the Equinoctial also cutteth, if it be justly described; now through the point B, with the Center of this hour of 6, draw the infinite line C B D, both wayes perpendicular to the Meridian Z S B, for upon this line shall stand all the Centers of the other hour circles, which to designe you are to work thus:

Make a second decimal scale, equal in length to B P, the Semidiameter of the hour of 6, then by help of the Canon of Tangents, take out of this scale, first, the Tangent of 15 gr. or 1 hour, which will be 26794, and prick it down upon the infinite line C D, both wayes from B to F, and from B to G; Again, seek the Tangent of 30 gr. which is 57735, and take it in the same scale, and prick it down upon the line C D, both wayes to H and I; Thirdly, seek the Tangent of 45 gr. 100000, which set as before, will just reach to C and D. Then fourthly, the Tangent of 60 gr. 173205 will so reach to K and L, Lastly, set the Tangent of 75 gr. which is 373205, from B to R, and from B to T. This done, if now you set one foot of your Compasses in F as a Center, and open the other to the point P in the Instrument, you shall describe the hour Circle of 7, on the East side of the Meridian, and carrying one foot with the same extent unto G, will reach unto P again, which swept on the other side, will describe the hour of 5. So also in the same manner may you describe from H & I, as two Centers of the hour, of 8 and 4, passing through the same point P; And upon the Centers C and D as before, you may describe the hours of 9 & 3; * And from K and L, the hour of 10 and 2 Lastly, from R and T, the hours of 11 and 1, all which are exactly to passe through P the Pole; And so the like is to be done for the half hours, &c.

** The line* C D *is supposed to be infinitely extended from* B, *both wayes, and therefore (for want of room in this place) the letters* R K *and* L T, *could not be in this Figure placed according as their true Tangent distances doe require.*

Thus have we done with those lines which doe properly belong to the projection: But because the Instrument, if it passe thus, will not perform all the uses for which it is intended, I have therefore added other lines to it, which may well stand without defacing any of the work, the description whereof is in briefe, as followeth.

V. *The description of what lines are added to the Projection, which vary not with the latitude, as the Projection it self doth.*

COunt from N to ♋ and ♋, both wayes upon the limb, the Suns greatest declination 23½ gr. and draw the chord ♋ ♋, then look the Secant of 23 gr. 30 min. in the Canon of Secants, which will be found 109044, take this upon your first decimal scale, and prick it down upon the Meridian from Z to *h*, then again setting one foot of your compasses in *h*, open the other to ♋, and describe the arch ♋, ♈, ♑.

Again, Having obscurely drawn the crosse Diameter of East and West, take so much of it as may conveniently be used on both sides the Center, namely, Z ♋, and divide it into 11 equal parts, and of the same equal parts let Z ♈ upon the Meridian contain 12, then draw the two streight lines ♈ ♋, on both sides of the Meridian: these may be called the Triangular lines, to distinguish them from the rest: Thus are all the lines to be drawn, now follows the manner of their division.

VI. *The division of the Triangular lines.*

DIvide each of those lines first into two equal parts at *b* and *d*: then again divide each of those halfs *b* ♈ *b* ♋, *d* ♎, *d* ♋ into 45 such parts as a Tangent line of 45 gr. or a radius of that length would require, so shall each of the whole lines contain 90 parts, unto each 30 division whereof are che characters of the twelve Signes to be set.

VII. *The division of the Ark and Chord.*

ON the Limb of your Instrument, number the complement of latitude from N to *p*, and draw the infinite line Z *p*, then Prolong the Chord ♋ and ♑ both wayes, so shall it meet with Z *p*, at *n*; Now to the radius *b n*, describe the Semicircle *n k r*, cutting the Meridian extended at *k*, and divide this Semicircle into 180 equal parts, or degrees; which done, if you first draw right lines from each degree thereof to the Center *b*, and beyond, till it crosse through

through the Ark divided in the Instrument, (as you see each 30^{th} degree in the scheame doth) you shall by that meanes divide the Ark into its proper parts. Secondly, if from each degree on both sides *k*, you draw lines parallel to the Meridian of the Instrument Z *k*, till they cut through the chord, they will so divide the chord into its requisite parts.

The line c n, is a line of Versed Sines.

Hitherto have you neer had the whole description of the Instrument it self in every part, after which all superfluities being first drawn away, you must affix such characters and figures as are necessary to help you in your several accounts; to the effecting whereof, the Picture it self will be of sufficient direction, and much better then many words.

VIII. *Of the Ruler.*

THere remaines onely a Ruler to be fitted to the Instrument, the breadth whereof may be as you will, about the 10^{th} or 12^{th} part of the length, and the length to reach over as a Diameter to the whole Planisphere, you must take care that the fiducial edge be very streight, and at the middle of it a little Semicircle left, whose Center A, being truly placed, upon the very middle point of the fiducial line must be pierced through with a small hole, that so it may be fixed through it, to the Center of the Instrument at Z. Next of all, you are to fit the Ruler for the graduation, which is done by drawing two lines parallel to the fiducial edge; one very neer it, to receive the degrees, the other farther off, to receive the figures for distinction, and numeration.

For the graduation of it, set off the length A B both wayes from A, equal to Z N, the Semidiameter of the innermost Circle of the Planisphere, which is also equal to your first decimal Scale; then the easiest way to graduate it will be by the joynt help of the Canon of Tangents, and your decimal Scale; in this manner; look into the Canon for 45 gr. and it will be 100000, equal to the number of the whole Scale, and those are signified upon the Ruler already by A B, A B; then look half a degree lesse, namely, $44\frac{1}{2}$ whose Tangent is *98269*, take that out of your decimal Scale with your compasses, and setting one foot in A, with the other draw the first division, between the edge and the parallel line next to it, upon both sides the Center A; Then again,

Look

Look the Tangent of half a degree lesse, *viz.* 44, whose Tangent is 96568, which take off, and set it both wayes from A, as before, and thus proceed by half degrees till you have gone down through the forty five first whole degrees of the Canon, and then you shall find that you have inscribed twice 45 degrees, that is 90 parts upon each half of the Ruler, which represent such degrees as here are required, every 10th and 5th of them must be distinguisht from the rest with a longer line, and numbered inwards towards the Center by 10, 20, 30, &c. to 90, as in the Figure sufficiently appeareth. After this is ended, you are to pinne down the Ruler to the Instrument as is before shewed, and then will your Planisphere be fitted to the uses which now follow.

CHAP. II.

An explanation of the Circles, and lines in the Projection.

THe limbe of the Planisphere representeth the Horizon of the place for which it is made; The Diameter N S stand for the Meridian whose sections with the Horizon at N and S, signifie the North and South points of the same Horizon, and the points W and E, being each a quarter of the Circle distant from the former, doe represent the points of East and West, and a Diameter drawn through them and the Center, is the Prime vertical or East and West Azimuth; The Center noted with Z, signifies the Zenith or Pole of the Horizon. Tne Ruler therefore, being fixed thereto, shall represent any Azimuth, or vertical Circle, all which doe passe through the Zenith point, and the degrees and numbers upon the Horizon, will shew what Azimuth from North or South, the Ruler being fixed at any place doth represent. The degrees upon the Ruler denote the degrees of any, or all the Azimuths, and so perform the office of Almicanters, or parallels of altitude above the Horizon.

Within the Projection it self, the point P upon the Meridian signifies the North-Pole, and all the circular lines meeting there

there, (but spreading over the whole superficies) are the Meridians of the Sphere, such as stand for the hours of the place, according whereunto they have their figures set upon them, shewing what hour each for them stand for.

The parallels which crosse through the Meridian, or hour Circles, are the diurnal Arks of the Sun at several times of the year; There are so many of them drawn as the Instrument will well contain, the rest must be supplyed by imagination to passe between them that are drawn; even so many as may answer to every degree of the Ecliptick. And according to that supposition, each 30th. deg. or parallel hath such characters, or Signes annexed unto it, as it doth cut through in the Ecliptick, and the intermediate lines stand for those 30 parallels that passe through the 30 degrees of each Signe, and accordingly must be estimated, and numbred.

The other lines which are inesrted, are not properly of the Projection, neither shall any explication of them be needful, more then when it is treated to shew the use of them.

Chap. III.

The use of the Planisphere, digested into several Propositions.

Some of the Propositions of this Chapter have been delivered by others, what I have added of my own, or omitted of theirs, may easily be found by comparing their Books with this. Their only purpose being barely to perform these things upon the Instrument, and to go no farther; that use indeed may be made of this Chap. but my intent is beyond these, for that which is here performed is premised onely, and prepared for what is to be done in the next Chapter, which is the onely ayme and scope, which these three first Chapters drive at.

I *To find the degree of the Ecliptick that the Sun is in every day, and the parallel belonging to it.*

You may know the degree of the Sun in the Ecliptick (if no better way) well enough for your purpose by remem-

remembring the day upon which in every moneth it entreth into the several Signes, and allowing the motions of it, to be one degree every day, so shall you know how many degrees it is gone into any Signe, or how many degrees it wants to come to the beginning of the Signe, as *August* 7th I know the Sun entreth into ♍ the 13 day, and that the 7th day wants six dayes of the 13: therefore I conclude the Sun to want six degrees of ♍, and so to be in the 24th of ♌; And again, for *August* 16, because 16 is three dayes more then 13, (the day of the Suns entrance into ♍) therefore I say that the Sun is in the third degree of ♍: And these notes gives (neer) their beginning, *Jan.* 11 ♒, *Feb.* 8 ♓, *Mar.* 10 ♈, *Apr.* 10 ♉, *May* 11 ♊, *June* 11 ♋, *July* 13 ♌, *Aug.* 13 ♍, *Sep.* 13 ♎, *Octo.* 13 ♏, *Nov.* 12 ♐, *Dec.* 11 ♑.

Then to find the parallel for 7th of *August* is not hard; For if you remember every Signe hath 30 parallels upon the Planisphere, (either expressed or to be supposed, or supplyed by imagination) you may accordingly find where the 24th parallel from ♌ to ♍ is to be placed, and so imagine a line to run all along even with the rest, and the same shall be the parallel of the day, and the like may be done for all dayes of the year.

II. *At any time to find the Suns Azimuth.*

OBserve the Suns altitude, or in what Almicanter the Sun is by a Quadrant or otherwise, as is shewed in the fourth Chapter by the Semicircle, count this Almicanter, or Altitude upon the Ruler, and (keeping it upon the due coast from South, either Westward, or Eastward, according as you made your observation either in the Morning or Evening) move it till the altitude thereon numbred, doe meet with the parallel of the day whereupon your observation was made, and there fixe it, so shall it lye in the same Azimuth wherein the Sun at the time of observation was, and the numbers in the Horizon or limbe, will give you how many degrees that Azimuth is from the South, if it shall be required: Example, at *London*, latitude 51 $\frac{1}{2}$, observation made *Aug.* seventh, the Sun in ♌ 24, altitude 35 evening, parallel ♌ 24, Azimuth 65 $\frac{1}{2}$.

III. *To*

III. *To find at what Altitudes above, or profundities (or depressions) under the Horizon, every hour circle cuts upon any Azimuth.*

THe Ruler being laid to the Azimuth, as in the former Proposition, or otherwise, will shew the things required of it self. As supposing the Azimuth to be 65 $\frac{1}{8}$ from South toward the West, as was now found out, then shall the hours above the Horizon cut those number of degrees and minutes. Namely, 12 cuts 90, as it ever doth, 1 cuts 79, 2 cuts 63, 3 cuts 43, 4 cuts 17 $\frac{1}{2}$ deg. and these are to be accounted altitudes above the Horizon: then out of the other part of the Ruler, 5 cuts 7 deg. 6 cuts 27 $\frac{1}{2}$, 7 cuts 42 $\frac{1}{2}$, 8, 53, 9, 63, 10, 71 $\frac{1}{2}$, and 11 cuts 80 deg. below the Horizon. And note ever, that from the Center of this Instrument towards that part of the Ruler whereon the altitude of the Sun, and parallel for the day do intersect, I say on that halfe of the Ruler the intersections of the hours are to be accounted altitudes from the Horizon to the Zenith of those very hours that do intersect: On the other part of the Ruler the sections are to be esteemed depressions, or profundities under the Horizon down to the Nadir, not of those hours that doe intersect the Ruler on that coast as they are placed on the Instrument, but of their opposite hours in the contrary part of the Heavens; which they may well do because each opposite hours are equal in this respect, one to the other; and so in our Example though 5, 6, 7, &c. hours lie upon the North-East part of the Horizon upon the morning, yet by them you are to understand the same hours of 5, 6, 7, &c. on the South-West part of the Heavens on the evening tide.

IV. *To find what number of degrees each hour beareth from 12, upon the limb or Horizon of the Instrument.*

THis is easie to be done, for in a Projection for *London*; you shall find 11 and 1, to be distant from 12 at noon, 11 deg. 51 min, 10 and 2 are distant 24 deg. 19 min. 9 and 3, 38 deg. 3 min. 8 and 4, 53 d. 35 m. 7 and 5, 61 d. 6 m. 6 and 6 are distant 90 deg. So likewise, you may find 5 in the

the morning and 7 at night, to be distant upon the Horizon from the South 108 deg. 54 min. and 4 in the morning with 8 in the evening to be distant 126 deg. 25 min. the use of this Proposition will appear in the next Chapter.

V. *To find upon the Planisphere, 1 the parallel of the 12 Signes. 2 The parallel for every length of the day. 3 The parallel of every known declination.*

1 FOr the first, which are the parallels of the beginnings or any other part of every Signe, the Instrument it self will shew readily, because there are the characters of the Signes annexed to them, and these parallels are so framed that they answer to each degree of the Ecliptick as in the structure of the Instrument is declared, and in this Chapter, Proposition the first.

2 For the parallels noting out the just length of day, look into the Ark and chord mentioned Chap. 1 § 7, for those two lines will help you in this fully after this manner. Let the day be 14 hours long, take $\frac{1}{2}$ that length, *viz.* 7, that shewes the time of Sun setting; to this time reckoned in the same chord now mentioned apply your Ruler, so shall it shew you upon the Ark, the place in which the Sun is that time when the day is 14 hours long, which is ♉ or ♌, 29 at *London.* If then according to Proposition 1 in this Chapter, you look for that degree of *Taurus* 1, or *Leo* 29, amongst the parallels there may you affirm the parallel for that length of the day to passe along, so if the length of the day had been 10 $\frac{1}{2}$ hours half that length 5 $\frac{1}{4}$ shewes the time of Sun set. If therefore you look upon the chord of 5 hours 15 minutes of an hour, which is as much as 3 $\frac{3}{4}$ degr. and thereto apply the Ruler, you shall find it to cut upon the Ark of ♎ 22 or ♓ 8 degrees, which are the degrees of the Ecliptick wherein the Sun being makes the day of 10 hours and 30 min. length.

3 The manner to find the parallel upon your Instrument answering to any known declination, may be seen by an Example. Suppose the declination from the Equinoctial to be 15 $\frac{1}{6}$ deg. Northward, count that declination upon the limb of your Instrument from N towards W, and thereto lay the

the Ruler , which will also immediately shew you upon the Ark the Suns place to be ♉ 11 or ♌ 19. And if the same declination had been Southerly, then must you have counted it on the limb from N towards E, and the Ruler there laid shewes upon the Ark the degrees of the Ecliptick answerable, to be ♏ 11, ♒ 19, if then according to the first Proposition, you look the parallels of those degrees in that Instrument, they shall be the parallels of the fore-named declinations.

VI. *The intersection of any hour circle with any parallel being assigned, to find what altitude the same shall have above the Horizon.*

THis is useful for many purposes, (as hereafter is shewed) and most easie to be performed; For having your parallel given, you by the last Proposition, shall see quickly where every hour circle cuts through the same; uuto those intersections apply your Ruler, so shall the degrees of the same Ruler, being counted from the Horizon, shew you the Altitudes required: So in ♉ 1, or ♌ 29, when the day at *London* is 14 hours long, the Suns altitude at 9 or 3 a clock will be 36 deg. above the Horizon: And in ♌ 19 degrees, the Suns altitude at 9 and 3 a clock, will be 38 ½ degrees; and so of all other parallels and houres.

VII. *The descendent point of the Ecliptick being assigned, to find, 1 what point of the Ecliptick is in any hour circle, and 2 what altitude it hath.*

Example LEt the beginning of *Leo* descend at *London*, I would then know what degree of the Ecliptick is in the hour of 3, and in the hour of 10, at that very instant: First, I lay the Ruler upon the beginning of ♌ counted in the ark, where I shall find it to cut upon the chord 7 hours, and 9 degrees, which hours are to be taken for afternoon hours: Now from 3 a clock to 7, and 9 degrees afternoon, are 4 hours, and 9 degrees, which turned into degrees, makes 69 degrees: And so from 10 a clock, to 7 and 9 degrees afternoon, are 9 hours, and 9 degrees, or 144 degrees: These

being fore-known, go to the triangular lines, and because the signe descending is supposed to be ♌, lay your Ruler at ♌ in that line betwixt N and E, and mark where it cuts the limb, namely, at $2\frac{1}{5}$ degrees from 60 towards 50. Now from hence count upon the limb *secundum ser. signorum*, 69 degrees, your first number of degrees, which will fall between N and W upon $11\frac{1}{5}$ degrees, whereto again lay your Ruler, which you shall find to cut upon the triangular line ♎ the 12, almost; And this is the degree of the Ecliptick which is in the hour of 3, when the beginning of ♌ is descending at *London*; And if you apply the Ruler to ♎ 12, in the hour of 3, the altitude of it shall be $21\frac{1}{2}$. Secondly, lay your Ruler again at ♌ in the triangular line, that it may cut $2\frac{1}{5}$ degrees, from 60 towards 50, and from the Ruler so laid, count 144 degrees, which is the number for 10 a clock, so shall the number go from N towards W $84\frac{1}{5}$, whereto if you apply your Ruler, you shall find it to cut about the 25 deg. of ♐, and this is the degree of the Ecliptick that is in the hour of 10, when the beginning of ♌ is descending under the Horizon at *London*, and if you apply the Ruler to ♐ 25 deg. in the hour of 10, you shall find the altitude of it $10\frac{1}{4}$. The like may be done for any other signe, or degree; And remember that when ♌ is descending, then is ♒ the opposite signe ascending above the Horizon, and what is done for the descending of ♌, is likewise done for the ascending of ♒.

VIII. *The culminant point of the Ecliptick being assigned, to find at that time; 1 What point of the Ecliptick is in any hour circle. 2 What altitude it hath there.*

THe culminant point, is that point which is in the Meridian at any time. This work will be somewhat easier then the former, as will best appear by an Example: Suppose at *London* (or any where else, for this first part of the Proposition is general, and therefore a man may make Tables if he list, for this first part of the Propsition, which will serve for all Latitudes) the beginning of *Leo* were culminant, and I would know what degree of the Ecliptick is in the hourci rcle of 8 in the morning: Because from the Meridian to 8 a clock, is 4 hours, or 60 deg. and that forward *secundum*

seriem

seriem signorum, therefore first I apply the Ruler to ♌ in the triangular lines, where it cuts in the limb $2\frac{1}{5}$ deg. from 60 towards 50; from thence I count 4 hours or 60 deg. forwards towards N, which will fall in the limb upon the quarter N, W, or $2\frac{1}{5}$ deg. from N, and then the Ruler shewes upon the triangular line about ♎ 3 deg. to be in the 8 a clock hour: Now the altitude of that point in that hour, is 17 degrees, as the Ruler applyed to it will shew: Again, if the beginning of ♌ be supposed culminant, and I would know what degree is in the hour of 5 afternoon at that same time; because from the Meridian to 5 a clock, are 75 degrees, and that *contra ser. signorum*; therefore having first laid the Ruler upon the beginning of ♌ in the triangular lines, which cuts as before, $2\frac{1}{5}$ deg. from 60 towards 50, in the limb, from whence I count backwards towards E and S, in the limb 75 deg. which will fall upon $4\frac{1}{5}$ deg. from 50 toward 40 in the South Equater, and the ruler being laid here will cut upon the Triangular line on the opposite part of the Instrument about the 19 deg. of ♉, and such is the deg. of the Eclipticк, which possesseth the hour of 5 a clock afternoon, when the beginning of ♌ is in the Meridian.

Then for the second thing which is particular to every latitude, if you apply the Ruler to the 19 deg. of ♉, in the hour of 5, you shall find the altitude of that point to be 23 deg. in the latitude of *London*: The opposite points are in the opposite hours below the Horizon, at the same time when the beginning of ♌ is in culmination, or the beginning of the opposite Signe ♒ is in *Imo Cœli*, as is easie to be understood.

What Propositions soever are here done for hours (as what altitude any thing hath upon hour Circles,) doe the same also upon Azimuths, for there will be need of them hereafter, in putting the Furniture into refracted Water-Dials, &c.

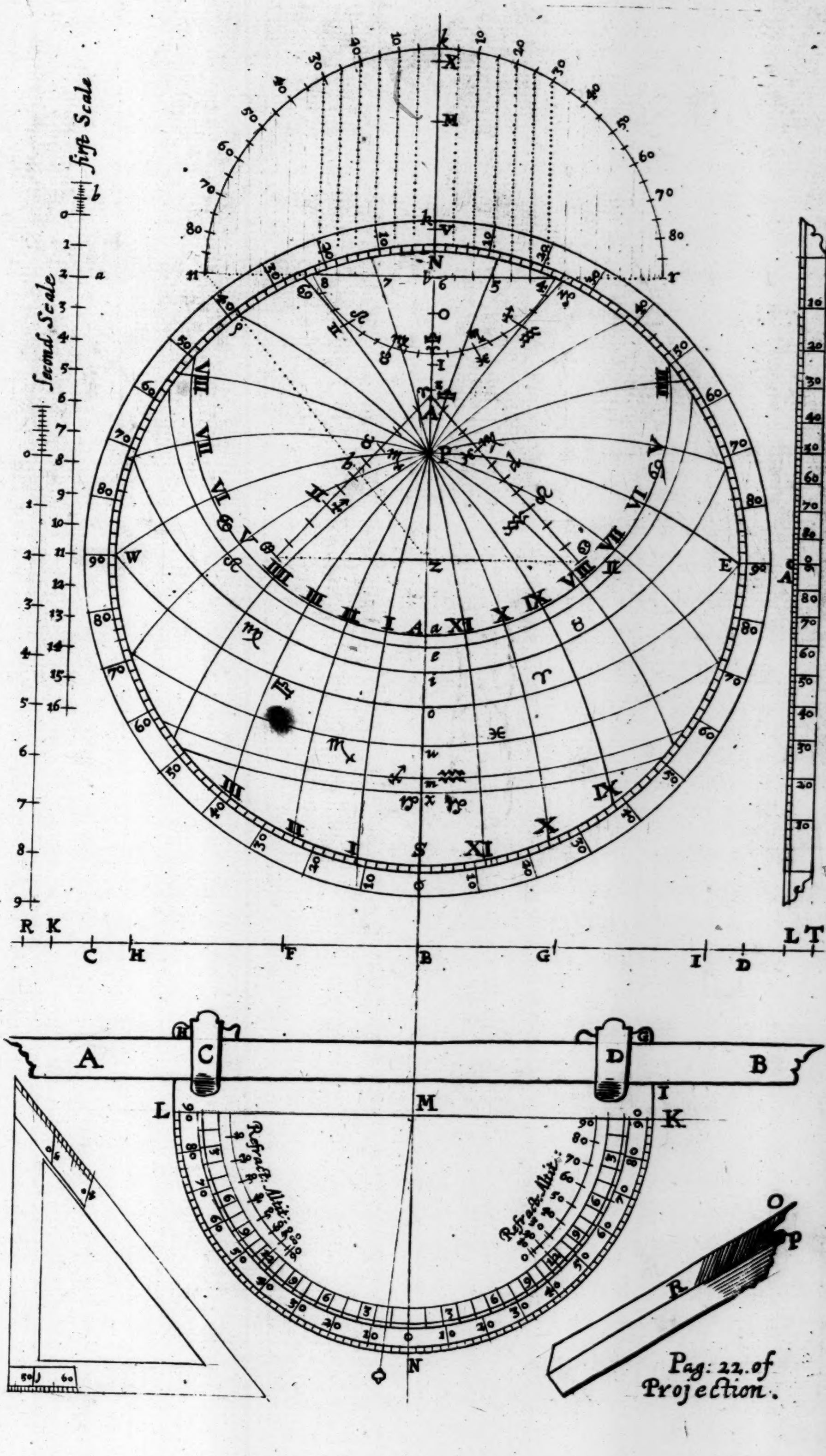
APPEN-

APPENDIX.

The description of the Semicircle.

THe Semicircle it ſelf, and the two Squares that are in it, are ſo commonly known that I ſhall not need to ſay any thing of the diviſion of either of them, eſpecially ſince the figure of it is here ready to repreſent the ſame as fully as can be required; onely remember that I call M N the Semidiameter of it, and L K the Diameter.

The difficulty that is, is in the contrivance of it. The limb above the Semicircle noted with I K, muſt be of ſuch breadth, that if the threed hang upon the Diameter L K, the plummet may have liberty enough without touching the Ruler A B at all: upon that breadth alſo, you are to ſet two loops as at E and F through which the Ruler muſt have juſt room to ſlip up and down as occaſion ſhall be: and that it may be faſtened from ſlipping when it is required it ſhould be fixed, you muſt either make two ſcrues at the back-ſides of thoſe loops, or two wedges, ſuch as are ſignified by G and H, which wedges muſt be ſo ſhaped, that though they be looſed, yet they ſhall not ſlip out, and to that purpoſe, at their leſſer ends they have little knots left as the figure declares. Yet if you draw out the Ruler to turn the other edge of it towards the Semicircle, (as ſometimes of force you muſt) then may the wedges be taken out if need require, and again, firſt, put in before the Ruler, that when the Ruler is put in they may be kept there, and not loſt. The Ruler being thin as of braſſe, or other metal ſuch as this figure repreſents, muſt be ſharpe at both ends of one of the edges, as the Picture ſhews; but if it be of wood, and ſo become of more thickneſſe, then muſt you line the two very ends of the edge of your Ruler with a little plate of braſſe like the figure R O P, laid in ſtreight and even with the end of the Ruler, and at the end of that plate make two ſharp points as O and P doe manifeſt,



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Projection.

manifest, standing even with the two very edges of the upper flat of the Rule; And so the other end of the Rule must be placed upon the same edge in the same manner.

Now insteed of this Semicircle in narrow places, and where room is wanting, may a Triangle of past-board be used for the elevation, or depression of any thing, the figure whereof appeares with the Semicircle.

Let the ruler be about 3 times the length of the Diameter of the Semicircle.

The use of this Semicircle is general, As, upon a line drawn any where, to project any altitude or depression above or below the Horizon, from a fixed point that stands at a distance from that line.

THe manner is easie. For if you hold the edge of your Ruler to the fixed point, and also apply the point of that edge to the line given, removing it higher, or lower, till the threed hanging down by the side of the Semicircle directed to it, at full liberty, doe fall upon the altitude intended, then doth the Ruler lye at the altitude or depth, and project it from the fixed point into the line, as is required: You must in this work (as occasion is) turne the Ruler, and remove your Semicircle, and so in other occasions.

It will be convenient to have 2 or 3 Semicircles of several bignesses.

¶ Note, That wheresoever in the following precepts I mention the Semicircle, a Quadrant so fitted with a Ruler, and divided on both sides, will sufficiently serve the turn.

CHAP. IV.

A general and most easie way to project Hour-lines upon all kindes of superficies without any regard had to their standing, either in respect of Declination or Inclination.

1 LEt a Gnomon, being first sharpned into a point, be shaped, and fastned in such wise, that it no way hinder either the draught of the horizontal line, or the point of the shadow from having free accesse to the Dial at all times of the year.

2 Draw an horizontal line, by help of your Semicircle in a true level both in regard of it self, and also to the point of the Gnomon, through the whole superficies on which the Dial

Dial is to be described. Or having two points in the same level with the point of the Gnomon, project it upon your superficies, if it be a rugged one. And if the superficies be more then one, or if any of them be very much inclined toward the Horizon, or else be very rugged, or far remote from the Gnomon, so that it will not at all, or not so well, receive an horizontal line upon it, you may *Either* set up some board or such like object upon which for a time you are to inscribe the horizontal line, and by help of which the Hours are to be projected upon the superficies; *Or* else (which perhaps will be better) you may extend a threed in the air (it matters not which way, nor whether from the Gnomon towards the Sun, or from the Sun: whether stretcht out in one length, or with returns, so long as it lieth justly parallel, in every point of it, to the Horizon, and in the same level with the point of the Gnomon:) which being fixed in this manner, will very well supply the use of the horizontal line: or the horizontal line may be partly threed, and partly drawn upon the superficies, as occasion shall be. And upon it may any point be transferred, and signed out by slipping knots of threed tyed upon it.

3 Upon the superficies of the Dial, observe the point of the shadow of the Gnomon (making a mark at it) and the Suns altitude, both of them at the same instant of time.

4 By the altitude observed, compute the Azimuth of the Sun from the Meridian.

5 The same Azimuth must be transferred unto, or projected upon, the Horizontal line by help of a perpendicular threed, covering to your sight (as it hangeth down) the points of the Gnomon and shadow both together; and at the same view cutting through the horizontal line: observe then punctually where it cuts through the same line, for that same section being signed thereon, shall be the Azimuth projected into the horizontal line.

6 Let any kind of board or past-board be now applyed to the point of the Gnomon, so, as that it may be staid; either upon the horizontal line (where it may so be conveniently) or at least so placed toward the horizontal line, that it may have a just respect unto it, and in that posture may have some stay for the edge of it to rest upon, that after it is furnished with

with ſuch neceſſary lines as muſt be drawn upon it, it may be placed in its former juſt poſture without any impeachment. Upon this plain ſo placed, let the point of the Gnomon be ſigned, which may be called the Center; and from this Center, to the ſigne of the Azimuth, before projected into the Horizontal line, draw a right line: this right line ſo drawn, ſhall repreſent upon the board or paſt-board, the ſame Azimuth which was before computed.

7 Then taking away the ſame plain, draw upon it the Meridian or line of 12; extending it from the Center before noted, at the true Angle that it hath from the Azimuth before computed and deſcribed, and alſo toward the true coaſt of the World. And let it be extended on both ſides the Center if need be.

8 To the Meridian ſo pitched upon the paſt-board, draw (from the Center) the lines of an horizontal Dial made to that latitude wherein you are.

9 Then again, let the plain board or paſt-board be applyed to its former ſituation, the Center of the horizontal Dial reſting upon the point of the Gnomon, and every thing elſe anſwering to the ſame juſt poſture that it had at the firſt. Which done, let a threed be fixed in the Center of the horizontal Dial, by help whereof you may transferre every hour from the paſt-board into the horizontal line. Let every hour be therein noted (by fixing marks upon the horizontal line where it is drawn, or by ſlipping knots ſet upon the threed, where a threed horizontal line is uſed) eſpecially mark out the hour of 12: For which (if it chance to run beſides the ſuperficies) ſome kinde of object (whereon the horizontal line is alſo to be drawn) or an horizontal threed muſt be faſtned, that may receive it, till ſuch time as your Dial be finiſhed.

10 After all this, take your plain away (for there will now be no more need of it) and conjecture where about the Axis of the world, would paſſe from the point of the Gnomon to the Poles of the World, for into that place is the Meridian to be projected. Which that it may be done more commodiouſly, if no object ſtand in the way that will receive it, you muſt place one there, it matters not whether above or below the Gnomon, chuſe that wch is moſt convenient. Or, a threed laid aſlope in the Meridian juſtly as it ought, will ſerve as well as may be.

If then you hold up a perpendicular threed, so that by your eye you may see the point of the Gnomon, and also the point of 12 in the horizontal line, both together, the same threed so hanging, shall shew where the Meridian is to be drawn. Or, you may extend a threed from the point of the Gnomon to the point of 12 in the horizontal line, which threed shall represent the line of 12: And staying your threed there, close to it, hang up two perpendicular threeds at a good distance, so shall the same two threeds, give you the tracke of the Meridian line.

11 The next work will be to project one of the poles of the world (that namely, which lyes the same way that this projected Meridian doth from the point of the Gnomon) into this Meridian. And this is done by elevating or depressing your Semicircle, from the point of the Gnomon towards the Meridian line, according to the latitude of your place; for so will the Ruler of the Semicircle, or a threed extended along by it, signe out the very pole point. If now you extend a threed from this pole point, to the point of the Gnomon, the same shall represent the Axis of the world.

12 Last of all; By these helps, all the hours may easily be projected. For if the eye do lay, or project, this threed or Axis upon each point of those hours that were inserted before into the horizontal line, the Axis upon an hour point, or a point upon the Axis, each one of those projections shall represent upon your Dial, each of the hours required, and will shew upon every object, that stands in the way, where the hours are to be drawn. Or, where convenient room is wanting to place the eye so as it may make this projection, there may two threeds be used for the same purpose, one whereof must be fastned to the point of the Gnomon, the other to the pole designed in the Meridian line. Then stretching one of the threeds to any of the points noted in the horizontal line, and holding it there, you may take the other, & extend it to the superficies, so as it may closely passe by the first threed, by which work you may make as many points upon your superficies as you please, through which each hour is to be drawn. Having thus traced the way before hand, you may afterward draw the hours without any difficulty, be the superficies never so irregular. Among which lines, the

the shadow of the point of the Gnomon, as it creepeth along, will shew the Time of the Day.

CHAP. V.

Of inserting the usual Furniture into Sun-Dials.

1 AZimuths, or 2 *Points of the Compasse*, may be projected into any Dial directly, as the hours were in this manner. Upon the Plain (whereon you drew the horizontal Dial, and from the same Center therein fixed, describe a Circle; and upon it, set off from the Meridian line, each tenth Azimuth by dividing each Quadrant of the Circle into 9 equal parts, or each point of the Compasse by dividing the several Quadrants into 8 equal parts; and applying the Plain to its first posture, by a threed from the Center of the Circle, project these Azimuths or winds into the horizontal line, making marks in the same line for each one of them, as you did before for the hours. After this, from the point of the Gnomon, set a threed perpendicularly either upward or downward, which may represent the Zenith line, and is therefore the Axis of all the Azimuths. By this threed then, & the points signed out in the horizontal line, you may project the Azimuths or Winds in the same manner as you did the hours before. Or thus: Stretch a threed from the point of the Gnomon, to the several points of the Azimuths in the horizontal line: and note the Nadir point directly under the point of the Gnomon, upon some object laid there for that purpose. Then if with your eye you repose the threed before extended upon the same Nadir point, the shadow or appearance of the threed will shew upon the Dial superficies, shew where the same Azimuth is to be drawn. The like must be done for every Aizmuth or point of the Compasse severally.

3 *Almicantars* may be projected by the semicircle it selfe, without any other help. For if you lift up the Semicircle to such a number of degrees as answers to the Almicanter which is to be inserted, and apply the Ruler of it, being in

that posture to the point of the Gnomon and to each hour-line, or to the several Azimuth lines, or else to any part of the superficies which you will, the same Ruler will signe out points, through which the Almicanters are to be drawn.

4 *Such Almicantars as shew the Proportions of shadowes (cast upon horizontal plains) to their upright bodies*, may be projected in the self-same manner, by elevating the Semicircle to such numbers in the Geometrical Square (which is upon the Semicircle) as answer to the proportions that shall be required. That point of the Square which is $3\frac{9}{10}$ answers to 18 gr. and is the Crepusculum line.

¶ These four particulars may be inserted in this manner generally in all latitudes alike, and are therefore as universal as are the former Precepts for the hours. The rest that follow must have particular Tables framed for them, agreeable to every latitude. The computation of which Tables may be in such manner as is hereafter shewed.

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5 *Parallels of the Suns declination*, 6 *Parallels of the Length of the Day*, 7 *Parallels of the beginning of the twelve Signes*, must first be known what parallels they are from the Equinoctial, or what declination they have, and likewise what altitudes each of them have upon every hour in your own latitude. The parallels of declination are soon found if you determine which of them to put in, as every fifth, or tenth from the Equinoctial, for their declination is according to their number. The parallels of the 12 Signes are these 11 g. 30 m. for ♉ ♍ ♏ ♓ : 20 gr. 12 m. for ♊ ♌ ♐ ♒ : 23 gr. 30 m. for ♋ and ♑: the Equinoctial it self serving for ♈ and ♎. Only it must be remembred which Signes are North and which South, that so they may be placed either above or below the Equinoctial. The parallels for the dayes length of 16, 15, 14, 13, 12, 11, 10, 9, 8 hours, of what declination from the Equinoctial they are, must be searched out (as they shall agree to each particular latitude) in this manner: As the Radius, to the sine of half an hour, that is to the sine of 7 g. 30 m. So is the Co-tangent of your latitude, to the Tangent of the Declination of that parallel, which being

being North, makes the day 13 hours long, or being South makes it 11 hours long. So likewise, As the Radius, to the sine of two half hours or 15 gr. 00 m. So is the Co-tangent of your latitude, to the Tangent of that parallel that makes the Day 14 or 10 hours in length. And as the Radius to the sine of 3 half hours or 4 half hours, that is $22\frac{1}{2}$ gr. or 30 gr. So is the Co-tangent of your latitude to the Tangents of the declinations or parallels that make the Day of 15 and 9, and of 16 and 8 hours length.

Having found such parallels of declination as you mean to use for the three former purposes, you are then to compute upon each of them, the altitudes of the Sun for every hour. And amongst many wayes, let this be one, which is general to them all, and best wrought by the natural Canon, in this manner.

First, for the Equinoctial, which is the line that passeth through the beginning of ♈ and ♎, and from whence all declinations are counted, as also the line upon which the Day is every where 12 hours long, the altitudes for each hour may be found by this Proportion. As the Radius, is to the Co-sine of your latitude; So are the sines of 1, 2, 3, 4, 5, 6, hours, to the sines of the altitudes of the hours 7, 8, 9, 10, 11, 12, in the morning, or of 5, 4, 3, 2, 1, 12, in the afternoon, when the Sun is in the Equinoctial. At 6 the Sun is just in the Horizon. Now for inserting the Equinoctial line upon a plain superficies any two altitudes for two such hours as are at a convenient distance, will serve turn; because the Equinoctial being a great Circle of the Sphere, is projected upon a plain into a streight line, and two points are sufficient to direct where to draw a streight line upon a plain. But if the superficies be manifold or uneven, all the altitudes must be made use of, or two altitudes and the point of the Gnomon will shew the Equinoctial superficies, and so it may be projected with a threed.

Secondly, for all other parallels this course may be taken.

1 Find out the sines of the altitudes of 6 a clock in all North parallels by this Proportion; As the Radius, to the sine of your latitude; So is the sine of every declination, to the sine of the altitude of 6 a clock in that parallel of declination. By this sine found, and entred into the Canon of sines, you may get the altitude of 6 for every parallel.

2 For the same North parallels; adde the declination of your parallel to the complement of your latitude, the sum will be the altitude of the Sun for 12 a clock in that parallel. Then out of the sine of this altitude of 12, take the sine of the altitude of 6, reserving the *Difference.*

3 As the Radius, to this Difference; So the sines of 1, 2, 3, 4, 5, hours, to several fourth numbers, or sines.

4 To every one of these fourth numbers, adde the sine of the altitude of 6; So shall the several sums produce the sines of the altitudes for every hour between 6 and 12.

5 Take as many of those fourth numbers as you can, out of the sine of the altitude of 6; so shall the several remainders make the sines of the altitudes of such hours as are between 6 and Sun-rising, or Sun-setting.

6 Take the sine of the altitude of 6, out of all such of the fourth numbers, as are bigger then it, so shall the remainders give the sines of the altitudes of the Sun upon such South parallels which have the like declinations from the Equinoctial, that these North parallels have.

¶ Thus having found out the altitudes required in each kind, they must be ordered into Tables, and reserved for use. And if according to the usual manner of working by the Semicircle, you insert from the point of the Gnomon into the particular hours such altitudes as your Tables afford, you shall find pricks through which to draw each requisite parallel.

Of Signes of the Ecliptick Ascending, Descending, and Culminating.

IF you would insert the Signes into the hour lines, you must find out what altitudes the intersections of the Ecliptick have with the hour Circles (two of them at the least, to set them upon a plain, but more are better, that they may serve in all cases, and to all superficies) at that moment of time, when the beginning of any Signe is Ascending, Descending, or Culminating, which will be found a hard calculation. It would be as easie to find what altitudes the Ecliptick hath at those times with some chief Azimuths.

But

But the most easie way, that I know, will be to find out what amplitude the beginning of every Signe, rising or setting, hath; and what altitude the Ecliptick at the same time cutteth upon the Meridian. And for Signes Culminating, it must be enquired what altitude the beginning of each Signe hath when it is in the Meridian, and what amplitude also it hath at the same time upon the Horizon.

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8 Then for *Signes Ascending*. If ♈ ascend, then is the amplitude 00, and ♑ is in the Meridian, and so the Meridian altitude of ♑ is the altitude of the Ecliptick upon the Meridian whilest the first point of ♈ is ascending. So if the first point of ♎ be ascendent, then likewise the amplitude will be 00, and ♋ will be in the Meridian; so that the Meridian altitude of ♋ is the altitude of the Ecliptick upon the Meridian, whilest the beginning of ♎ is ascending. For the other Signes, to know what altitude the Ecliptick cuts upon the Meridian at their ascent above the Horizon, there must be inquired, 1 their Amplitude; 2 the Oriental angle, or the angle made berween the Ecliptick and Horizon at the same time.

1 The Amplitude is thus known; As the sine of the latitude, is to the sine of the declination of the beginning of any Signe; So is the Radius, to the sine of the amplitude from the East. This for North signes being added to 90, for South signes subducted from 90, produceth the amplitude reckoned from the South.

2 The Oriental angle, is thus found. As the co-sine of the declinalion of the point ascending, is to the sine of your latitude; So is the Radius, to the sine of the angle made between that Meridian that passeth through the point ascending, and the Horizon. This angle added to the angle made by the same Meridian and Ecliptick, gives the true Oriental angle. Now the angles made by the Ecliptick and Meridians that passe through the beginning of each Signe, are these ♈ 113 gr. 30 min. ♉ ♓ 110 gr. 38 min. ♊ ♒ 102 gr. 16 m. ♋ ♑ 90 gr. 00 m. ♌ ♐ 77 g. 44 m. ♏ ♍ 69 g. 22 m. ♎ 66 gr. 30 m. these, I say, are the angles before mentioned, which in these Northern latitudes and while they are in the ascendent,

ascendent, doe look upwards from the Horizon toward the Zenith and North Pole, or towards the ark included between them. But their supplements must be taken in South latitudes. And although the Oriental angle doe fall out to be obtuse, and the Tangent of it is used in the next work, whereas Tangents serve no further than 90, it is to be remembered here that any ark and the supplement thereof have one and the same, as Sine, so Tangent, and Secant also.

3 As the Radius, to the Tangent of the Oriental angle; So the sine of the amplitude from the South, to the Tangent of the Eclipticks altitude upon the Meridian. Now these altitudes upon the Meridian being computed for ♈ ♉ ♊ ♋ ♌ ♍ ♎ will be sufficient; for ♓ ascending, the Ecliptick hath the same Meridian altitude that it hath when ♉ ascends: and ♒ the same with ♊, and ♑ with ♋, ♐ with ♌, ♏ with ♍.

¶ The two Tables then of amplitudes and Meridian altitudes being framed, you may by them insert the 12 Signes ascending in this manner with least trouble, though enough too. Piece out your horizontal line by a returning threed where need is; and upon it project the amplitudes of the ascending Signes from the South, amongst the morning hours. They must be protracted first upon a plain or past-board as the hours and Azimuths were before, and from thence transmitted to the horizontal line, and marks or knots set thereunto. Then if the Meridian line be there all is well; but if it be not upon the Dial superficies, you must, for a time, draw or stretch one in the aire by a threed placed in the plain of the Meridian in such manner as that it may receive what is now to be inserted into it. Into the same Meridian therefore, by help of your Semicircle, insert the several Meridian altitudes of the Ecliptick, and set marks at them. After this, you may without any great difficulty, project the several positions of the Ecliptick, thus: Stretch a threed, fixed at one end to the point of the Gnomon, to the several marks set in the horizontal line, and at every such extent let your eye repose the threed upon that point in the Meridian which answers there to the same Signe that the threed was extended

unto

unto in the horizontal line, so shall the shadow of the threed shew you upon the Dial, where the line for that ascendent Signe is to be drawn. And so having projected them all (12 in number) you may at the East end, among the morning hours, write, *Signes Ascending* with the characters of those set upon each of them, which properly belong unto them: and, among the evening hours, write *Signes Descending*, setting upon each line the characters of those Signes that are opposite to the former, because when any Signe is ascending, the opposite is descending.

Descending Signes then are put in by the same work that ascending are.

Note that in Dials that look towards the North, you must by your Semicircle project the same Meridian altitudes upward, above the horizontal line, and not downwards as in Dials looking towards the South.

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9 For *Signes Culminating*. You must first find their Meridian altitudes, which is easily done for the beginnings of every Signe. For having their declinations before set down, you must, if they be North Signes, adde their declinations to the height of the Equinoctial, or to the complement of your latitude, or in South Signes, subduct the declinations out of the complement of your latitude, so the numbers produced will be the Meridian altitudes of the beginnings of the twelve Signes. Secondly, you must seek what amplitudes the Ecliptick hath, when the beginnings of the twelve Signes are in the Meridian. To which purpose also, the acute angle made between the Meridian that passeth through the beginning of each Signe, and the Ecliptick, must be had in readiness: and they are these, ♈ ♎ *66* gr. 30 m. ♉ ♍ ♏ ♓ *69* gr. 22 m. ♊ ♌ ♐ ♒ 77 gr. 44 m. ♋ ♑ *90* gr. 00 m. And likewise it must be noted, that any Signe from ♋ to ♑ being in the Meridian, the *Ortive Amplitude* of the Ecliptick from the South is lesse then *90* gr. the *Occasive* more. But any Signe from ♑ to ♋ possessing the Meridian, the *Ortive amplitude* is from the South more than *90* gr. the *Occasive* lesse. Now then the amplitude is found by this proportion; As the Radius, is to the sine of the Meridian altitude of the beginning of any Signe; So is the Tangent of the angle at the Meridian

(ſet down before for every Signe) to the tangent of the Eclipticks amplitude at that time from the South. The amplitudes *ortive* of the Ecliptick when ♋ & ♑ are in the South are alwayes 90 gr. and if you enquire the ortive amplitudes of ♌ ♍ ♎ ♏ ♐, their ſupplements are the ortive amplitudes for ♊ ♉ ♈ ♓ ♒, remembring the cautions given before. And the ortive amplitude of the Ecliptick from the South when any Signe is culminating is equal to the occaſive amplitude of the Ecliptick from the South when that Signe that is as much diſtant from ♋ as the fore-named Signe was, is culminating.

¶ The Tables of the Eclipticks amplitudes from the South, and Meridian altitudes being fitted, you muſt now accommodate your horizontal and Meridian lines as you did before for aſcending Signes; and then among the morning hours from a plain board or paſt-board, project your amplitudes into the horizontal line for the 12 Signes, and their Meridian altitudes into the Meridian line by your Semicircle. And being thus prepared you may project the Eclipticks ſeverally into your Dial ſuperficies, and character each line with that Signe that belongs unto it, and with the character of the oppoſite Signe that is in *Imo Cœli* at the ſame time.

By help of the Parallels of the length of the Day may be inſcribed theſe that follow.

10 *Hours from Sun-riſing.* 11 *Hours from Sun-ſetting.* 12 *Planetary hours.* 13 *The ſix Houſes that are above the Horizon.*

The Eaſterne part of the Horizontal line is the beginning of the hours from Sun-riſing, as the Weſtern part is the beginning of the hours numbred from yeſter-days Sun-ſet. Look then for any two parallels of the Dayes length that are of equal or even number (and not odde) as 8, 10, 12, 14, 16: and if your Dial be deſcribed upon a plain, count upon any two of thoſe parallels the firſt hour from the horizontal line, and draw a ſtreight line through thoſe

two

two points: the ſame line if it be from the Eaſt part of the Horizon is the firſt hour from or after Sun-riſing, if from the Weſt it is the 23 hour from yeſterdayes Sun-ſet. So the right line drawn through theſe two ſecond points from the Eaſt part of the Horizon is the ſecond hour from Sun-riſing, or from the Weſt part it will be the 22 hour from yeſter dayes Sun-ſet which are accordingly to be figured. And ſo of all the reſt.

For the Planetary hours, chooſe out the parallels of the dayes length 15 and 9 hours; and in the firſt take each 5 quarters from the Horizon, in the ſecond each 3 quarters, and draw ſtreight lines through them if the ſuperficies be plain, the ſame lines are the Planetary hours, the Meridian being 6, the Weſt horizon 12. But in all theſe, if the ſuperficies be not plain, but either many plains together, or one curved and irregular, you are to ſtretch a threed ſo as that you may ſee the two points for each hour before menttioned, and the point of the Gnomon all together upon the threed; then ſhall the ſhadow of the threed in that poſition expreſſe where every ſuch hour-line muſt be drawn.

For the Houſes, find out by your Semicircle, that point in the hour of 12 that is level with the point of the Gnomon. If then your Dial be upon a plain ſuperficies, draw ſtreight lines from the fore-named point through each ſecond hour point in the Equinoctial line on both ſides 12; the ſame lines ſhall be the 6 houſes above the Horizon, and the Meridian line is the tenth houſe. But if the ſuperficies be curved, hold a threed ſo as that you may ſee through it the fore-ſaid two points of each houſe, together with the point of the Gnomon; for then the ſhadow of the threed will ſhew to your eye where each Houſe is to be drawn.

14 *Of the Riſing, Culminating, and Setting of any fixed Star.*

Suppoſe the ſtar to be *Lucida Pleiadum.*

The declination of the Star Northward is 23 gr. 00 min. the right aſcenſion 51 gr. 42 m. Firſt then, get the Semidiurnal ark of the ſtar by this proportion, As the Co-tangent of your latitude, to the Tangent of the ſtars declination: So is the Radius, to the ſine of the ſtars aſcenſionall difference:

difference, which being added to 90 gr. (because the declination is North, else it should be subtracted) gives the stars semidiurnal ark. For *London* it would be 122 gr. 15 m. This taken out of the stars right ascension leaveth (289 g. 27 m.) the right ascension of *Medium Cœli* when the star is rising. Or the semidiurnal ark added to the stars right ascension, gives (173 gr. 57 m.) the right ascension of *Medium Cœli* when the star is setting. Then lastly also, the right ascension of the star, is the right ascension of *Medium Cœli* when the star culminates. Now having gotten these right ascensions, you may get the points of the Ecliptick, their declinations, and the angles of Ecliptick and Meridian answerable, in this manner. As the Radius, to the sine of $66\frac{1}{2}$ gr. So is the Tangent of right ascension, to the Tangent of the point of the Ecliptick answerable. As the Radius, to the sine of right ascension, So the Tangent of $23\frac{1}{2}$, to the Tangent of the declination of that point to which the right ascension belonged. As the Radius, to the sine of $23\frac{1}{2}$ gr. So the co-sine of right ascension, to the co-sine of the acute angle made by the Ecliptick and Meridian.

Then note, that if the right ascension of *Medium Cœli* be in the second or third quarters of the Equator, the *Ortive amplitude* of the Ecliptick from the South is lesse than 90 g. the *occasive* more. But if the right ascension be in the first or last quarters, then is the *Ortive amplitude* more than 90, the *occasive* less. —— Having found *Medium Cœli*, say, As the Radius, to the sine of $23\frac{1}{2}$; So the sine of *Medium Cœli*, to the sine of the declination of *Medium Cœli*. By this declination compared with the altitude of the Equator, you may also find the altitude of *Medium Cœli*, which is the Meridian altitude of the Ecliptick. Then again, say, As the Radius, to the sine of the Eclipticks Meridian altitude; So is the Tangent of the angle between the Ecliptick and Meridian, to the Tangent of the Eclipticks amplitude.

In this manner also may the appulse of any fixed star to any Azimuth, or Almicator, or Meridian, or any other standing Circle, be computed and inserted; If namely, the situation of the Ecliptick at that same moment be projected upon the Dial.

These

These being found, will help to put in such lines as shew the stars ascension above the Horizon, Descension, and Culmination. The manner of putting them in, is the very same that was used before for inserting the Signes of the Ecliptick *Ascending*, *Descending*, *Culminating*, so that more words about it will be needlesse.

These are the principal things wherewith Sun-Dials are usually furnished. If these be well understood, it will not be hard to insert the Cosmical, Acromychal, or Heliacal rising and setting of stars, or any such like requisite. All the several uses of each kind of lines is shewed by the shadow of the point of the Gnomon, as it creepeth along through them.

Chap. VI.

Let this stand as a briefer and lesse troublesome way, than the former: The Problem may be propounded, more generally then before in this manner.

If a point be assigned upon any superficies flat or curved, one, or more, wherein the hour-lines and Axis shall concur, how to project the hours to that point, and to set up an Axis after the ordinary manner to give shadow to them without any knowledge how the Dial standeth, in respect either of declination or inclination.

Remember to doe it to a horizontally laid superficies, where the point assigned is below the horizon, and so no horizontal threed can be fixed in æquilibrio to it.

1 To the point assigned (upon any side of it) by direction of your Semicircle or other level, stretch out an horizontal threed, serving for the horizontal line; this horizontal line need not be one direct line, but may be turned at one or more angles, provided that it lie totally in the superficies of the Horizon.

2 With a perpendicular threed held up, project the Sun into the assigned point, and into the horizontal threed, and tie a little mark of threed upon the same horizontal, through which the shadow cutteth, at the same instant also take the Suns altitude.

3 By

3 By the altitude taken, find out the Azimuth; This Azimuth, what ever it be, is represented by the knot.

4 Apply a paſt-board to the aſſigned point, and hold it flat that it may anſwer to the horizontal threed alſo, and upon this paſt-board protract your Azimuth by a threed extended from the point aſſigned for the Center, to the mark upon the horizontal threed. This done,

5 By help of that Azimuth upon your paſt-board, protract the Meridian line, obſerving the true coaſt, and quantity of the angel from the Azimuth: and to the Meridian deſcribe an horizontal Dial.

6 Applying the paſt-board to its place again, all things ſtanding right as before, project all the hours into the horizontal threed from off the paſtboard, and ſet marks upon the ſame for the points of each ſeveral hour which marks may be little movable knots to ſlip too and fro upon the ſame threed.

7 Project the Meridian point by a perpendicular threed upon ſome object into that place whereabouts you imagine the Axis of the world would paſſe, above or below from the point aſſigned for the Center.

8 With your Semicircle elevated or depreſſed (as it ſhall be required) from the point aſſigned for the Center, according to your latitude project the pole of the world.

9 Extend a threed from the point aſſigned for the Center to the poles of the world, which ſhall repreſent the Axis.

10 By the point upon the horizontal threed, and this Axis (either by your eye, laying the Axis to the hour-points, or laying the hour knots to the Axis) you may project all the hours and draw them, Or elſe you may let the Axis alone, and content your ſelf with the pole-point projected into the Meridian, for if from the point aſſigned to be the Center or meeting of the hours and Axis, you extend a threed to each hour point in the horizontal line, and do repoſe (with your eye) the ſame threed upon the pole-point, then ſhall the ſhadow of the threed give you that hour-line, and doe ſo in all the reſt.

11 Your threed or Axis lying in its true ſituation, you may eaſily fit an Axis to the ſame poſture. If your Dial be deſcribed upon a plain ſuperficies you may then (by one ſide of

of a Normal Square, applyed to a threed or Axis, and the other side lying upon the plain) find out the substile, and measure from it the elevation of the Axis above the plain: But if the Dial be described upon a curved superficies you mustbe content to set up your Axis by the direction of the threed onely.

12 This point assigned for the Center being a point of the Axis, is as it were the Apex of the Gnomon, unto which all the worke is projected. But if it be required to set up an Axis to such a superficies, upon which the Axis and hours will not meet in any tolerable manner, because perhaps the Axis may be but of very small elevation above the superficies, and yet an Axis is required: in this case, set up any point (of wire, or such like) of such distance from the superficies, as that the Axis and hours may be distinct: And through that point let it be required to make the Axis passe, you have no more to doe but onely to project to this point, as before, by letting the shadow of a perpendicular threed passe through that point, and noting the same upon your horizontal threed, and counting that end of the wire as your Center, proceed as before, for the threed that lies to project the hours is a pattern for the Axis.

This way is as general as the former, serving to project the hours upon many superficies be they plain or curved, and however situate whether contiguous, or seperate, and that without any laborious inquisition of any of their situations, in respect of inclination or declination. If you will put in that furniture which is usual, you must make some mark (notch, or button) upon your Axis, unto which (as representing the Center of the world) by help of your Semicircle you are to project the altitudes of such great or lesser Circles as you intend to insert; For which purpose you may make use of an Astrolabe, or my Ruler, or rather you may calculate Tables for your own latitude, which shall supply you with such altitudes, as are requisite to put in, in each particular.

Propositions in the first way 1,2,3,4,5,6,7,8,9,10,11,12, These are to project to an Apex.

Propositions answerable in the second way, 1,2,3,4,5,6,7, 8,9,10,11,12, These are to an Axis.

Upon

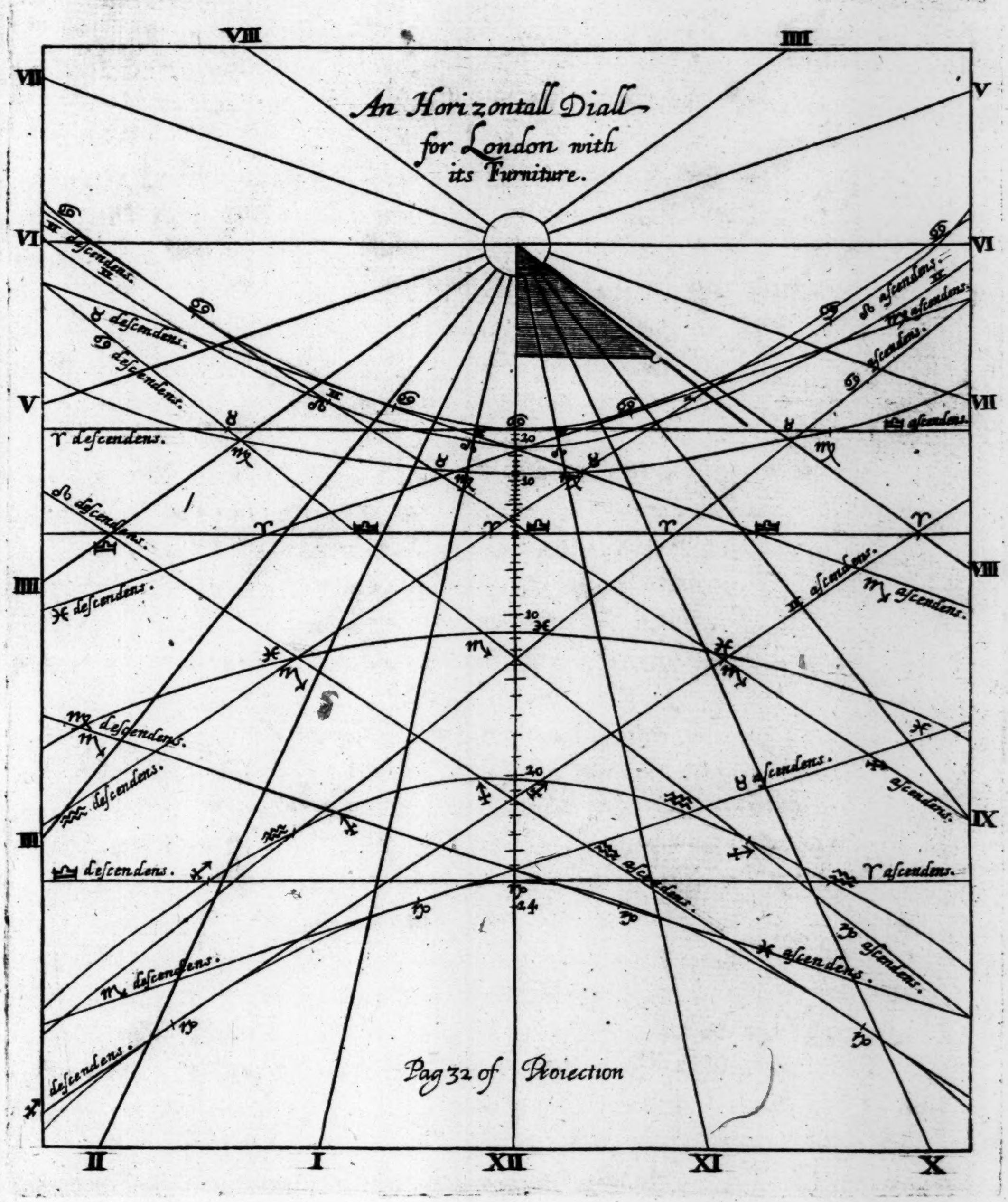
Upon a plain (but not upon a curved superficies) to make a Dial with an Axis, to any point assigned for the Center.

FIrst, project a Dial to the point of a Gnomon, the projective way; then having assigned your Center, from it, draw hours parallels to the projective ones, so are you furnished with hours. For the Axis doe thus:

Note, That it must passe through the Center of the Dial, and must be parallel to that Axis that was drawn from the point of the Gnomon.

Now to set it absolutely parallel, it must be remembred, that when the gnomonical Axis is reposed upon the Center of this new drawn Dial, it must also cover the new Axis, that is, to your sight, must lie just under it, and being limited to that superficies you may the more easie stretch a threed from the Center, parallel to the gnomonical Axis: Or use your Semicircle being elivated to your latitude, and kept in rhe fore-named superficies.

FINIS.



An Horizontall Diall for London with its Furniture.
VIII
IIII
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Mr. Samuel Foster his PRECEPTS, concerning REFRACTED DIALS.

THe Suns beames are refracted by any transparent body that they fall upon. If the same be more dense, or more thin and rare then the *Medium* through which they first shine. Such bodies are either *Solid*, or *Liquid*: in both which kinds the most common bodies are, Glasse, or Christal, and Water.

Refractions (as we are here to use them) may be divers wayes considered. First, in Solid bodies, the superficies refracting may be either Plain, or Curved, and this either truly regular, such as is Spherical, or such like; or else various, of no determinate regular form. And likewise the plain especially (but the other also in some sort) may be either Horizontal: or otherwise placed, upright, or leaning.

Again, In Solids Pellucid, the rayes of the Sun from the point of an Index standing without side the Pellucid between the Sun and it must passe, either first from the Index through the Aire, and then into the Solid, and so meeting with an opacous body, those joyned to the out-side of the transparent body, may there be terminated, and so suffer

but one refraction at its first entrance into the Pellucid; Or else the Opacous body (upon which the shadow of the Index stayeth, and pierceth no farther, but is made visible) may stand at some distance from the Pellucid, and so the Suns rayes passe out of the Pellucid into the Air again, before they come to the Opacum. By which meanes they suffer a double refraction, one at their entrance into, the other at their going out of, the Pellucid body.

Or further, The point of the Index may be within the Pellucid, and so the Suns beames must first enter into the Pellucid, and suffer one refraction, before it comes to the point of the Index, and afterwards, either meet with an Opacous body, close joyned to the outward superficies of the Pellucid, and so suffer no more refraction, but be there terminated; Or else, If the Opacum stand at a distance from the Pellucid, the Suns beames must again passe through the Air, and suffer a second refraction (at their going out of the Pellucid) before they meet with the Opacous body, or dark superficies that stayes them.

All these cases are varied according as the Index and opacous, and pellucid bodies doe stand.

Or again. The point of the Index may stand without the Pellucid, and the Suns beames be twice refracted through both superficies before they come to the point of the Index, the Pellucid being interposed between the Sun and the point of the Index, and the point of the Index standing between the Pellucid and Opacous bodies.

Secondly, For Water, or any such transparent liquid, the varieties are not so many, Because the superficies of it, is alwayes level with the Horizon, and because likewise the liquid applies it selfe contiguously to the Opacous body or Vessel that containes it; onely besides one fraction, the irregularity of the Vessel that containes the Water is troublesome. How the refraction by Water alone can be but one, which is at the Suns beames entrance into the water; But the variety of projecting the lines of the Dial is two-fold, according as the Index-point may stand either within or without (that is above) the Water.

Many varieties besides the irregularity of the opacous bodies that are to receive the lineaments, which of themselves are infinite.

But if water be put into a Glasse or any such Pellucid Vessel, then may the varieties be as many as were the former of Solids in respect of the situation of the Index, Pellucid, and Opacum. Yea, and more, because before, the Pellucid was

was simple and simular, but this Pellucid mixt or dissimilar; So that the refractions are here multiplyed into four varieties, or breaches (whereas the other had but two) *causa ipsius mixiones vel compositiones duorum pellucidorum:* the first fraction is at the entrance into the Glasse; The second, at the going out of the Glasse into the liquid; The third, at the going out of the liquid, and entrance into the Glasse; The fourth, at the going out of the Glasse into the Aire.

Now all these complications of infinite varieties, gather such an incomprehensibility, or innumerable number of difficulties in drawing hours, so many wayes, and quantities refracted, that it will be thought to exceed the comprehension of humane reason to accomplish it, especially being so infinitely varied by the irregularity of those superficies that are to receive the lineaments: If all the cases mentioned were intermingled, there would be no end of varieties.

And thus may the varieties be still augmented, by making refractions through more pellucides at once.

And because the quantity of the several refractions, at their several incidences are unknown, and although they were known, yet by reason of the irregularity of most Pellucid solids, the angles and coasts of incidence would be altogether unknown, and in that regard, the refractions both in quantity and coast unknown also; In all these regards, it is altogether impossible to give any Rule, either by Calculation, or Geometrically by drawing lines, how the hours should be delineated.

By the way note, that the Index must of necessity be a point, not a line or Axis.

In Water (indeed) where the superficies is both a true plaine, and also lying truly Horizontal, the varieties will be fewer, and so the work more easie. But of this, I will speak afterward peculiarly, because things necessary in this kind may be more vulgarly had (being more obvious) and the way much more easie in it self, though commonly also thought to be exceeding difficult, being esteemed as a rarity above the common apprehension and performance. And if this that is easiest be so esteemed off, what shall the former (so difficult) be accounted off, being involved in such a innumerable number of various varieties.

Of refracted Sun-Dials in Water.

How to draw them by the Semicircle, and Plainisphere, joyntly together.

1 THe refractions to all inclinations or altitudes in water must be had, as I have framed a Table for that purpose, which is here inserted.

A Table of Refracted Altitudes to each 5th. gr. of the Quadrant.

True Altitudes.	*Refracted Altitude.*	
gr.	gr.	′
0	41	28
5	41	42
10	42	26
15	43	37
20	45	14
25	47	13
30	49	32
35	52	08
40	54	58
45	58	00
50	61	12
55	64	32
60	68	00
65	71	32
70	75	09
75	78	49
80	82	31
85	86	15
90	90	00

2 The Vessel that holds the water, may be of any fashon, regular or irregular, it matters not, but it must be furnished with every 10th. or 5th. Azimuth as need shall be; the manner whereof in briefe may be this. Set the Vessel so upright as it must stand when the water is in it; And assume a point for the South, andover against it (in the same Horizontal level) an other for the North, both opposite to each other in respect of the point of the Gnomon, which must first of all be fixed; that is, having taken one point for the South, in the same level with the point of the Gnomon: (for to it, an Horizontal line, is first to be drawn in the Vessel, or else,extended by a threed)from that South-point extend a threed, out right over the point of the Gnomon: which will find the North-point on the other side of the Vessel.

Afterwards in the Horizontal line (drawn round about the Vessel, or otherwise represented with threeds conveniently) by help of a Past-board set upon the Gnomons top, and by help of the North and South-points, you may project each 5th. or 10th. Azimuth, and make marks in the same Horizontal

zontal line for each of them. This being done, by the Semicircle, find out the Zenith-point in the Vessel, that is, apply the Ruler to the Gnomons-point, and holding it upright there, the foot of it will shew the point required, for the Vessel now standing (and as it must be justly afterwards placed) in this posture. Then lastly, if you lay a threed to each Azimuthal point, and to the apex of the Gnomon: and so to the opposite point of each Zenith (two being alwayes opposite, one to another, and may well go together) you may repose this threed upon the Zenith-point lately found; So shall the umbrage of the threed shew all along the Vessel, where the same Azimuth is to be drawn; And the same is to be done in all others. Or without drawing, or projecting either, threeds may be fixed for Azimuths from the points in the Horizontal line to the Zenith.

3 These things being thus prepared, it is left to choice whether the point of the Gnomon shall lye alwayes hidden in Water, or else stand above the Water. These two cases are very different, and therefore must be treated of in several, as two distinct cases.

When the Gnomon *is hidden all under Water.* I.

IN this case you are not tyed at all how full to make your Vessel, onely be sure to cover the Gnomons point, it matters not how much, whether more or lesse, for both are as one. Then for the line of 12, that is already drawn, being the same with the North and South Azimuth, but the rest must be inscribed by points severally fixed into each particular Azimuth, the manner whereof may be this.

Upon your Plainisphere lay the Ruler to any Azimuth, (as the $60^{th.}$) from the South, and there see what degrees of the Ruler (or what Altitudes) each hour-circle cutteth, and write them down in a Table; Thus doe upon every $10^{th.}$ or $5^{th.}$ Azimuth (as you shall think fit) making a Table of all those Altitudes, or intersections, Then coming to the Table of Refractions, and to each particular Altitude of your Table, finde amongst the Refractions, how much belongs to each of them, and adde the same to the Altitude (before found by the projection) particularly, so shall you have turned the direct Altitudes into refracted ones.

After

After all this, come again to your Vessel, and with the Semicircle insert each particular refracted Altitude into his proper Azimuth whereto it belongeth, so shall you have points in each Azimuth, for so many hours as the same Azimuth is capable off: Having then these helps, through each point belonging (in every particular Azimuth) to the same hour, as suppose the hour of 9, draw one continued curved line, which must serve for the hour of 9 a clock, so through all the points in every Azimuth serving for 8, drawn one continued line, which must in like manner serve for the hour of 8 a clock; and so do for all the rest. The Horizontal line will be about 37 gr. below the point of the Gnomon, so much, namely, as the Horizontal refraction cometh unto, and up to this Horizontal line (and not any higher) must the curved hour-lines be drawn. The coasts of North and South will be opposite (in the Vessel) to those of the Heavens, in the same manner here, as they are in other Dials. This work cannot be done by projecting the hours with help of an Axis, as in other projections, for neither the rayes from the eye, can possibly fall upon the Water, to project in the same manner that the Suns beames doe, (which in direct projections is not requisite, but in refracted it is) nor the projections made by the Suns beames themselves, (though of the same hour-circle) will be the same in fashion, the Sun standing in several positions to make this projection, as in one instance in a right Sphere will sufficiently appeare: For in a right Sphere the Axis (as all know) must lye parallel to the Horizon, or superficies of the water, and the hour of 6, will be the same with the Horizontal line; If therefore we suppose such an Axis in a round Spherick concave Vessel, full of water to be laid from one side of the Vessel to the other, and the Sun to rise or set in the Equinoctial, which is proper to the Axis, then shall the hour of 6, or the one half of the Horizon be projected dipping down (from each point of the Axis projected by the parallel raise of the Sun) so much as the Horizontal refraction comes to (about 37 gr.) whence it must follow that this projection of the Horizon must dip most under the Axis in the projected Equinoctial Circle, & nothing at all under the two ends of the Axis, which concur with the sides of the concave Vessel, whence the Sun

being

being in the Æquinoctial, as we now suppose it to be, the Axis and Horizon or 6 a clock line, in a round Vessel would appear as in the first Figure.

Fig. 1.

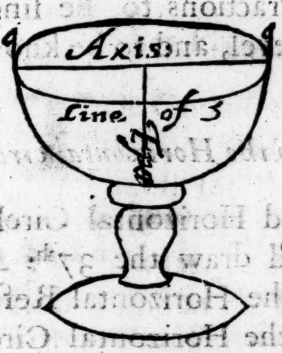


Fig. 2.

But again, if the Sun being out of the Æquinoctial, as in one of the Tropicks, and there be supposed to rise and set, then shall the Horizon or hour of 6, be projected so by the Sun, as that a ray from the Sun upon the middle point of the Axis shall project the Horizon in the lowest point; which lowest point will be in the projected Tropick, and not in the Æquinoctial, as the former projection was. So that now the Axis, and the line of 6, will appear in the same Vessel in the form of *a o b*, whereas before it was like *a æ b*, whence first it is evident, that an Axis cannot project (in all positions of the Sun to the Axis and water) one single line for each hour, as they ought to be. And therefore that way by an Axis is in this work to be rejected as unservicable. Secondly, It follows also, that (though the Sun should (by reason of infinite remotenesse) makes requisite lines yet) the eye cannot in this kind perform what the Sun doth, because it being alwayes neer to the superficies of the water, doth receive distances from several parts of the same superficies of different quantities greater and lesser, & so the rayes passing from the eye doe make several angles of inclination, and consequently several refractions, which the Sun by his immane distance doth not, but is thereby freed from it; So that the hour-lines cannot be projected (by help of an Axis) with the eye in the same fashion that the Sun requires, nor yet if they could, would they be of any use, as is before said.

ſaid Thirdly, Again it follows, that a point onely (and not a line, or part of the Axis) is to be uſed for a Gnomon. Fourthly, That the inſcription of the hours, muſt needs be done by finding certain points through which they are to be drawn; One way of which protraction is now delivered, This former way ſuppoſeth the refractions to be ſingle upon a Meridian lying in a Horizontal level, and to be known alſo, according to all inclinations.

How to make the hours cloſe right with the Horizontal circle.

YOu muſt firſt draw the refracted Horizontal Circle, which is all one as if you would draw the 37th. Almicanter (for about 37 gr. is the Horizontal Refraction, and ſo much therefore muſt the Horizontal Circle dip under the point of the Gnomon in the water) ſo that I need ſay no more of that. Then may you divide this Horizontal Circle into ſuch parts or degrees as the ſpaces of an Horizontal Dial will require, aud into thoſe diviſions muſt the ends of your hour-lines run. Alſo above this Horizontal line nothing needs be drawn, for it is of no uſe, the point of the Gnomon will never grow higher. Likewiſe it will be moſt convenient to fill the Veſſel with water up to the brim, in this caſe here propounded where the Gnomon lyes hidden under water, and ſo alſo to make the brim 37 gr. (at moſt, but fewer degrees is beſt) above the point of the Gnomon, which your Semicircle will doe; for by theſe means the Sun ſhall have free acceſſe to the Dial ſo long as it is above the Horizon, which otherwiſe will not poſſibly be.

And here note, That if the Refracted altitudes be inſerted into your Semicircle, out of the Table of Refractions in water, and ſo made into a Scale or Limbe; if this (I ſay) be done, then may you immediately, (without turning your direct altitudes into refracted, according as is preſcribed in the precedent pages) put in the ſame things in the ſame manner and quantity, if you count theſe altitudes in your refracted ſcale (and not in the common limb) and accordingly doe inſert them all your threed and plummet hanging upon the altitudes taken in the ſame ſcale; So will the former labour of turning one into the other

other be taken quite away, And so much will serve for this first case, when the Gnomon is quite covered under water: The second follows, which is

When the point of the Gnomon stands above the Water. II.

1 Per Planisph. 2 Per project. Ocularem.

1 THe Gnomon being set, and the Vessel fitted (as is before prescribed) with Azimuths convenient, you must set the Vessel upright according to the self-same posture that you intended it should have when it is filled with water, and in that situation let it be fixed, till your work be done at the least.

2 Next you are to consider how high you will fill it with water, for to that altitude you must draw an exact true Horizontal line upon the sides of the Vessel, the very same that the edge, or superficies of the water will make when it is filled up to it; This is necessary to be done first, as also you must draw another horizontal line about the sides of the Vessel, which must be in equilibrio with the point of the Gnomon, and this will be (most conveniently) the very edge of the Vessel, that so the Sun (all the time that it is above the Horizon) may have accesse to the Gnomons point, and shew the hour too, both which cannot be, unlesse the Vessels brim be just in equilibration with the Gnomons point.

3 Between this brim of the Vessel, and the water horizontal line, is part of the Dial to be drawn, by direct projection; And below this, namely, where the water filleth up, is to be drawn the rest of the Dial by refracted projection; And accordingly we are to give distinct Rules for both.

4 For the upper part, it may be delineated either by the horizontal Planisphere, and the Semicircle, or else by projecting it with an Axis.

I

BY the Planisphere, you may find what altitudes are due to every hour, upon every Azimuth: And by the Semicircle, you may put them into the right Azimuths, and so from point to point draw the hour-lines till you come done to the water

horizontal line; And for the upper ends of the hours to make them fall true into the brim of the Vessel, you must doe as before in the former work was done. That is, you must describe (in the brim or horizontal line of the Dial) the spaces of an horizontal Dial, and in those points or spaces must the hour begin to issue forth. So again, for the lower ends of the direct hour-lines to find the very points into which they are to run upon the water horizontal-line, the work will be either harder or easier according as the Vessel, and standing of the Gnomon are regular or irregular. — For if the Vessel be round, and the point of the Gnomon doe stand just in the Center of it, then it will be easie to doe it, for then the water horizontal-line is a true Almicanter; And by your Semicircle you may know what Almicanter it is. If accordingly therefore you consider upon your Planisphere how many degrees of that Almicanter are comprehended between 12, and each hour, and insert the same spaces, or degrees, into that same Almicanter, or water horizontal-line, those points shall be the terms of the hours into which they must come. But if the Vessel be not regular, or though it be, if the situation of the Gnomon be not regular to it, then it will be difficult; And indeed so difficult, that it is not *operæ pretium* to use it, I will together referre it to this next projective way of putting in this superiour part of the hours which will perform this thing easily. But by the way, after you have inserted the one part of the hour-lineiby drawing it, as I now shewed, you may continue it whether you will by projection as I have heretofore shewed, and so you may continue it downward unto the water horizontal-line.

2

THe second way is (without the Planisphere) by projection. And this is done in the same manner that I have often heretofore shewed, either by an Axis, and horizontal points, or else by the Æquinoctial points, and for these you need draw no Azimuths, or else by Azimuthal points, put into two Azimuths, which only are necessary to be done; I need not therefore make repetition of it here again. So then the upper part of the Dial above the water is described.

5 The

5 The lower refracted part which lyes within the water, may alſo be done two wayes, either by the Planiſphere, and Semicircle, or elſe by Projection alone.

1. *By the Planiſphere.*

SEek how much each hour is elivated upon every ſuch Azimuth as is deſcribed in the Veſſel, & by the Table of Refractions turned into refracted altitudes, as was before ſhewed; So theſe two altitudes may be called, The firſt, *The direct*, and the later, *The Refracted altitude.* Or when you come to inſert theſe by your Semicircle, for the direct altitudes, you may count them upon that limb which is divided into equal degrees, and the refracted altitudes, you may inſert by that limb, which is made for refracted altitudes by water. And ſo you muſt underſtand me when I bid you to put in the direct altitude, and the refracted altitude, that is, to count the ſame altitude in the direct, or equally graduated limb, and in the limb of refractions, and ſo you ſhall need no Table of refractions, becauſe this new inſerted limb performs the uſe of the ſame Table immediately, without any turning of one altitude into another. Both altitudes we are here to uſe. Firſt, therefore, we ſuppoſe the Veſſel ſet as it muſt ſtand when it is filled with water, and in this ſituation, look what Azimuth you mean to deal with, or into which you intend to inſert the hour-points, from the ſame Azimuth noted in the water horizontal line, and in the true horizontal level, and juſt alſo under the point of the Gnomon, which is to ſay juſt in the Zenith line (or from the Azimuthal point in the water horizontal line to, or directly towards the interſection of the water horizontal plain with the Zenith line) ſtretch a threed, and (having firſt put upon it a bead that may ſlip up and down, or elſe a ſlipping knot may be put on afterwards) there faſten it: After this is done, by help of the Semicircle applyed to the point of the Gnomon, put upon that threed (as being the Azimuth) the direct altitude which you mean to inſert, and thereto ſlip your knot or bead; then again from this bead down unto the ſame Azimuth, drawn upon the Veſſel ſides, project (with your Semicircle or Rulers edge applyed thereto) the refracted altitude, and there make

a mark, for in that mark must the hour (whose altitude you now insert) run, the same work you are to doe for all the hours and their altitudes that passe through this Azimuth; and the like must be done in other Azimuths, also for the same hour-points. Then lastly, having found points for every hour, you may through those points draw the hour-lines, and so finish up the Dial in every particular.

2 *Without the Planisphere, by projection.*

BUt now the Vessel must be filled up to the water horizontal line, & be in all points fitted as when it is really to shew the hour of the Day, which being so prepared you shall need to inscribe no Azimuths at all into the Vessel, but as in other projecting of hours, so here do thus. Make two points for North and South, and set the hour-points upon the brim of the Vessel which you take to be in æquilibrio with the Gnomons-point (however put those points into the horizontal line which is in equilibrio with the Gnomons point, or if there be none drawn in the Vessel, set threeds there round about it, as the manner of other Dials hath been, and into them insert knots or hour-points) and erect an Axis as in other Dials; Then project (as you use to doe) the Axis upon those points, and with some stile or dent make a mark where the point of the Gnomon is reposed through the water, upon the side of the Vessel, which mark shall serve for one point through which to draw that same hour. Then removing your eye a little higher or lower, still repose the Axis upon the same hour-point, and mark again the place upon which the point of the Gnomon seems to lie, for this also will be another point through which the same hour is to be drawn: Thus remove the place of your eye so often, and doe the same work over, until you have found points sufficient to finish the draught of the whole line; In the same manner you must find points, and through them draw each of the other hours. This kind of work is necessary for that part of any hour which lyes under the water, but for the part above the water, that is projected at one view, as hath been before shewed; for that not going down into the water at all is freed from refraction.

Remember

Remember also that your Axis must alwayes go above the Gnomons-point, and keep in the Aire, but at no hand goe down into the water. To the water it may goe, and be fastned below too in the water, but my meaning is, you must not then project by that part of it which is within the water, because the refraction will deceive you. And be careful that the projecting part of the Axis (namely, all that which lies above the water) do lye at the true Elevation of your Pole, and that you project onely by that same part. And thus have we finished these two cases, which were to shew: How to draw hours in a Vessel of water, where the Gnomon lyes within the water, or where it stands above it. Now if besides the hours any shall, in these two cases, desire

To put in the other furniture also.

THey may in breife doe it thus. It must be remembred that all furniture is to be put in by the Planisphere, and Semicircle, as I have already shewed; And that all things that way are put in by altitudes, such as in each kind the Planisphere will help unto. The very same manner of work is here again to be used, onely in the first case you must altogether use refracted altitudes; and in the later case, you must use both direct, and refracted altitudes, one after the other.

For the first case then it will be as easie as if you were to work in the way heretofore taught by the Planisphere, applying the Ruler of the Semicircle to the point of the Gnomon, and to the hours, onely you must remember to count all your altitudes in the refracted limb of the Semicircle and not in the common limb of equal degrees, because all, both Gnomons-point, and hours, are under water: and this will be enough to admonish concerning the first case, where all things are totally refracted; Or you may put all in by the Semicircles projecting upon the Azimuths, not hours, as followes in the other way. Then again, in the second case, where the work is partly direct, and partly refracted. So much of your work as is above water may be furnished with direct projection as hath been shewed heretofore in the use of the Planisphere and Semicircle. But for the

the other part which is below in the water, there are ſeveral wayes to be uſed; but the beſt will be to project all upon the Azimuths that were at firſt preſcribed to be drawn upon the Veſſel ſides and ſo all will be eaſie, whereas otherwiſe they will be very hard. Having then by the Planiſphere found ſuch altitudes upon the Azimuths as are requiſite, you are then prepared to put the ſame in; but it muſt be by uſing the ſame way that was before put in practice for the inſcription of the hours; namely, thus. Let your Veſſel have no water at all in it, but yet ſet true, as hath been before preſcribed; Then from any Azimuthal point in the water horizontal-line, to the interſection of the water horizontal ſuperficies, with the Zenith-line falling from the point of the Gnomon, ſtretch out a threed and faſten it there; and upon it let be put a ſlipping knot or bead: Then look what altitudes you have to put in, (for parallels of Æquinoctial, or Almicanters, or Sections of the Ecliptick with the Azimuths, or any ſuch like) the ſame muſt be put into the threed firſt, by applying the Ruler of the Semicircle to the Gnomons-point, and fitting it up till (the ſide of it alſo touching the threed) the Plummet hang at the direct altitude of the equal limb, then to that point of the threed, where the edge of the Ruler croſſeth it, ſlip your knot or bead, afterwards again, apply the edge of the Semicircle to this knot, and keeping it ſtill there cloſe to it, lift it up till the Plummet, and the end or point of it, keep alſo in the Azimuth whereto the threed is annexed, that part of it I mean which goes up to the ſide of the Veſſel into the water, ſo ſhall the end or point of the Ruler, give you the point of the refracted altitude required. Thus doe till you have found all the points of ſuch things as you mean to put upon that Azimuth, and then goe to another Azimuth and do ſo there too, untill you have done as much as you deſire; Then laſtly, through every correſpondent point, draw ſuch lines as you require. This is the ſum of what is to be done in this Caſe.

And note here, that if it be ſo that the Altitudes of ſome things cannot be had upon the Azimuths by the Planiſphere (ſuch as are thoſe things that concern the motions of

of the Ecliptick) in such a case you may (by the Planisphere) find in what part the Ecliptick cuts any hour Circle as is shewed before in the use of the Planisphere, and thereto apply the Ruler of the Planisphere, which will shew you in what Azimuth this shall happen; this Section (I say) of the Ecliptick with the same hour. If therefore, you put in that same Azimuth into the horizontal line, and project it into the Vessel, you shall find the same point of intersection with the hour, and through that point must the Ecliptick Circle passe; The like may be done for all the other points of intersection. And this you may doe without finding what altitudes the Ecliptick hath upon any Azimuth, which I beleeve the Planisphere will not doe very well: Therefore, in such cases this direction may be ready, or else take that way which is adjoyned to this, if you think not much of your labour, whereto that way will put you.

This way I have given as the best, but I feare the Planisphere will not doe (as I said) his part. I have therefore added this note to help forward the businesse, & that all may be the better known, or understood at least, I will add another direction here, which every one may refuse or use at his pleasure as he shall like. And this is projectively without the Planisphere. Therefore, now again, fill your Vessel with water as full as it must be, and having the Axis and the hour points found and placed as even now they were, you may on the projecting side (that is on that side on which you do stand when you project any point from the Gnomon to the Vessel, or on that side which the Sun is on when it casteth this shadow) from some superiour point of the Axis (or from the supream point of the whole axis) stretch out a threed, and with your eye repose it, and the Axis and the point of any one hour, all three in one, & in that same position fasten your threed; This done, find upon this hour (by your Planisphere) such altitudes as you require, and from the Gnomons point insert them into the new fastned threed by help of your Semicircle, and there tie knots upon the same threed for marks; Then come to project the same knots, which is done by reposing with your eye those same knots upon the point of the Gnomon, and in that position, both those

those points will be reposed also, upon the sides of the Vessel within the water, Observe therefore, where those points are reposed by the eye, and upon the Vessel sides (with some bodkin or dent) make a mark, for that must be the projected point, answering to that upon the threed from whence it was projected: In the same manner are the other points to be projected, and marked, and so you are to deal with other hours too, onely for each of them you must place a new threed, and furnish it with knots, as before was done. This may serve for direction in this way; Other wayes a man may find out of himself as necessity shall put him to it, and therefore I will mention no more here.

As concerning Azimuths, it may be observed that they onely, of all other Circles, suffer no refraction by water, because they all stand perpendicularly to the superficies of it; And therefore they are already put in, as is prescribed in preparing the Vessel for the rest of the work, both for hours, and other furniture, for in both these their help is requisite. But when all inscriptions are made, if they prove cumbersome to the rest of the work, by filling it over full, they may then in such cases be wiped out.

FINIS.

☞

THE WHOLE ART OF REFLEX DIALLING,

Shewing the way to draw all manner of Dialls which ſhall ſhew the hour by a Spot of light reflected from a Glaſſe upon any Cieling, or other Object whatſoever, without any reſpect had to the Axis of the World, either projected or reflected.

AS ALSO

Whether the Glaſſe lie parallel to the Horizon, or oblique unto it.

TOGETHER WITH ALL NECESSARY FURNITURE BELONGING THEREUNTO.

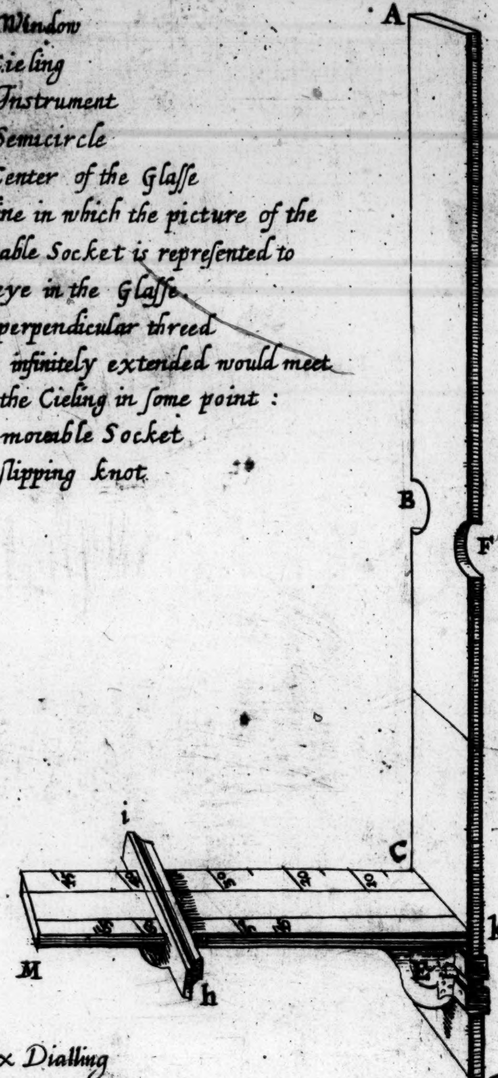
All performed by an eaſie Inſtrument fitted with lines to that purpoſe.

By JOHN TWYSDEN, *M.D.C.L.*

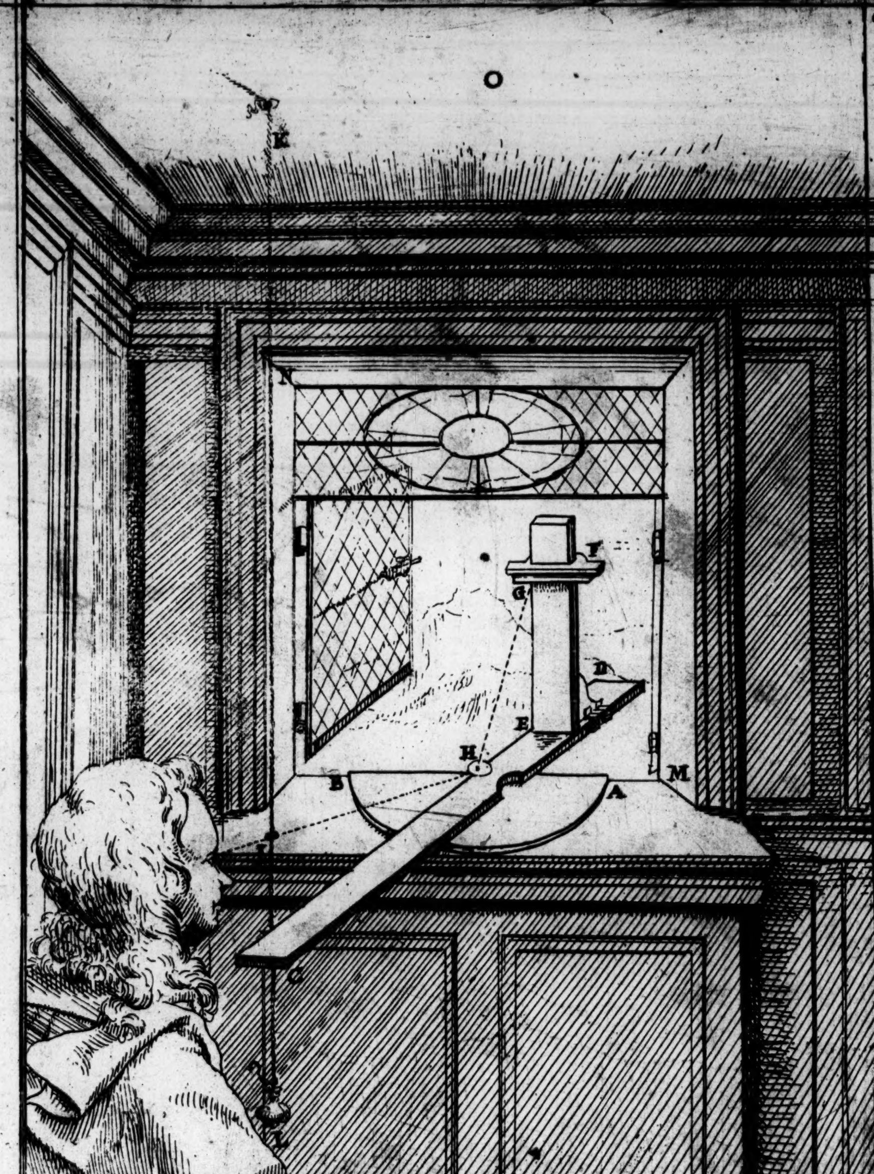
LONDON,
Printed by R. & W. LEYBOURN.

M. DC. LIX.

M.N . the Window
O . the Cieling
C.D . the Instrument
A.B . the Semicircle
H . the Center of the Glasse
H.I . the line in which the picture of the moveable Socket is represented to the eye in the Glasse.
K.L . the perpendicular thread
H.I . being infinitely extended would meet with the Cieling in some point :
F.G . the moveable Socket
I . the slipping knot



The figure of the Instrument



CHAP. I.

The Description of the Instrument.

LEt there be a streight Ruler of Wood, or Brasse made A G, the length, breadth, and thicknesse, at discretion: about the middle of it, or neerer to the end A, let the hollow B be made large enough to encompasse a socket of Brasse, into which the Glasse must be fitted, and so that the fiducial edge A B C, may be imagined to passe through the Center of the Glasse, when it is fixed. On the other side, as at F, may be made another hollow, like that at B, to the end you may use either edge of the Ruler, as occasion may serve, to the end of this Ruler must be added another at right angles C M, made moveable, yet so supported by a bracket E, behind, that it may stand steady at right angles, and unto this let there be fitted a slipping socket with a fiducial edge *h i*; let the piece C M be divided as a tangent line to the Radius B C, and of that length that it may contain about 47, or 48 degrees, which you need not divide beyond 45. On the other side K M, to a shorter Radius, let the tangent line be continued to 64 degrees, or thereabout; which will be farre enough for most Dials of this kind, the whole representing two sides of a Rectangular Paralellogram, or Carpenters square, the one legge longer than the other, all which by the figure annexed, is easily understood.

CHAP. II.

Precepts for the ready Use of this Instrument.

FIrst, in the place where you intend the Glasse shall lye, make fast some piece of Wood or Brasse, exactly Horizontal, unto which you may joyn some other large piece of Board, Pastboard, or other, it matters not, so as it be made to stand firm, and Horizontal, till the Dial shall be finished, and then taken away.

Secondly, Having upon any part of your fixed piece of Wood made a mark, over which precisely shall be the Center of your Glasse, upon this mark as a Center describe so much of a Circle as is necessary, to as large a Radius as the Pastboard will give way, and then the Sun shining hold up a threed, so that the shadow of it may passe through the Center of your Circle, and mark where it cuts the Circumference, and at the same instant take his altitude, and find his Azimuth, either trigonometrically, or by some Astrolabe: (of all projections of the Sphear, I know none so exact for the performance of all things necessary for the making these Dials, and the solution of all other Astronomical Problemes, as that commonly called *Blagraves* Jewel, now put out, every way much amended, and altered by Mr. *John Palmer*, Rector of *Ecton* in *Northampton* Shire my especial friend.)

Thirdly, Having found his azimuth, set off now the South or East line, by help of a Scale of Chords made to the Radius of your formerly described Circle, we will take the Example of an East Dial; As for Example, in the latitude of 52 deg. 15 min. I observed in the Tropick of *Cancer* the Suns altitude 15 deg. 00 min. By my Astrolabe I find his azimuth, then from the East, or six of clock line was 19 deg. or 71 deg. from the Meridian or Midnight line Northward, but because in this Example the Meridian could not be expressed, I set off 19 degrees upon my Circle to the right Coast, and there through the Center draw a line which shall represent the East azimuth.

Fourthly, Your East or Meridian line, if it may be, being thus drawn, have recourse to your Astrolabe, or by Trigonometry find these ensuing things. First, for all necessary houres which will come upon the Dial, find the Suns azimuth, and likewise what altitude it hath iu that hour, and azimuth, do this for the Tropick, the Horizon (in Dials made to Oblique Glasses) the Æquinoctial, or for as many of the Suns Parallels as you please, I have made choice of the distance upon the Horizon, and Tropick of *Cancer*, for in a flat roof two are enough, because the hours will be streight lines, otherwise if the roof be concave, convex, or any way uneven, it will require the finding of more points, write these down, as in the Table ensuing. *In*

In the Latitude of 52 degrees, 15 minutes.

Distances from the East on the Horizon.			
Hours	*deg.*	*min.*	
4	36	00	*From East Northward.*
5	18	40	
6	00	00	
7	18	40	*From the East Southward.*
8	36	20	
9	51	40	
10	65	30	
11	78	20	
12	90	00	

In the Tropick of Cancer.					
Hou.		*Azim. from East.*		*Suns Altit.*	
H.	*m.*	*D.*	*m.*	*D.*	*m.*
4	00	37	30	02	00
5	00	25	40	10	00
6	00	15	00	18	30
7	00	03	30	27	30
8	00	09	00	37	00
9	00	22	30	45	30
10	00	40	00	53	30
11	00	62	30	59	20

The Suns Azimuth, Altitude, and Amplitude, for every hour in the Æquinoctial and Tropicks, calculated from 50 to 56 gr. of Latitude.

t.50 d.00′ Tro.♋			*Æquinoctial*		*Tropick ♑*		*Horiz*	*Lat.51 d.00′ Tro.♋*			*Æquinoctial*		*Tropick ♑*		*Horiz*
urs	*Azim*	*Altit.*	*Azim*	*Altit.*	*Azim*	*Altit.*	*Amp*	*Hours*	*Azim*	*Altit.*	*Azim*	*Altit.*	*Azim*	*Altit.*	*Amp*
4	37.24	00.37					37.00	4	37.24	1.13					36.38
5	26.19	8.48					19.17	5	26.10	9.14					19.02
6	15.36	17.47	00.00	00.00			00.00	6	15.18	18.03	00.00	00.00			00.00
7	4.53 *n*	27.15	11.36	9.35			19.17	7	4.20 *n*	27.20	11.45	9.22			19.02
8	6.49	36.53	23.52	18.45		☉ *set*	37.00	8	7.31	36.46	24.09	18.20		☉ *set*	36.38
9	20.12	46.15	37.28	27.02	49.16	6.24	52.33	9	21.20	45.53	37.50	26.25	49.40	5.38	52.09
10	37.26	54.41	53.00	33.50	62.04	11.30	66,09	10	38.37	54.04	53.25	33.02	62.10	10.57	65.50
11	60.38	61.02	70.44	38.23	75.45	15.18	78.24	11	61.31	60.09	70.57	37.26	75.49	14.20	78.14
12	90.00	63.30	90.00	40.00	90.00	16.30	90.00	12	90.00	62.30	90.00	39.00	90.00	15.30	90.00

.52 d.00′ Tro.♋			*Æquinoctial*		*Tropick ♑*		*Horiz*	*Lat.53 d.00′ Tro.♋*			*Æquinoctial*		*Tropick ♑*		*Horiz*
urs	*Azim*	*Altit.*	*Azim*	*Altit.*	*Azi.*	*Alti.*	*Amp*	*Hours*	*Azi.*	*Alti.*	*Azi.*	*Altit.*	*Azim*	*Alti.*	*Ampl.*
4	37.23	1.50					36.14	4	37.21	2.26					35.52
5	26.01	9.41					18.47	5	25.52	10.07					18.33
6	14.59	18.19	00.00	00.00			00.00	6	14.40	18.34	00.00	00.00			00.00
7	3.43 *n*	27.25	11.55	9.10			18.47	7	3.18 *n*	27.28	12.05	8.58			18.33
8	8.19	36.37	24.28	17.56			36.14	8	9.03	36.28	24.45	17.31			35.52
9	22.16	45.31	38.14	25.48	49.24	4.53	51.46	9	23.12	45.08	38.37	25.11	49.27	4.07	51.23
10	39.41	53.26	53.46	32.13	62.15	10.04	65.32	10	40.42	52.47	54.08	31.25	65.19	9.11	65.15
11	62.20	59.16	71.13	36.29	75.53	13.12	78.05	11	63.05	58.23	71.27	35.33	75.56	12.24	77.55
12	90.00	61.30	90.00	38.00	90.00	14.30	90.00	12	90.00	60.30	90.00	37.00	90.00	13.30	90.00

.54 d.00 Tro.♋			*Æquinoctial*		*Tropick ♑*		*Horiz*	*Lat.55 d.00′ Tro.♋*			*Æquinoctial*		*Tropick ♑*		*Horiz*
urs	*Azim*	*Altit.*	*Azim*	*Altit.*	*Azim*	*Altit.*	*Ampl.*	*Hours*	*Azim.*	*Altit.*	*Azim*	*Altit.*	*Azim.*	*Altit.*	*Ampl.*
4	37.19	3.03					35.31	4	37.16	3.39					35.11
5	25,42	10.33					18.19	5	25.32	10.59					18.07
6	14.20	18.49	00.00	00.00			00.00	6	14.00	19.04	00.00	00.00			00.00
7	2.49 *n*	27.31	12.14	8.45			18.19	7	2.13 *n*	27.34	12.23	8.32			18.07
8	9.47	36.14	25.02	17.05			35.31	8	10.29	36.08	25.19	16.40			35.11
9	24.06	44.44	38.58	24.34	49.30	3.21	51.02	9	25.00	44.19	39.19	23.56	49.31	2.36	50.41
10	41.40	52.08	54.29	30.36	62.23	8.18	64.58	10	42.36	51.28	54.50	29.47	62.28	7.24	64.41
11	63.48	57.29	71.40	34.36	75.59	11.25	77.46	11	64.28	56.35	71.53	33.39	76.02	10.27	77.37
12	90.00	59.30	90.00	36.00	90.00	12.30	90.00	12	90.00	58.30	90.00	35.00	90.00	11.30	90.00

Lat. 56 d. 00' Trop. ♋			*Æquinoctial*		*Tropick* ♑		*Horiz*
Hours	*Azimut*	*Altitu*	*Azim.*	*Altit.*	*Azim.*	*Altit.*	*Ampl.*
4	37.13	4.15					34.51
5	25.21	11.25					18.55
6	13.40	19.18	0.0	00.00			00.00
7	1.31*n*	27.36	12.32	8.19			18.55
8	11.10	35.57	25.35	16.14			34.51
9	25.53	43.53	39.40	23.17	49.33	1.50	50.20
10	43.31	50.46½	55.9	28.58	62.31	6.31	64.25
11	65.6	55.41	72.5	32.42	76.5	9.29	77.29
12	90.00	57.30	90.0	34.00	90.0	10.30	90.00

CHAP. III.

Having gone thus farre, your next work will be to fasten your Glasse in its socket, to what obliquity you please, at adventure, and so to order all things that the Center of your Glasse may be directly over the Center of your formerly described Circle, and the heighth of the Center of your Glasse equal to the thicknesse of your Instrument, so that the hollow part of the Ruler encompassing the socket, the fiducial edge may passe through the Center of your Glass, which you may mark with a little speck of ink, till your Dial is done.

The hours are to be drawn in this manner: First, get the points where the hour-lines shall cut or touch the Horizon in the cieling, by which points the Horizon it self may at the last be drawn. These points you shall get, as in this example in the latitude of 51 deg. 00 min. when the Sun riseth at four, I find by the Table annexed in the Column belonging to that latitude, that his amplitude or distance from the East Northward is 37 deg. 19 m. Place therefore the Radius of your Instrument to that amplitude or Azimuth marked before in your circle upon the horizontal board, & the socket being set to the Suns altitude, which is 00 deg. 00 min. observe with your eye where the fiducial edge of the socket in the point of intersection with the altitude, will be reflected from the middle of the Glasse, which you shall find alwayes in the same Azimuth if the Glasse be horizontal: but if the Glasse be oblique to the Horizon, the reflection will swerve toward the Pole Zenith of the glasse more or lesse as the obliquity is. Hang a threed or fasten it in any place, so that holding

holding it between your eye and the glasse, it may catch this reflected socket where ever it comes, and where it cuts the threed tie a slipping knot. Now a threed extended from the Center of the Glasse, by this knot, to the cieling, shall touch the point where the hour-line of four is to cut the Horizon. In like manner, you shall find the points for 5,6,7,8, if need be, and if you will also, for 9,10,11, and 12, working by the Amplitudes of the several hour-lines, as you did by the amplitude of four. A line drawn through these points shall represent the reflected Horizon, if you shall have a desire to draw it.

Then lastly, go to your Table for the Tropick of *Cancer*, and in the Azimuths marked in your Circle, and belonging to every hour you inteud to draw, place the Radius of your Instrument, as before you did for the intersections of the hours with the Horizon, and move the socket in the upright Ruler of your Instrument to the degree of altitude belonging to that hour you intend to draw, which you shall find in your Table calculated for the elevation of the Pole from 50 deg. to 56 deg. and with your eye reflect it by help of a threed hung up any where, and held between your eye and the Glasse in the same manner as you did the reflected Horizon, and where a threed extended from the Center of the glasse by the knot touches the cieling, that is the point for that hour, and a line drawn from thence to its correspondent in the Horizon, shall represent the line where the reflected spot of light will be for all the yeer.

As for example: In the latitude of 51 deg. 00 min. I find by my Table that the Suns amplitude or azimuth from the East Northward in the Tropick of ♋ is 37 deg. 19 min. at the hour of four. There I place the Radius of my Instrument, and move the socket to 1 deg. 13 min. the Suns altitude in that hour, then the Instrument remaining in this situation, I reflect the socket as before was shewed. This you must repeat for such hours as you intend to draw, and finish your Dial if you think fit.

❡ Note, When you cannot readily find the image of the socket in the glasse being narrow, you shall lay a broader piece upon the narrower, and having found it in the broader (which will soon be done) keep your eye upon it till some

body

body removes the broader Glaſſe, and you ſhall eaſily find it in the narrower, for there about it will paſſe.

Note alſo, That if you find not your latitude in the Tables, you muſt work a proportional part, in this manner: Suppoſe I deſire to draw a Dial in the Latitude of 51 d. 32 m. and would find where the hour of four interſects the Horizon I find not that latitude, but find 50 d. 00 m. and 51 d. 00 m. In 50 d. 00 m. I find the amplitude at 4 h. 00 m. is 37 d. 24 m. In 51 d. 00 m. it is 37 d. 19 m. their difference is 5 m. As therefore 1 d. 5 m. :: 32 m. will be to 2′ 40″; which being ſubducted out of the amplitude belonging to the latitude of 50 d. 37, 24, ſhall give you 37 d. 21′ 20″, the amplitude required. Or, adding it to the amplitude of 51 d. 00 m. you ſhall find the ſame thing.

CHAP. IV.

THe parallels of Declination, of Altitude, the Azimuths, Proportions of the ſhadowes to their gnomons, and the like, commonly called, *The Furniture of Dials*, may be eaſily inſerted by this Inſtrument, if any man ſhall deſire it. Though to ſpeak my own judgement, I think theſe kind of additions rather for ornament then uſe. Firſt, becauſe they are many of them in their own nature difficult to deſcribe, being ſections of a Cone, and muſt therefore be drawn from many points which hath ſome difficulty in the performance, except where they fall out to be Circles, which caſe will only happen where the plain paſſing by the vertex of the Cone makes right angles with the Axis, there the common ſection is a Circle. If the plain touch the Cone, it will be a Parabola. If it cut it, an Hyperbole. Laſtly, If it neither makes right angles with the Axis, and neither cuts, nor touches the Cone, it will be an Ellipſis, or ſtreight lines, as the Azimuths in a flat roof.

Secondly, becauſe when they are drawn, every Aſtrolabe will reſolve the problems more truly then they will.

I might adde a third reaſon, because the multitude of lines often hinders thoſe that are not uſed to them, to tell the houre of the Day, which is the chiefe uſe of Sun Dials, eſpe-

especially in those of this kind where the shadow of one point of the Axis gives the hour.

Yet, lest any should think this Instrument imperfect, I shall shew the Description of some of them, and leave the rest to the Industry of every Man.

CHAP. V.

The Parallels of Declination.

THese are of as great use as any, because the two Tropicks being the parallels of the greatest Northern, or Southern Declination may serve to limit or bound the Dial, and for them I need adde no new Precept, having before in the third Chapter taught you the description of the Tropick of ♋. The Tropick of ♑ is described in the same manner by help of your Table, placing your Instrument to the Azimuth belonging to every hour, and marked in your horizontal Circle, and reflecting the socket being before placed to the due altitude. If you desire the intermediate parallels, either you must take the pains to Calculate Tables, or by any Astrolabe, you may perform it exactly enough for this purpose.

CHAP. VI.

THe parallels of altitude are inserted after this manner, not much differing from the former. Suppose, I would insert the 20th. parallel of altitude. Move the slipping socket to 20 degrees in the Ruler, and the Radius being place d in any part of the horizontal boord, reflect with your eye, by the help of a threed, and a slippinng knot, the image of the socket, and carry it to the cieling, do thus till you have found as many points as you please, through which a line drawn, shall represent that Almicanter.

CHAP. VII.

The Proportion of the shadows to their Gnomons.

THese are no other then Circles of altitude to a determined propor tion,& may thus be set on. Consider first, what proportion you desire to expresse. As for example, I desire to know when the shadow is double to the Radius. I take in my Compasses the lenght of the lesser Radius of my Instrument, and upon the upright Ruler from 00 d.00 m. measure that length twice, you will find the Compasses to fall upon 63 deg. 30. m. to that degree and minute, set your moveable socket, then your Instrument being placed as before is taught. *viz.* That the fiducial edge of it, passe through the Center of the glasse, remove it upon the horizontal boord, from place to place, and reflect several points through which draw a line, At all times when the spot is in that line the shadow of all upright thing whatsoever, shall be double to their length; by which means you may find what heigth any Steeple or the like is, by measuring the shadow of it. In the same manner may all other Proportions be inserted.

CHAP. VIII.

To put in the Azimuths.

LOok what Azimuth you desire to expresse: as for example, I desire to put in the 10th. Azimuth from the Meridian. First, upon your horizontal Circle, mark that Azimuth, and next examine what altitude the Sun hath in that Azimuth, in any parallel you think fit, or which is most proper to be made use of, and to that altitude set the socket, and place your Radius in the said Azimuth, then reflect the image of the socket, and carry it to the cieling, it will meet with the parallel if you have wrought truly, there make a mark. Do this for the Horizon, where the Sun hath no altitude, and mark the reflected point, through those two, draw a streight line, if the Roof be flat, otherwise you must seek more points. After the like manner may the unequal hours,

the degree of the Sun that culminates, and ſuch like, be inſerted, which I leave to the induſtry of every Practiſer to perform. I ſhall now ſhew a ready way by this Inſtrument, to make Dials to a flat Glaſſe, theſe precepts hitherto being fitted to glaſſes that lye aſlope or oblique, whether convex, flat, or concave.

CHAP. IX.

How to draw the hour-lines to a Glaſſe that lies parallel to the Horizon.

DO as you are directed in the foregoing precepts, only inſtead of reflecting with your eye, you may now place the Radius of your Inſtrument, ſo that the upright Ruler may be within the Room, then applying it over in the Azimuth given for that hour, move the ſocket to the altitude of the Sun in that hour, and from the Center gently extend a threed, which ſhall ſhew you one point, do this for as many parallels as you deſire, if the Roof happen not to be flat, otherwiſe two are enough.

For example, in the latitude of 51 deg. 30 m. I draw a Meridian if I can, which is likewiſe an Azimuth, and find that in the Tropick of *Cancer*, the Sun will then be 62 deg. 00 m. high, to which I move the ſocket, and gently extend a threed by it to the Roof which ſhall give the point required. Do this for the Æquinoctial, and through the points found draw the hour-lines.

FINIS.

A SHORT TREATISE OF FORTIFICATIONS.

Written by *J. T.* M.D.

CHAP. I.

A table for the easie and ready Delineation of all regular Fortifications, wherein the number of Bastions exceed not 15, and may farther be continued. Appliable likewise, to irregular Forts. Wherein the Curtine is assumed 360 such parts as the face is 240. That is, the Curtine to the face is as 3 to 2.

I Have made choice of those numbers for the denomination of the measures of the Curtine and face, rather then what I find usually done to determine them by yards, or feet; for this reason among others. Because Authors generally agree not what number of feet should be allowed to the Curtine, some assigning more, some lesse. Yet most giving the proportion of the Curtine to the face as 3, to 2. Now that proportion being observed in these Tables, it matters not what their measure is in yards, or feet, but let that be more or lesse, it may still be called 360, and the whole Fort described by a line divided into 100 parts, which may serve as a Scale for that purpose, in which all fractions of yards or feet are avoided.

Polygons

Polygons	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.
Semidiameter.	385	484	587	694	802	913	1024	1130	1249	1357	1463	1573
Polygon interior	544	569	587	602	614	624	633	640	647	649	652	654
Neck or Gorge.	92	104	114	121	127	132	136	140	143	144	146	147
I Wing.	77	87	95	101	106	111	114	117	120	121	122	123
II Wing	71	112	129	138	143	147	149	150	151	159	166	169
Capital line.	197	207	216	223	230	236	241	245	249	253	256	260
	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.	d. m.
Angle of the Poligon.	90 00	108 00	120 00	128 34	135 00	140 00	144 00	147 16	150 00	152 18	154 17	156 00
Angle between the wing prolonged, and the face.	75 00	73 0	67 30	65 20	63 45	62 30	61 30	60 41	60 00	58 51	57 51	77 00
Angle between the Curtine and Capital line.	45 00	54 00	60 00	64 17	67 30	70 00	72 00	73 08	75 00	76 09	77 08	78 00
Angle made by the two Polygons	90 00	72 00	60 00	51 26	45 00	40 00	36 00	32 44	30 00	27 41	25 42	24 00

CHAP. II.

The use of the preceding Table in the delineation of a regular fort of five Bastions.

FIrst, Having prepared a Scale of equal parts A B. Take upon it 484, which I find in the cell of Semidiameters, and underneath v in the row of Polygons. With this length as Semidiameter, describe the Circle A F B E: then on the same Scale take 569 in the rank of Polygons interior, & underneath 5, which will reach on the Circle from B to B, & shall divide it into five equal parts according to the number of Bastions proposed. After set off from the point B, B I, equal to 104, which shall be the Gorge, or Neck, from the point I, erect a perpendicular I H, I H, to the line B B, make I H 87 parts in your Scale, which is in the cell belonging to I Wing. Lastly, make B G 207, which is your Capital line, and drawing H G complete your Bastion. After the same manner may you finish all the several Bastions of this, or any other Fort, working according to the measures belonging to that Polygon you designe to fortifie. *Fig.* 4. *Fig.* 1.

But because it may sometime fall out that you cannot readily find the Center of your Figure, by reason some house or other obstacle may intervene, so that you will be troubled to draw the line A G, upon which your Capital line is taken, nor perhaps know what angle it makes with the Polygon interior continued: though in truth, that angle is alwayes equal to half your angle of circumference. You may make

use of that cell in your Table that shews the angle made between the Wing continued, and the face (*viz.*) G H F in the figure, which in our example will be found 70 d. 30 m. this being set off on both sides, you may finish your Bastion without help of the Capital line. Which done, draw lines parallel to the face of your Bastion, delineate your ditch, which is to be made round about your Fort as in the second figure appears, which is the side of a Pentagon fortified.

CHAP. III.

HAving thus finished your Fort, you may farther strengthen it with Half moons or Ravelins, Horn-works, Redoubts, and the like, according as the place shall require, or the number of Souldiers you have for defence will permit.

A half Moon hath either relation to the Curtine, or Bastion, and in the second Scheme are marked ᴰ. That which hath relation to the Curtine, is properly called a *Raveline*, and that at the point of the Bastion a Half moon. A Raveline is drawn in this manner. Divide your Curtine into two parts by the perpendicular *n o*: from *p* the middle of your Wing draw *p o*, *p o*, and from *o* their point of Intersection set off *o q* equal to the Wing of your Baston H I, or thereabout, and is left in part to the discretion of the Engineer, as also at what distance they shall be made from the Curtine or Bastion: but discretion is to be had, not to make the angle of the Raveline too acute, but so to proportion the distance, that the angle may be neer, or equal to the angle of the Bastion.

Fig. 2.

The half Moon is alike with the Raveline, but that it hath relation to the Bastion, and is drawn after the same manner, all which is plain in the second figure the half Moons being marked ᴰ.

Touching your Horn-work, it is most conveniently defended from the face of your Bastion, which if you intend to do, then make H F equal, or neer equal to G L your line of defence, and draw F F which shall be equal to I I, your Curtine, & let *m* L the Curtine of your Horn-work, be about a third part of I I, but somewhat more (*viz.*) 130 such parts as your

Curtine I I, is 360, or more if you please, to 150, and from *n* the middle of F F set off *n o*, *n o*, equal to 65 of your Scale, & draw *o m*, *o* L, right angled at (*o*) then set off the Angles F *k* S, each of them 65 degrees as you would have the angle of your Horn-work contein: draw the lines F S, F S. Bisect the angles S F F which shall cut off F *k* equal to *m* L the Curtine of your Horn-work. Of which *k m*, *k* L shall be the Wings.

CHAP. IV.

Of Irregular Forifications.

TOuching irregular Fortifications, I can give no new Precept; only in general it is to be known, that it is best, if possible, to reduce them to regular forms. But if the place will not permit it. Take the plot of your place, and observe what angles are made between the several sides thereof. Then look in your Table in the cell of the Angles of Polygons, and see to what Polygon the observed Angle comes neerest, and make that Bastion according to the measures belonging to that Polygon, and so of the rest: so shall you have unequal Bastions, but alike defensible. Wherein likewise it is to be noted, that if the two sides of your interior Polygon be of unequal length, the Bastion is best framed according to the measures taken upon the shorter side.

As for Example, Let it be required to draw Bastions to the sides of an inordinate Hexagon, whose angles are as they are marked in the third Figure. First, I observe 134 the angle comprehended between two of the sides comes neerest to the angle of an Octagon in my cell of the angles of Polygons. *Fig.* 3. I conclude, that Bastion is to be drawn according to the measures belonging to an Octagon, and taking my measures from the shortest side A B. Because therefore the measure of the Polygon interior belonging to an Octagon is 614, I open my Sector in the line of equal parts, in the term of 614, or els frame a Scale, of which 614 parts shall be equal to *A B*, but a Sector is much more convenient, because it is a ready Scale to all lengths. In the next place, I set off the Neck, or Gorge A E, *B* E, 127 such parts as A *B* is 614, then I likewise measure 106 for the Wing. Lastly, because the Center of this figure is not known by which the measure of the Capital line

line ſhould be taken, I take the angle between the Wing prolonged, and the face, and take off 240 ſuch parts as my Curtine is 360, and finiſh that *B*aſtion belonging to the angle 134. Then I go to the next whoſe angle is 113, neereſt to a Hexagon, therefore that *B*aſtion muſt be framed from the ſhorter ſide according to the meaſures of a Hexagon. The acute angle 73 comes neereſt to a Tetragon, and therefore muſt be framed according to thoſe meaſures. *B*ut in all irregular Fortifications much is attributed to the judgement of the Engineer either to increaſe or diminiſh the angles, as he finds moſt convenient, but in ſuch manner, that the lines of defence may ſcoure the face of the *B*aſtion, and that one part thereof may defend, and be defended by the other.

Here much might be inſerted to this purpoſe. I ſhall only adde one Probleme, as a taſte of the reſt, which may be of good uſe.

CHAP. V.

How to make unequal Baſtions upon an irregular figure, yet in due Proportion to the meaſures of your Table.

LEt the given irregular figure be the ſides of an inordinate Hexagon. The ſeveral ſides whereof let be as in the figure following A *B* 100, *B* C 70, AD 78, &c. call them Perches, Yards, Feet, or what you pleaſe; and the parts are required in proportion to the ſides of a regular ſix ſided figure. The parts to be found out, are the Gorge, or Neck, the firſt and ſecond VVing, which in the Table under the 4th. column you find, as followeth. Firſt, the Polygon interior in a Hexagon is 587. The Gorge 114. The firſt VVing 95, the ſecond VVing 129. Say then by the Golden Rule. As 587 the tabular ſide, is to 100 the ſide given; ſo 114 the tabular Gorge, is to 19$\frac{2}{5}$ Gorge ſought, which ſet on both ſides of the line from A to E, and from *B* to F. Then after the ſame manner ſeek the two wings, and ſet them reſpectively off. Thus do for all the ſides, and from point to point draw the lines, as in the following Scheme.

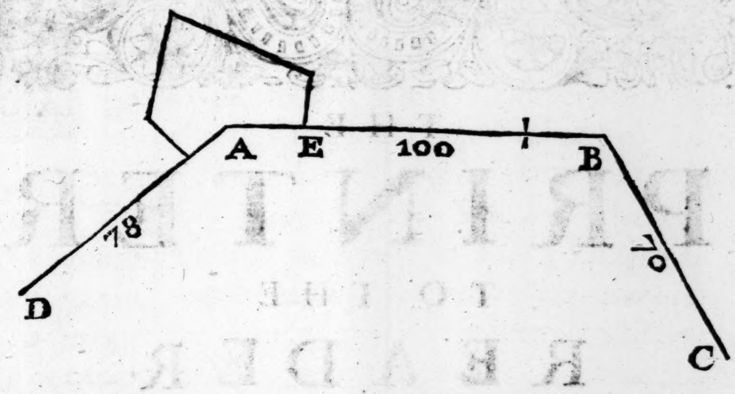
The operation for the ſide A *B*.

587 . 100 :: 114 . 19.$\frac{247}{587}$ The Neck.
587 . 100 :: 95 . 16.$\frac{108}{587}$ The Firſt Wing.
587 . 100 :: 129 . 21.$\frac{573}{587}$ The ſecond VVing.

For

For the side AD.

587 . 78 :: 114 . 15 . $\frac{87}{587}$. The neck.
587 . 78 :: 95 . 12 . $\frac{366}{587}$. The first Wing.
587 . 78 :: 129 . 17 . $\frac{83}{587}$. The second Wing.



CHAP. VI.

IT now remains to treat of the manner of making ditches about Forts, of the quantity of earth required to make a Fort with wals of any determined breadth, and heigth, as also what inclinations they ought to have both within side and without. Within of that slopenesse that the Souldiers may without much difficulty go up and down. But without sloping so little that they may not be scaled by the enemy, yet so much that the Foundation may be strong. But this I shall omit for the present: my designe not being to write all necessary to be known in that Art, but only so much as might enable any man of ingenuity to go on upon his own strength, and was at first written by me being then beyond Sea for the use of a young Gentleman who took delight in this study, and desire to have some insight in it. In all which I suppose the Reader not ignorant of the necessary precognita, as also the ordinary Geometrical Problems, to wit, to let fall a perpendicular, set off an Angle, and the like.

FINIS.

THE

PRINTER

TO THE

READER.

Courteous Reader.

THese pieces following came unto my hands fitted for the Preſs in the abſence of Dr. TWYSDEN from this place. I have notwithſtanding adventured to print them in that manner that they may paſs as an Appendix to theſe things of Mr. FOSTER, and be bound with them. Preſuming that what I have done will not be unpleaſing to any which is intended for the general good of all, by

W. LEYBOURN.

The Extract of a Letter written by Master IM. HALTON, from *Grayes-Inn*, in *May* 1650.

SIR,

“ BUt that my occasions doe and will detaine me
“ yet for some time in Towne, I should not have
“ given you this trouble of a Letter; for I purposed
“ in my first Vacation from bussines to have seen
“ you; yet because in our last discourse, there was something
“ started of *Reflexed Dialling*, the Theorie whereof I told
“ you, I thought I could manifest in 2 or 3 Diagrams, and
“ we not having opportunity *propter locum ambulandi da-*
“ *tum*, to designe the same, whereof you seemed a little earnest,
“ is the occacion of this, and the rather for that I am not
“ certaiu of seeing you.

First therefore, you are to take notice of this general Synopsis of Dials, or Plaines whereon Dials may be described.

All plain Superficies's whereon Dials may be described, are

- *Horizontal* — *Horizontal* 1
- *Perpendicular*
  - *Direct*
    - *Meridian* or *Polar.*
      - *East* 2
      - *West* 3
    - *Prime Vertical.*
      - *South* 4
      - *North* 5
  - *Declining*
    - *South*
      - *East* 6
      - *West* 7
    - *North*
      - *East* 8
      - *West* 9
- *Re- / In-clining*
  - *Direct*
    - *Meridian*
      - *East* 10
      - *West* 11
    - *Prime Vertical*
      - *Æqtinoct.* 12
      - *Polar* 13
      - *Neutral* 14
  - *Declining*
    - *South*
      - *East*
        - *Polar* 15
        - *Non Polar* 16
      - *West*
        - *Polar* 17
        - *Non Polar* 18
    - *North*
      - *East*
        - *Polar* 19
        - *Non Polar* 20
      - *West*
        - *Polar* 21
        - *Non Polar* 22

So that the names the Dial Plaines are in number 22. The first, admits of no variety; in others the same direction or calculation will serve for two of one kind, in some for four.

Now for the particular description of each, so many have made

made it their businesse and ingenuity, that here shall be no more said then what is already evulgated, which is more then sufficient, although I could *Symbolum offerre*, and that as currant as some of the rest; But because the occasion is Reflexed-Dialling, and that from a Plaine and Reflecting superficies howsoever posited, know that this superficies must necessarily be some one of the 22 varieties abovesaid, this is proved *inductivè*, by a perfect enumeration of all the singulars; for all plains must singularly be in some one of these 22 Positions.

2 Again, the plain of the Glasse (considered as an ordinary plain for a Dial) must be taken as a plain in a Counter-position to that ordinary plain, as for instance in the horizontal, an horizontal Glasse reflects an horizontal Dial, as should ordinarily be made to the Antipodes of the same place reflected; and so the like in rest of the plaines, where you must be still sure to apprehend the plain of the Glasse to make an Antipodical Dial to the same plaine taken with a reflection.

3 As all Dials in their delineations or traceing of their hour-lines, respect their proper Axis, Horizons, and Æquators, so likewise doe these the same in their reflexed posture; and there how you are to proceed to argue and state these, so as you may take your practice in them, as in the ordinary plaines, I shall be so free with you as to give you my conceptions, and therefore,

4 Because the Poles of the Æquator and Horizon are so called in the common Nomenclature, as they are perpendiculars, so for that reason shall I call the perpendicular of the Glasse, the Pole of the Glasse, concerning which Pole, take these 4 Theorems.

That the Azimuth	*upon which the Pole of the Glass is found cōpared with the*	1 *Meridian, is the Declination* 2 *Horizon, is the* ${}^{Re}_{In}$*-clination*	*Of that Glass.*
That the Hour-line.		3 *Meridian, is the Inclination of meridians, or difference of longitude.* 4 *Æquator, is the latitude or altitude of the Stile, viz. if it lye on the under or South-side of the Æquator South latitude, otherwise North latitude.*	

5 How

The names of all the severall lines contained in the table pag: 2.

A B. the Semidiameter
B B the Polygon interior
B I the neck or gorge
I H the . 1 . wing
I L the . 2 . wing
G H F The angle made between the wing and the face
I B I The angle of the Polygon
H G H the angle of the Bastion
B A B the angle at the center equall to the angle M B I made by the two Polygons
G B M The angle between the capitall line and the Polygon .

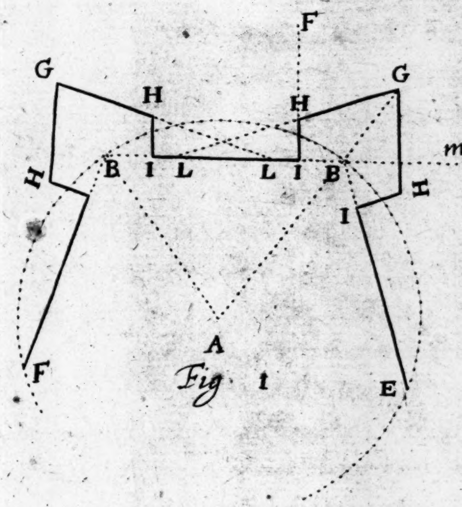


Fig 1

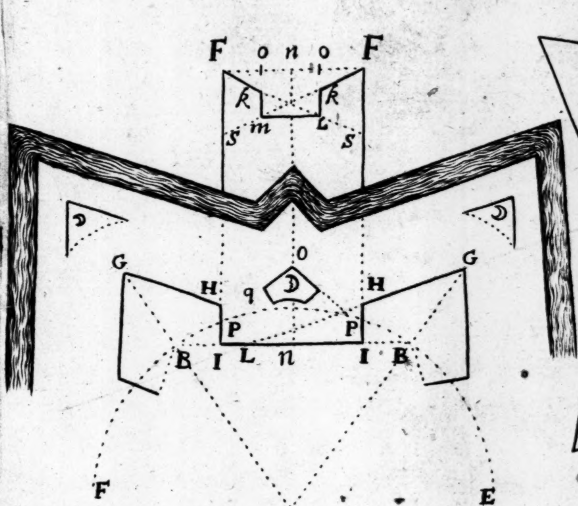


Fig 2

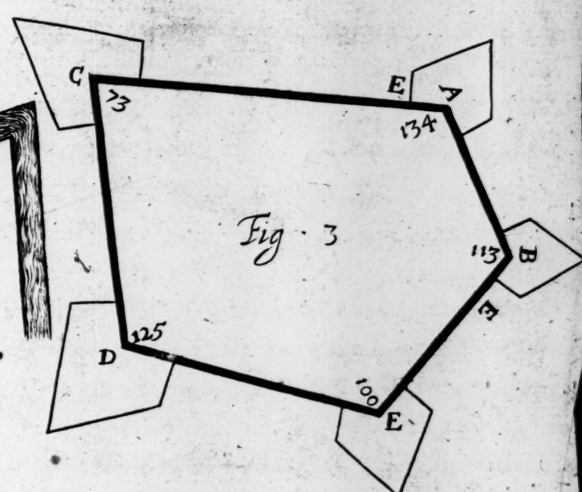


Fig · 3

The Scale

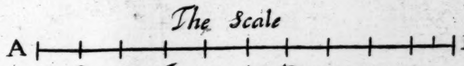


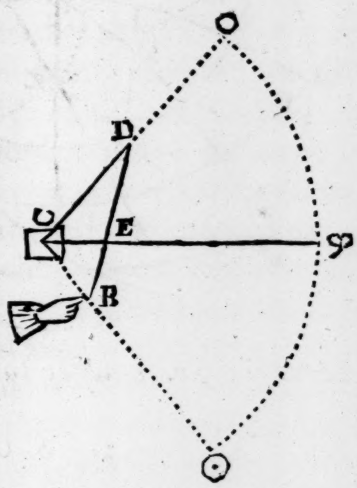
Fig 4

Pag 5 of Fortifications

5 How to find this Pole of the Glaſſe, the Glaſſeit ſelf being ſo ſmall, and ſet within a ſocket, as no Inſtrument can be applyed to the plain of it, there are two wayes.

1 Geometrically; For ſuppoſe the Sun ſhining on the glaſs at C. the ſpot or reflection ☉ at D. A Ruler, finger, or ſuch like thing, held up at B, ſo as the ſides of a Triangle B C D may be meaſured. Then the ſide D B cut in E by the 6 lib. *Euclide* prop. 3, ſhall give C E the Pole of the Glaſs. Then an horizontal paſtboard applyed to C (the Meridian being firſt found thereon) will by a perpendicular threed hung up, give you the Pole of the Glaſſe, and a Quadrant applyed to that Pole his altitude or depreſſion in reſpect of the Horizon.

2 By Trigonometry, or Calculation; For the Sun ſhining upon your glaſs take his altitude, and at the ſame time mark with a pencil the ſpot or reflection; for by this (without the Sun ſhining any more) there is enough to draw the whole Dial with all the lines of the Globe; for ſuppoſing C in the Center of the Earth (as all Nodus's of Styles are ſuppoſed) and C ☉ the ſpot or reflection to paſſe into the Heavens into a certain part as C ☉, which comes from thence. It is not to be doubted that the

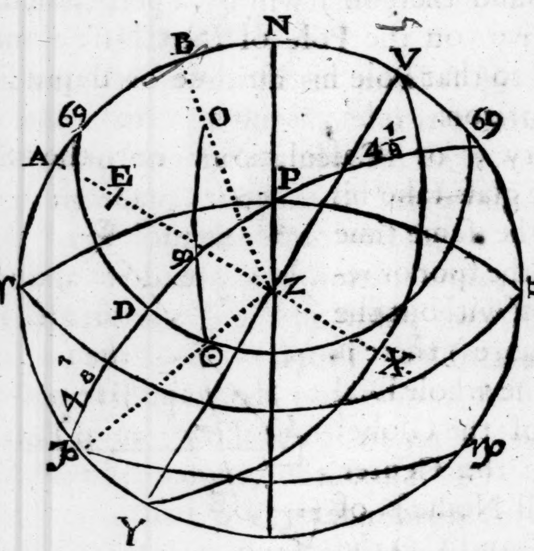


Arch of a great Circle O ☉ would be made thereby, the $\frac{1}{2}$ whereof would be C ♈ the pole of the glaſs, for the angle of incidence and reflection are equal, both in reſpect of the perpendicular, as alſo the plain of the glaſſe; And ſo from the Azimuth and altitude of the Sun taken, as alſo of the ſpot, the Azimuth, Altitude, Hour-lines, and Diſtance from the Æquator of the Pole of the glaſſe (the four things which are before directed) are eaſily found.

6 For the ſpeedy finding of the reflected Axis both of the Æquator and Horizon, (for without theſe the reflected hours and Azimuths are not to be drawn) as alſo the reflected Horizon, Æquator and Tropicks, there are two wayes.

1 By Instrument, 2 By Calculation. The first way directs the second, and so of the second I shall say but little, that my Letter may not swell; And for the first, for an Example, I will propound the Horizontal instrument of the place, where these Dials are intended be drawn, suppose at *London*, where the latitude is 51 d. 32 m. So then let this be the Question, which is propounded in Centesim's of degrees.

PROBLEM.



In the latitude N. 51 *d.* 53 *c.* $\frac{1}{3}$ *the Sun being in the first scruple of* Cancer, *having post meridional Azimuth from the South* 50 *deg.* 85 *c. and Altitude* 53 *deg.* 75 *c. casts from a Glasse a reflection* O *of post meridional Azimuth frõ the North* 21 *deg.* 74 *c. and of altitude* 26 *deg.* 69 *c. I desire to know the Plain?*

1 SOLUTION.

IN the Horizontal Diagram of the latitude proposed, let the Sun ☉, the spot O, the Zenith Z, the Pole of the glass φ, and the Azimuths and altitudes be laid down according to the Data's in the proposition, and the manner of the Diagram ☉ O, the $\frac{1}{2}$ whereof ☉ φ, and so Z φ.

And again. *P* φ and *P* Z φ equal to the declination of the glass, *viz.* N A.

Also twice Z φ = Z E the reflected Zenith of the glass, and E X = 90 deg.

And V X Y the proper Horizon or plain of the glass.

Lastly, twice *P* φ gives you *P* D the reflected Axis, &c. for the reflected Zenith and the reflected Axis of the glass, and those whereby the Hours and Azimuths are to be drawn, which

which together with the Æquator and Horizon (becauſe they only are great Circles and biſect the Globe) will be ſtraight lines in plano.

But becauſe, as perhaps, through haſte(and the ſhort contraction of this, which I had rather have diſcourſed then thus made up into a Letter) any error may have happened in the deſigning of the Triangles upon the Horizontal Diagram, take this ſecond Solution by the Globe.

2 *SOLUTION.*

THe *Pole* of the glaſs being as by the fifth found, by his Azimuth and altitude aſſigne a point upon the Globe, by ſome piece of white paper or other thing clapt thereon, from that point which your Quadrant of Altitude uſually made therewith, or rather Semicircle of ſteel, braſſe or Whale-bone, application being made to the Pole of the World, Zenith, two points in the Æquator, and Horizon (becauſe great Circles) Tropicks, &c. the oppoſite equal Arches thereof, ſhall give you the reſpective reflexed points; having alwayes a regard from this point or Pole of the glaſs aſſigned that you make the Angles of reflection equal to the Angles of incidence. From hence now ſome neate Concluſions may be deduced, ſuch as theſe:

1 First, That the Sun being at the point D (that is having North declination 19 d. 43 c. $\frac{1}{3}$; poſtmeridional Azimuth 80 d. 31 c. $\frac{2}{3}$; altitude 32 d. 53 c. $\frac{1}{3}$; and Hours 4 $\frac{1}{8}$) ſhall give a reflection to *P*, that is parallel to the Axis of the World, and ſo by conſequence the Sun in his own poſition to the glaſſe (if by obſervation you watch that moment) ſhall ſhew you the reflected Axis of the glaſs.

And ſo at *London* this reflected Axis is found, when the Sun is in the Meridian having North declination 13 d. 06 c. $\frac{2}{3}$, the plain of the glaſſe lying horizontal.

2 By this, the ſuperficies of the glaſſe, or the plain of the glaſſe, appeares to be one of the 22 variety of plains, and that it declines 60 deg. and reclines 54 gr.

3 That by the plain of the glaſſe repreſented in the horizontal Diagram by V X Y, you have the hour-lines expreſſible upon that plain, or which can be reflected by that

that glaſſe, and alſo the time of the year when the Sun will firſt ſhine thereon, and the continance.

4 That without any glaſſe, you may from a point taken, aſſigne a reflected Axis where you pleaſe, by an Azimuth and altitude taken to your own fancy; as ſuppoſe at D, then will φ the pole of your glaſſe be found as before, and you muſt be careful in bringing the Center of your glaſſe into this point, and ſo place it, which is alſo very feazible ſeveral wayes.

7 For the practice, or making of theſe Dials with all the Furniture thereof (the Theorie being thus laid down, I ſuppoſe you are well enough acquainted therewith; I ſhould propound for my own practice any one of theſe.

1 Having got the reflected Axis which will alwayes paſſe through the Center of the glaſs both into the Air and into the Room [if the tranſome of your window lye not directly in the Meridian] and having erected a paſtboard, or ſuch like thing at Right-angles thereto, parallel to the reflected Æquator, you may by threads deſigne the hours, as is now a very familiar practice in making of String-Dials, which ſerve both for the hour of the day by the Sun, and the houre of the night by the Stars.

2 By a plain ſet parallel to the ſuperficies of the glaſſe, at a convenient diſtance, whereon you are to deſigne what you intend to be put on this Dial, and if the parallel plain be of paſt-board or paper, a thread faſtned at the Center of the glaſſe ſtrikes your whole Dial *de morsâ papyro*. The diſtance of the plain from the glaſſe, will be as you pleaſe, *viz.* the diſtance of a plain from a Nodus.

3 The pole or perpendicular of the glaſſe being drawn out and deſigned, you can eaſily propound to your ſelfe, what, and in what poſition the Suns rayes will make an Angle of incidence with that perpendicular, and ſo by a Semicircle or Tangent of fine paſtboard faſtned to that perpendicular, you can, on the other ſide, aſſigne the like Angle for reflexion. And for the Horizon, which is to be reflected, two points may many times eaſily be got by the eye, looking into the glaſſe, and ſo between the eye and the glaſſe interpoſing a marke as p. 5. §. two points are ſufficient to deſigne that.

And

And thus by one or other of these wayes you shall be sure to hit of your purpose.

And to conclude, I shall tell you of an Instrument or Dial for my own use, which by one single hour-line designed within a Room (and that at pleasure, which will prevent the soiling of Hangings, Cupboards or such things in a Room) shall most readily give you the hour, and actually (if your Room be large) every day in the year. The Instrument may be of neat use in Gardens, being set neer the North side of a Wall or Tower, yet so that the Sun may shine thereon, and the reflection be made in the shadow. Of this Instrument I have given Mr. *Anthony Thompson* directions for the making it. It is very plain and ready, and the hours upon the Æquinoctial naturally divide themselves into 7½ deg. a piece, and the reason thereof (that is, the demonstration) is very apparent.

"But I cease to give you further trouble at this time, desiring rather your pardon for this confidence I take, in "adding to your *Mathematical* store, wherein, and in the "right use of your other fortunes, you are *Crœso ditior*, And "hoping your occasions will, &c.

An Extract of a Letter of a later date, written by the said Mr. Im. Halton to his said Friend, in which he intimates the Construction of an Instrument for taking of Altitudes.

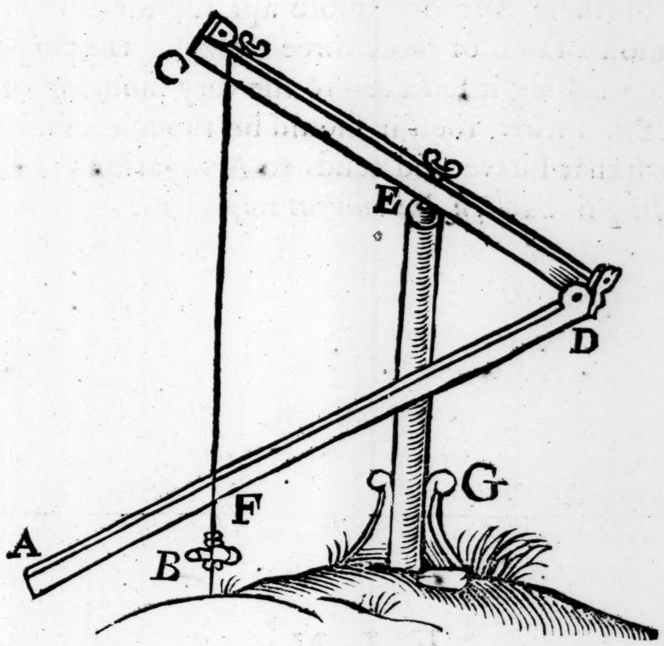
AND whereas having (by reason of some businesses) not had conveniency for the using of that great brasse Quadrant, neer four foot Radius, which in 1652 I had made for me, and which indeed with the great Ball, and socket, and Appendixes was importable, for those reasons I took occasion to part with it; yet now I fancy to my selfe I may have so much of a Vacation, as by turns to observe the Suns Somer Solstice, and Æquinoxes, and being destitute (as I have told you) of my great Quadrant, I went to thinking again, and have designed an Instrument, with sights and plummet, as the Type annexed will inform you, more portable, cheaper, larger, and possibly may be made the exactest of any yet produced; In briefe, you have two Rulers C D, A D. And C B is such a strong wyre of Brasse, or Copper, as a great Ball or Plummet as heavy as the wyre will well sustain, and the wyre marked at P, so as that C P may alwayes be equal to C D, which is exactly known by application. Then there is a second Ruler A D moveable upon a Pin at D, divided into equal parts, so as C D be one, and being at the time of observation carefully applyed to the mark at P, in the wyre C B you have the complement of the Altitude in parts: for the Triangle *P* C D is an Isosceles, and the Base is *P* D.

The Ruler A D may be so ordered on the pin at D, that either with the broad side, or (which is better) the edge, you may by application to *P* take an exact division.

And if your wyre C *P* (which is but the perpendicular leg of your Isosceles) should after some time shrinke or extend, beyond his just length, have a little skrew neere C, to order it to his due length.

If you intend to have A D of that length, as to serve you the whole Qradrant, his length will be 1.4142. C D being

being 1.0000. Then C D being ten feet (which with glasse sights, and Ball, and Socket at E, moved upon a stable foot fixed firme as G,) you have an Instrument both the largest and exactest: for the whole Beam C D being ten foot (and one foot is capable of 100, yea by Diagonals of 1000 divisions,) so shall C. D be of 10000, that is 1.0000.



Now a Quadrant containes but 5400 minutes, or 9000 centesimes of a degree. Every actual division without Diagonals gives you 5.4 minutes or 9 centesimes of a degree.

Now then D A being divided into equal parts as above said, beginning at D, and suppose the point *P* cut upon the Ruler A *D*.4838 the practice would be thus

4838	*Chorda à Vertice.*
2419	*Medietas chordæ.*
gr. m.	
14 00	*Arcus medietatis.*
28 00	*Arcus duplicatus.*
62 00	*Complementum & altitudo.*

From hence it is to be understood that D A may be graduated actually into degrees and parts from a Table of

Natural

Natural Sines, and the Complements of all Altitudes be designed.

Lastly, upon the whole matter you may judge by this of the Instruments called *Regula Ptolomëi Parallactica*, described by *Ptolemy* himself, as also by *Copernicus Revolutionum* lib. 4. cap. 15. and *Tycho* in his *Mechanicks*, where he shews two sorts: But this I conceive (not without a great esteeme of these Authors) more apt for use, being with the omission of one of those three Rulers, the perpendicular wyre correcting it self even to the very moment of observation. But rather then it should be thought of any, that thus much that I have said tends to Arrogation; Let it be called *The parallactick Instrument improved*.

FINIS.

Æquations arising from a Quantity divided into two unequal parts: And the Second Book of *Euclides Elements*, Demonstrated by species

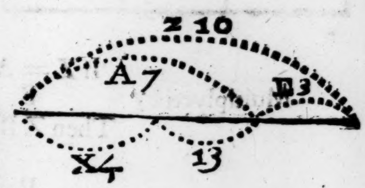
By JOHN LEEKE.

If Z 10 be divided into A 7 and E 3,

1 Then $\begin{matrix} Z = A + E, \\ 10 = 7 + 3 \end{matrix}$ Because the whole is equal to all its parts.

2 And $\begin{matrix} Z - E = A \\ 10 - 3 = 7 \end{matrix}$ by transposition.

3 And $\begin{matrix} Z - A = E \\ 10 - 7 = 3 \end{matrix}$ by transposition.



Because A is supposed the greater part, from it take I = E the lesser part, and let the remainder, which is the difference of the parts, be noted with X.

And I + X = A, because all the parts are equal to the whole,
But E = I by supposition.

4 Therefore $\begin{matrix} E + X = A \\ 3 + 4 = 7 \end{matrix}$ by interpretation.

5 And $\begin{matrix} E = A - X \\ 3 = 7 - 4 \end{matrix}$ by transposition.

6 And $\begin{matrix} X = A - E \\ 4 = 7 - 3 \end{matrix}$ by transposition.

But A — X = E by the fifth æquation.
And Z — A = E by the third æquation.

7 Therefore $\begin{matrix} A - X = Z - A \\ 7 - 4 = 10 - 7 \end{matrix}$ by interpretation.

8 And $\begin{matrix} 2A - X = Z \\ 14 - 4 = 10 \end{matrix}$ by transposition.

9 And $\begin{matrix} 2A = Z + X \\ 14 = 10 + 4 \end{matrix}$ by transposition.

10 And $\begin{matrix} A = \frac{1}{2}Z + \frac{1}{2}X \\ 7 = 5 + 2 \end{matrix}$ by division.

11 And $\begin{matrix} X = 2A - Z \\ 4 = 14 - 10 \end{matrix}$ by transposition.

But E + X = A by the fourth Æquation.
And Z — E = A by the second Æquation.

12 Therefore $\begin{matrix} E + X = Z - E \\ 3 + 4 = 10 - 3 \end{matrix}$ by interpretation.

13 And $\begin{matrix} 2E + X = Z \\ 6 + 4 = 10 \end{matrix}$ by transposition.

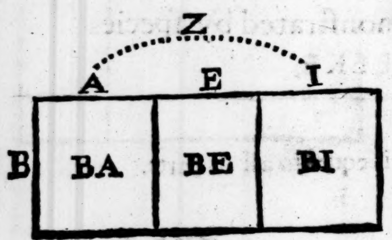
14 And $\begin{array}{l} 2E = Z - X \\ 6 = 10 - 4 \end{array}$ by transposition

15 And $\begin{array}{l} E = \frac{1}{2}Z - \frac{1}{2}X \\ 3 = 5 - 2 \end{array}$ by division.

16 And $\begin{array}{l} X = Z - 2E \\ 4 = 10 - 6 \end{array}$ by transposition of the 13 equation.

PROPOSITION. I.

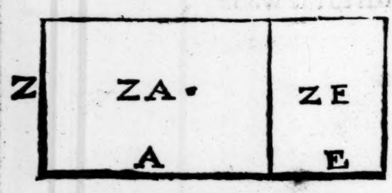
If there be two right lines, and the one of them be divided into any parts. The rectangle comprehended by the two right line is equal to the rectangles comprehended by the undivided line, and each segment of the divided line.



If $Z = A + E + I$ by supposition
Multiplyed by B
Then $ZB = BA + BE + BI$ by multiplication.

PROP. 2.

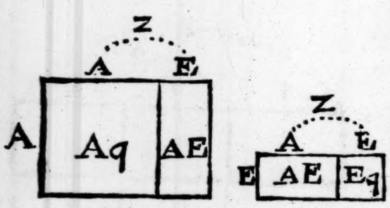
If a right line be divided at pleasure, the rectangles comprehended by the whole, and each segment, is equal to the square made to the whole.



If $Z = A + E$ by supposition.
Z
Then $Zq = ZA + ZE$ by multiplication.

PROP. 3.

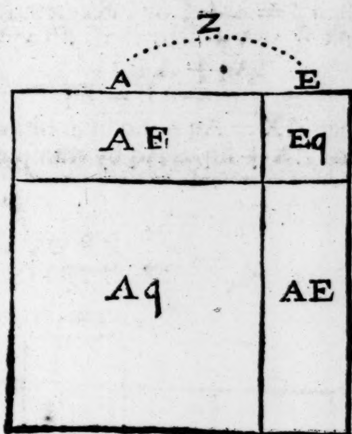
If a right line be divided at pleasure, the rectangle comprehended by the whole and one segment, is equal to the rectangle of the segments, and the square of the said segment.



If $Z = A + E$ by supposition.
A
Then $ZA = Aq + AE$
And $ZE = AE + Eq$ by multiplication.

PROP. 4.

If a right line be divided at pleasure, the square which is described on the whole line, is equal to the squares of the segments, and to twice the rectangle of the segments.

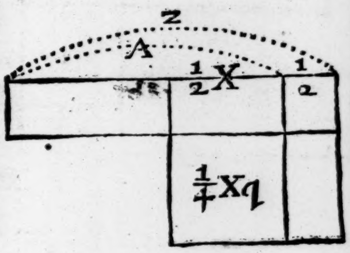


$$\begin{array}{l} \text{If } Z = A + E \text{ by supposition} \\ \phantom{\text{If }} Z = A + E \\ \hline \phantom{\text{If }} Aq + AE \\ \phantom{\text{If } Aq + } AE + Eq \\ \hline \end{array}$$

Then $Zq = Aq + 2AE + Eq$. by multiplication.

PROP. 5.

If a right line be divided into equal and unequal parts, The rectangle comprehended under the unequal segments of the whole, with the square of the intermediate part, is equal to the square of the half line.



For,

If $Z = A + E$ by supposition.

Then $A = \frac{Z + X}{2}$ by the tenth equation

And $E = \frac{Z - X}{2}$ by the 15 equation

$$\begin{array}{l} \hline Zq + ZX \\ ZX - Xq \end{array}$$

Then $AE = \frac{Zq - Xq}{4}$ by multiplication.

And $AE \frac{+Xq}{4} = \frac{Zq}{4}$ by transposition

PROP. 6

If a right line be divided into two equal parts, and unto it another right line be added, The rectangle comprehended by the whole, with the part added, and the part added; together with the square of the half; is equal to the square which is made of the half and part added, as of one line.

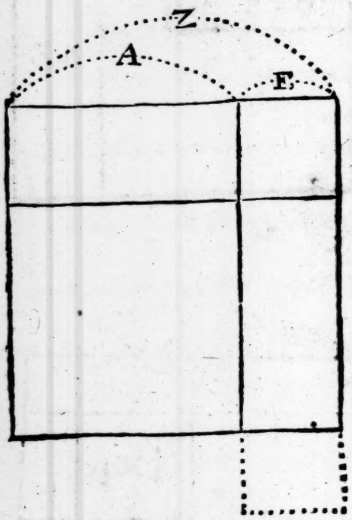
If

If $2E + X = Z$ by supposition.
And $E + X = A$ by the fourth equation
Then $Z = A + E$ by interpretation.
And $X = A - E$ by the sixth equation.

$$\begin{array}{l} Aq + AE \\ \quad - AE - Eq. \end{array}$$

Then $ZX = Aq - Eq$ by multiplication.
And $ZX + Eq = Aq$ by transposition.

PROP. 7.



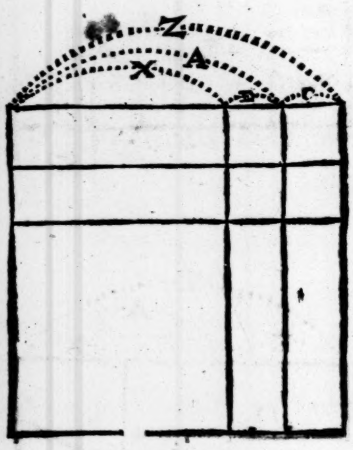
If a right line be divided at pleasure, The squares of the whole line, and of one of the segments together, are equal te twice the rectangle of the whole and the said segment, and square of the other segment.

If $Z = A + E$ by supposition,
Then $A = Z - E$ by transposition.

$$\begin{array}{l} A = Z - E \\ \hline Zq - ZE \\ \quad - ZE + Eq \end{array}$$

And $Aq = Zq - 2ZE + Eq$ by multiplication.
And $2ZE + Aq = Zq + Eq$ by transposition.

PROP. 8.



If a right line be divided at pleasure, The four rectangles comprehended by the whole, and one of the segments, with the square of the remaining segment, is equal to the square which is made of the whole line, and the segment as one line.

If $Z = A + E$ by supposition.
Then $2A = Z + X$ by he ninth equation.
And $2E = Z - X$ by the 14 equation.

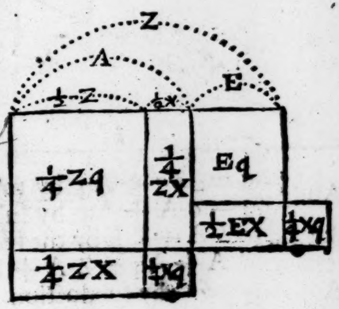
$$\begin{array}{l} Zq + ZX \\ \quad - ZX - Xq. \end{array}$$

$4AE = Zq - Xq$ by mutiplication.
And $4AE + Xq = Zq$ by transposition.

PRO-

PROP. 9.

If a right line be divided into two equal, and into two unequal parts. The squares which are made of the unequal segments of the whole line, are double to the squares of the half line and intermediate segment.



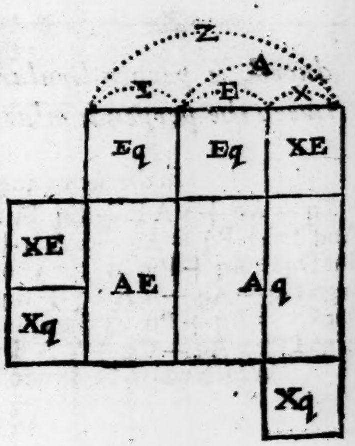
If $Z = A + E$ by supposition.
Then $\left.\begin{array}{l} X = A - E \\ X = A - E \end{array}\right\}$ by the 6 Equation.

$$\begin{array}{l} AA - AE \\ \quad - AE + EE \end{array}$$

Then $XX = AA - 2AE + EE$ by multiplication.
And $ZZ = AA + 2AE + EE$ by the 4 Prop. El. 2.
Therefore $ZZ + XX = 2AA + 2EE$ by addition.
And $\frac{ZZ + XX}{2} = AA + EE$ by division.

PROP. 10.

If a right line be divided into two equal parts, and unto it be added another right line, the squares which are made of the whole line and the part added, and of the part added, both together shall be double to the squares which are made of the half, and of the half and part added.

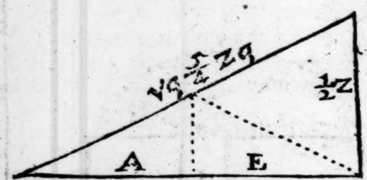


If $Z = A + E$ by supposition.
And $X = A - E$ by the sixth Æquation.

Then $Zq + Xq = 2\,Aq + 2\,Eq$ as before.

PROP. 11.

To divide a right line by extream and mean proportion, that is, so as the rectangle comprehended by the whole, and one of the parts may be equal to the square of the other part.

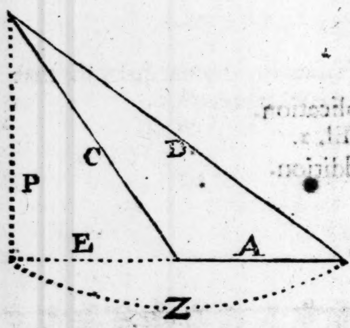


If $Z = A + E$
And $\sqrt{q}\,\frac{5}{4}Zq - \frac{1}{2}Z = A$ by construction
Then $ZE = Aq$.
For $\sqrt{q}\,\frac{5}{4}Zq = \frac{1}{2}Z + A$ by transposition
$\sqrt{q}\,\frac{5}{4}Zq = \frac{1}{2}Z + A$

$\frac{1}{4}Zq + \frac{1}{2}ZA$
$+ \frac{1}{2}ZA + Aq.$

And $\frac{5}{4}Zq = \frac{1}{4}Zq + Aq$ by multiplication
$Zq = ZA + Aq$ by substracting $\frac{1}{4}Zq$
But $Zq = ZA + ZE$ by the second Prop.
Therefore $ZA + Aq = ZA + ZE$ by interpretation.
And $Aq = ZE$ by subduction. Which was to be proved.

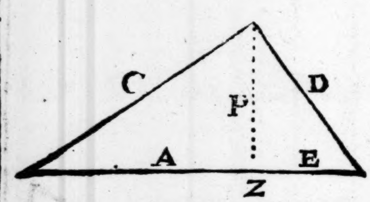
PROP. 12.



In obtuse angled Triangles, the square which is made of the side subtending the obtuse angle is greater then the squares which are made of the sides comprehending the obtuse angle, by twice rectangle comprehended by one of the sides which are about the obtuse angle (on which, being produced, a perpendicular falleth) and the part which is between the perpendicular and the obtuse angle.

In the obtuse angled triangle CDA.
$Zq = Aq + 2AE + Eq$ by the fourth Prop.
And $Zq + Pq = Aq + 2AE + Eq + Pq$ by addition
But $Dq = Zq + Pq$ by the 47 of the first.
Ergò $Dq = Aq + 2AEq + Eq + Pq$ by interpretation
But $Cq = Eq + Pq$ by the 47 of the first.
Ergò $Dq = Aq + Cq + 2AE$ by interpretation:
Which was to be demonstrated.

PROP. 13.



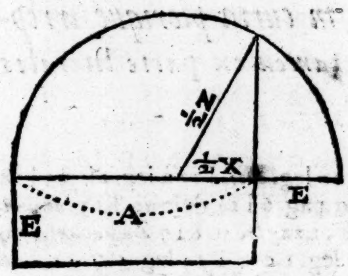
In accute angled Triangles the square which is made of the side subtending the accute angle, is lesse then the squares which are made of the sides comprehending the accute angle, by twice the rectangle comprehended by one of the sides which are about the accute angle (on which a perpendicular falleth) and the part which is between the perpendicular and the accute angle.

In

In the accute angled triangle CDZ.

$Zq + Eq = 2\,ZE + Aq$ by the ſeventh Prop.
And $Zq + Eq + Pq = 2\,ZE + Aq + Pq$ by addition,
But $Eq + Pq = Dq$ by the 47 of the firſt.
Ergò $Zq + Dq = 2\,ZE + Aq + Pq$ by interpretation,
But $Cq = Aq + Pq$ by the 47 of the firſt.
Ergò $Zq + Dq = Cq + 2\,ZE$ by interpretation.
Which was to be proved.

PROP. 14.



To make a ſquare equal to a right-lined figure given.

$Æ + \frac{1}{4}Xq = \frac{1}{4}Zq$ by the fifth Prop.
$Æ = \frac{1}{4}Zq - \frac{1}{4}Xq$ by tranſpoſition.
But $\frac{1}{4}Zq - \frac{1}{4}Xq = Pq$ by the 47 of the firſt,
Ergò $Æ = Pq$ by interpretation.
Which was to be proved.

FINIS.

Ex absentia nostra , & Correctoris in curia plerique irrepsere errores Typographici maximâ tamen ex parte literales quos equus sic corrigat.

Astroscopium: Pag. 8 lin. 19 lege Gyra.

Instrument. Planetar. Pag. 3 l 24 dele *non*. p 4 l 20 leg. *dici*, l 33 à leg. *ad*. p 5 l 18 leg. *excerpenda*, l 26 leg. *proxime* l 31 *completi*, l 34 *ut*. pag. 6 l 12 dele *ut*, l 13 *lamina*, lin. 13 leg. *semel ut*, l 14 *singulis*, l 32 *soluta*. pa. 7 l 31 *unum*, p. 8 l 7 *Zodiacum*, l 30 dele *ei*. pag. 9 l 32 *vero*. pag. 10 l 23 leg. 1.08, l 26 leg. 1.00, l 34 leg. 587.52, l 36 leg. 55.42. pag. 11 l 16 & leg. *à*, l ult. *proprium*. pag. 13 l 28 leg. *rite*. pag. 14 l 5 *versantur*. pag. 16 l 7 *longitudinem*, pag. 17 l 13 leg. *Venerem & Mercurium*, ex l 27 *opposita*, l 28 dele 5. pag. 18 l 4 leg. *Præcipuum*. pag. 20 l 11 *cursum*, l 12 *progressionis*, l 15 *dies*, l 22 *intraturus*, l 24 lege *Planeta à motu directo*. pag. 21 l 5. leg. *certum*. pag. 22 l 16 *sextili*, l 23 *sextum*. pag. 25 l 25 dele *magis*. pag. 29 l 11 *Consule*, li. 12 *tabulam*, pa. 30 l 3 *Anomalas*, l 6 leg. *tab.* C, l 10 leg. *quæ in prædicta tabula exhibentur*, & dele *dani*, l 29 leg. *tab.* C, l 36 leg. *in*. pa. 31 l 15 leg. *respondentes*, l 25 leg. *schematibus*. pa. 32 l 12 leg. *inscribuntur*. pa. 33 l. pen. leg. *inserendos*. pa. 35 l 10 dele *tres partes*, ex pa. 36 l 8 leg. *observatu*. pa. 39 l 4 lege *Cancrum*, l 34 lege *Cancer cum Capricorno*. pa. 40 l 8 lege *Cancri*. pa. 43 l 10 leg. *Capricor*. dele *Cancer*, l 16 leg. *Nodi*, l 26 dele *min*. pa. 44 l 7 leg. *distantia*.

Observat Eclipsium. Pag. 19 lin. ult. lege *Goesano*.

Probl. Geom. varia. Pag. 4 l 6 leg. 1296000 *tales anguli*. pa. 4 l 25 leg. 50289, l 33 leg. *differentia*. pa. 10 l 15 leg. *ita Sphara*. p. 10 l 30 leg. *trilineum*. pa. 12 l 29 leg. *quadruplus*. pa. 16 l 18 B x A leg. B in A.

Probl. quorundam. Pag. 3. l 14. leg. *duplam*, l 17 leg. *duplam*. p. 6. le. *octavum*. p. 8 l 12 leg. *æquatur*. pag. 9 l ult. leg. $\frac{2rrt}{rr - tt}$ pa. 10 l 19 leg B *m* dele q. pa 14 l. pen. *major* leg. *minor*. pa 15 li. 17 leg. *minor erit a* — 1 li. 18 lege 2*a* — 1.

In Appendix. Pag. 3. l 25 read C O, l 31 read *angles*. pa. 4 l 30 read *Diagram*. Quæritur ⊙ O, *the ½ whereof, &c.* pag. 4 l 37 read *are*. pag. 5 l 15 read *with*. pag. 9 in the Diagram for F read P.